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Catalogue of Minimal Cyclic Polygonal Designs in Two Different Block Sizes 4 and 2

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Abstract: The cyclic polygonal designs with unequal block sizes have many practical applications. In agricultural, environmental and ecological surveys, the presence of unequal sized blocks is useful, particularly in large experiments. In this article, a catalogue of polygonal designs in blocks of two different sizes 4 & 2 is presented with distance α = 1, 2, …, 8.

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INTRODUCTION

In a large part of literature on the design of experiments, the use of balanced sampling plans excluding contiguous units is useful when the contiguous units are similar. Studies in ecological and environmental sciences, often a balanced sampling plan is used for generating samples from the population by avoiding the neighboring (contiguous or adjacent) units which provide redundant information. Polygonal designs are a specialization of partially balanced incomplete block designs (PBIBD). A polygonal design with minimum distance α is denoted by PD (v, k, λ, α) [1] defined a polygonal design with minimum distance α as an incomplete block design in which v treatments are arranged in b blocks of size k if (i) every treatment {0, 1, …, v-1} appears in r blocks, (ii) every pair of treatments which differs more than the distance α appears in exactly λ blocks, whereas other pairs do not appear at all in any block and every block contains k distinct treatments. A polygonal design with parameter v, b, r, λ and α must satisfy the two conditions (i) bk = vr and (ii) $\lambda[v-(2\alpha+1)] = r(k-1)$. [2, 3] developed balanced sampling plans excluding contiguous units (BSEC). [4] constructed CPDs in the form of PBIBDs. Further work on PDs (in the form of BSECs, BSAs or PBIBDs) has been appeared in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. Generalized polygonal designs with block sizes 3 and λ = 1 by [21].

The cyclic polygonal designs with unequal block sizes are required in many practical problems. In agricultural, environmental and ecological surveys, the presence of unequal block sized is useful particularly in large experiments. [22] gave some examples of unequal block sizes which are useful in plant and tree breeding trials. The CPDs in blocks of unequal size can be used to investigate the abundance and diversity of species, when the area in the forests supposed as strata and from each stratum unequal sized sample is selected. The existence and construction of minimal CPDs for unequal block sizes has been attempted by [23] using method of cyclic shifts for λ = 1 and distance α = 1, 2, …, 6 with blocks of sizes $k \in [3, 2]$. In this article, a catalogue of CPDs in two different blocks sizes 4 & 2 is presented with α = 1, 2, …, 8.

MATERIALS AND METHODS

Method of Cyclic Shifts: Proposed designs are constructed by Method of cyclic shifts introduced by [24]. It is briefly explained here for CPD. However, for more detail see [21]. Let $S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$ be a set of shifts where $1 \le q_{ji} \le v-1$.

- If each element 2, …, v-2 appears an equal number of times in a new set of shifts but 1 and v-1 do not appear then this design is CPD with α = 1.

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- If each element 3, …, v-3 appears an equal number of times in a new set of shifts but 1, 2, v-1 and v-2 do not appear then this design is CPD with $\alpha = 2$.
- If each element α+1, …, v-(α+1) appears an equal number of times in a new set of shifts but 1, 2, …, α, v-1, v-2, …, v-α do not appear then this design is CPD with joint distance α.

Here new set of shifts consist of (i) each shift of S along with its complement, (ii) sum (mod v) of two, three, …, k-1 consecutive shifts along with their complements. Complement of q_i is v-q_i.

For a circular polygonal design with $\alpha = 1$, the concurrence matrix will be;

$$NN' = \begin{pmatrix} r & 0 & \lambda & \lambda & \cdots & \lambda & 0 \\ 0 & r & 0 & \lambda & \cdots & \lambda & \lambda \\ \lambda & 0 & r & 0 & \cdots & \lambda & \lambda \\ \lambda & \lambda & 0 & r & \cdots & \lambda & \lambda \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda & \lambda & \lambda & \lambda & \cdots & r & 0 \\ 0 & \lambda & \lambda & \lambda & \cdots & 0 & r \end{pmatrix}$$

For a circular polygonal design with joint distance $\alpha = 2$ the concurrence matrix is:

$$NN' = \begin{pmatrix} r & 0 & 0 & \lambda & \cdots & \lambda & 0 & 0 \\ 0 & r & 0 & 0 & \cdots & \lambda & \lambda & 0 \\ 0 & 0 & r & 0 & \cdots & \lambda & \lambda & \lambda \\ \lambda & 0 & 0 & r & \cdots & \lambda & \lambda & \lambda \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \lambda & \lambda & \lambda & \ldots & \cdots & r & 0 & 0 \\ 0 & \lambda & \lambda & \lambda & \cdots & 0 & r & 0 \\ 0 & 0 & \lambda & \lambda & \cdots & 0 & 0 & r \end{pmatrix}$$

Example 1: A circular polygonal design for $v = 11$, $k_1 = 3$, $k_2 = 2$, $\lambda = 1$ and $\alpha = 1$ is constructed through the sets of shifts [2, 3] + [4]. The required design is:

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}
0	1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	0	1
5	6	7	8	9	10	0	1	2	3	4
B_{12}	B_{13}	B_{14}	B_{15}	B_{16}	B_{17}	B_{18}	B_{19}	B_{20}	B_{21}	B_{22}
0	1	2	3	4	5	6	7	8	9	10
4	5	6	7	8	9	10	0	1	2	3

And the concurrence matrix is:

$$NN' = \begin{pmatrix} 5 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 5 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 5 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 5 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 5 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 5 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 5 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 5 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 5 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 5 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 5 \end{pmatrix}$$

From the above design we can see that the treatment 0 and 1, 1 and 2, 2 and 3,...,8 and 9, 9 and 10, 10 and 0 do not occur together within the blocks. Therefore, the above design is a PD with $\alpha = 1$.

Catalogue of Circular Polygonal Designs: In Table 1, 2, …, 8, some polygonal designs in blocks of sizes $k_1 = 4$ & $k_2 = 2$ are presented with $\alpha = 1, 2, \ldots, 8$ respectively which are given in Appendix A.

Table 1: Catalogue of CPDs in Blocks of Sizes $k_1 = 4$ & $k_2 = 2$ with $\alpha = 1$

v	k_1	k_2	Sets of shifts
31	4	2	[3, 4, 5]+[2, 6, 10]+[11]+[14]
32	4	2	[2, 4, 5]+[3, 7, 8]+[12]+[13]+[16]($\frac{1}{2}$)
33	4	2	[2, 4, 7]+[3, 5, 9]+[10]+[12]+[15]
34	4	2	[3, 5, 10]+[9, 4, 7]+[6]+[12]+[17]($\frac{1}{2}$)
35	4	2	[2, 4, 7]+[3, 5, 9]+[10]+[12]+[15]
36	4	2	[2, 4, 7]+[3, 5, 9]+[10]+[12]+[15]+[16]+[18]($\frac{1}{2}$)
37	4	2	[2, 4, 5]+[3, 7, 8]+[11]+[12]+[13]+[14]+[16]+[17]
38	4	2	[2, 4, 7]+[3, 5, 9]+[10]+[12]+[15]+[16]+[18]+[19]($\frac{1}{2}$)
39	4	2	[2, 4, 7]+[3, 5, 9]+[10]+[12]+[15]+[16]+[18]+[19]
40	4	2	[10, 10, 10]($\frac{1}{4}$)+[2, 4, 7]+[3, 5, 9]+[12]+[15]+[16]+[18]+[19]
41	4	2	[2, 4, 7]+[3, 5, 9]+[10]+[12]+[15]+[16]+[18]+[19]+[20]
42	4	2	[2, 4, 7]+[3, 5, 9]+[10]+[15]+[16]+[18]+[19]+[20]+[21]($\frac{1}{2}$)
43	4	2	[2, 4, 7]+[3, 5, 9]+[10]+[12]+[15]+[16]+[18]+[19]+[20]+[21]
44	4	2	[11, 11, 11]($\frac{1}{4}$)+[2, 4, 12]+[3, 5, 9]+[7]+[10]+[13]+[15]+[19]+[20]+[21]
45	4	2	[2, 3, 6]+[4, 8, 7]+[10]+[13]+[14]+[16]+[17]+[18]+[20]+[21]+[22]
46	4	2	[2, 4, 7]+[3, 5, 9]+[10]+[12]+[15]+[16]+[18]+[19]+[20]+[21]+[22]+[23]($\frac{1}{2}$)
47	4	2	[2, 8, 11]+[3, 4, 5]+[6]+[13]+[14]+[15]+[16]+[17]+[18]+[20]+[22]+[23]
48	4	2	[12, 12, 12]($\frac{1}{4}$)+[2, 4, 7]+[3, 5, 9]+[10]+[15]+[16]+[18]+[19]+[20]+[21]+[22]+[23]
49	4	2	[2, 4, 7]+[3, 5, 9]+[10]+[12]+[15]+[16]+[18]+[19]+[20]+[21]+[22]+[23]+[24]
50	4	2	[2, 3, 4]+[6, 8, 10]+[11]+[12]+[13]+[15]+[16]+[17]+[19]+[20]+[21]+[22]+[23]+[25]($\frac{1}{2}$)

Table 2: Catalogue of CPDs in Blocks of Sizes $k_1 = 4$ & $k_2 = 2$ with $\alpha = 2$.

v	k_1	k_2	Sets of shifts
31	4	2	[3, 4, 5]+[6]+[8]+[10]+[11]+[13]+[14]+[15]
32	4	2	[3, 7, 8]+[4]+[5]+[6]+[9]+[11]+[12]+[13]+[16] ($\frac{1}{2}$)
33	4	2	[3, 5, 9]+[4, 6, 12]+[7]+[13]+[16]
34	4	2	[3, 5, 10]+[9, 4, 7]+[6]+[12]+[17] ($\frac{1}{2}$)
35	4	2	[3, 5, 7]+[4, 6, 11]+[9]+[13]+[16]
36	4	2	[9, 9, 9] ($\frac{1}{4}$)+[3, 4, 6]+[5, 11, 8]+[14]+[15]
37	4	2	[3, 5, 7]+[4, 6, 11]+[13]+[14]+[18]
38	4	2	[3, 4, 5]+[6, 10, 8]+[11]+[13]+[15]+[17]+[19] ($\frac{1}{2}$)
39	4	2	[3, 5, 7]+[6, 4, 9]+[11]+[14]+[16]+[17]+[18]
40	4	2	[10, 10, 10]+[3, 4, 5]+[6, 8, 11]+[13]+[16]+[17]+[18]
41	4	2	[3, 4, 5]+[6, 8, 11]+[10]+[13]+[16]+[17]+[18]+[20]
42	4	2	[3, 4, 5]+[6, 8, 11]+[10]+[13]+[15]+[16]+[18]+[20]+[21] ($\frac{1}{2}$)
43	4	2	[3, 4, 5]+[6, 8, 11]+[10]+[13]+[15]+[16]+[18]+[20]+[21]
44	4	2	[11, 11, 11]($\frac{1}{4}$)+[3, 5, 9]+[4, 6, 13]+[7]+[12]+[15]+[16]+[18]+[20]
45	4	2	[4, 8, 7]+[6, 14, 9]+[3]+[5]+[10]+[11]+[13]+[17]+[18]+[22]
46	4	2	[3, 4, 6]+[5, 9, 8]+[11]+[12]+[15]+[16]+[18]+[19]+[20]+[21]+[23] ($\frac{1}{2}$)
47	4	2	[4, 5, 6]+[7, 10, 8]+[3]+[12]+[13]+[14]+[16]+[19]+[20]+[21]+[23]
48	4	2	[3, 4, 13]+[6, 8, 11]+[5]+[9]+[10]+[15]+[16]+[18]+[21]+[22] ($\frac{1}{2}$)
49	4	2	[3, 5, 9]+[4, 6, 12]+[7]+[11]+[13]+[15]+[16]+[19]+[20]+[21]+[23]+[24]
50	4	2	[3, 10, 11]+[4, 12, 15]+[5, 9, 8]+[6]+[7]+[18]+[20]+[25]($\frac{1}{2}$)

Table 3: Catalogue of CPDs in Blocks of Sizes $k_1 = 4$ & $k_2 = 2$ with $\alpha = 3$

v	k_1	k_2	Sets of shifts
31	4	2	[4, 5, 6]+[7]+[8]+[10]+[12]+[14]+[15]
32	4	2	[4, 5, 6]+[7]+[8]+[10]+[12]+[13]+[14]+[16] ($\frac{1}{2}$)
33	4	2	[4, 6, 8]+[5]+[7]+[9]+[11]+[12]+[13]+[16]
34	4	2	[4, 6, 8]+[5]+[7]+[9]+[11]+[12]+[13]+[15]+[17] ($\frac{1}{2}$)
35	4	2	[4, 5, 7]+[6, 8, 10]+[13]+[15]
36	4	2	[9, 9, 9,] ($\frac{1}{4}$)+[5, 11, 8]+[4]+[6]+[7]+[10]+[13]+[14]+[15] ($\frac{1}{2}$)
37	4	2	[4, 6, 7]+[5, 9, 12]+[8]+[15]+[18]
38	4	2	[4, 6, 8]+[5, 7, 9]+[11]+[13]+[15]+[19] ($\frac{1}{2}$)
39	4	2	[4, 5, 7]+[6, 8, 10]+[11]+[13]+[17]+[19]
40	4	2	[4, 6, 8]+[5, 7, 9]+[11]+[13]+[13]+[15]+[17]+[20]
41	4	2	[4, 6, 8]+[5, 7, 9]+[11]+[13]+[15]+[17]+[20]
42	4	2	[4, 6, 8]+[5, 7, 13]+[9]+[11]+[15]+[16]+[19]+[21] ($\frac{1}{2}$)
43	4	2	[4, 6, 8]+[5, 7, 9]+[11]+[13]+[15]+[17]+[19]+[20]
44	4	2	[11, 11, 11]+[4, 6, 8]+[5, 7, 9]+[`3]+[15]+[17]+[19]+[20]
45	4	2	[4, 8, 7]+[6, 14, 9]+[5]+[10]+[11]+[13]+[17]+[18]+[22]
46	4	2	[4, 6, 7]+[5, 9, 11]+[8]+[12]+[15]+[16]+[18]+[19]+[22]+[23] ($\frac{1}{2}$)
47	4	2	[4, 5, 6]+[7, 10, 8]+[12]+[13]+[14]+[16]+[19]+[20]+[21]+[23]
48	4	2	[12, 12, 12] ($\frac{1}{4}$)+[4, 5, 6]+[7, 10, 8]+[13]+[14]+[16]+[19]+[20]+[21]+[22]
49	4	2	[4, 5, 6]+[7, 10, 8]+[12]+[13]+[14]+[16]+[19]+[20]+[21]+[22]+[23]
50	4	2	[4, 12, 14]+[5, 10, 13]+[6]+[7]+[8]+[9]+[11]+[17]+[18]+[19]+[21]+[25]($\frac{1}{2}$)

Table 4: Catalogue of CPDs in Blocks of Sizes $k_1 = 4$ & $k_2 = 2$ with $\alpha = 4$.

v	k_1	k_2	Sets of shifts
31	4	2	[5, 6, 8]+[7]+[9]+[10]+[13]+[15]
32	4	2	[5, 6, 7]+[8]+[9]+[10]+[12]+[15]+[16] ($\frac{1}{2}$)
33	4	2	[5, 6, 7]+[8]+[9]+[10]+[12]+[14]+[16]
34	4	2	[5, 6, 7]+[8]+[9]+[10]+[12]+[14]+[15]+[17] ($\frac{1}{2}$)
35	4	2	[5, 6, 7]+[8]+[9]+[10]+[12]+[14]+[15]+[16]
36	4	2	[9, 9, 9]+[6, 7, 8]+[10]+[11]+[12]+[16]+[17]
37	4	2	[5, 6, 7]+[8]+[9]+[10]+[12]+[14]+[15]+[16]+[17]
38	4	2	[5, 6, 7]+[8]+[9]+[10]+[12]+[14]+[15]+[16]+[17]+[19] ($\frac{1}{2}$)
39	4	2	[5, 7, 9]+[6, 11, 8]+[10]+[13]+[15]
40	4	2	[10, 10, 10] ($\frac{1}{4}$)+[5, 7, 9]+[6]+[8]+[11]+[13]+[14]+[15]+[17]+[18]
41	4	2	[5, 7, 9]+[6, 8, 10]+[11]+[13]+[15]+[19]
42	4	2	[5, 6, 7]+[8, 9, 10]+[12]+[14]+[16]+[20]+[21] ($\frac{1}{2}$)
43	4	2	[5, 6, 7]+[8, 9, 12]+[10]+[15]+[16]+[19]+[20]
44	4	2	[11, 11, 11] ($\frac{1}{4}$)+[5, 7, 9]+[6, 8, 10]+[13]+[15]+[17]+[19]
45	4	2	[6, 14, 9]+[7, 10, 13]+[8]+[11]+[12]+[18]+[19]
46	4	2	[5, 6, 8]+[7, 9, 12]+[10]+[13]+[15]+[17]+[20]+[22]+[23] ($\frac{1}{2}$)
47	4	2	[5, 7, 8]+[6, 10, 9]+[11]+[13]+[14]+[17]+[18]+[21]+[23]
48	4	2	[12, 12, 12] ($\frac{1}{4}$)+[5, 6, 8]+[7, 9, 12]+[10]+[13]+[15]+[17]+[18]+[22]+[23] ($\frac{1}{2}$)
49	4	2	[5, 7, 11]+[6, 13, 8]+[9]+[10]+[14]+[15]+[16]+[17]+[20]+[24]
50	4	2	[4, 9, 15]+[5, 6, 8]+[7]+[10]+[12]+[16]+[17]+[18]+[20]+[21]+[23]+[24]+[25]($\frac{1}{2}$)

Table 5: Catalogue of CPDs in Blocks of Sizes $k_1 = 4$ & $k_2 = 2$ with $\alpha = 5$.

v	k_1	k_2	Sets of shifts
24	4	2	[6, 6, 6]($\frac{1}{4}$)+[7]+[8]+[9]+[10]+[11]
25	4	2	[6, 7, 8]+[10]+[11]+[12]+[14]($\frac{1}{2}$)
26	4	2	[6, 7, 8]+[9]+[11]+[12]+[14]
27	4	2	[6, 7, 10]+[8, 8, 8]($\frac{1}{4}$)+[11]+[12]+[14]
28	4	2	[6, 7, 8]+[9]+[10]+[11]+[14]+[16]
29	4	2	[6, 7, 9]+[8]+[10]+[11]+[15]+[16]+[17]($\frac{1}{2}$)
30	4	2	[8, 9, 11]+[6]+[10]+[12]+[13]+[14]+[16]
31	4	2	[6, 8, 12]+[9, 9, 9]+[11]+[12]+[13]+[15]+[17]($\frac{1}{2}$)
32	4	2	[6, 7, 8]+[9]+[10]+[11]+[12]+[14]+[17]+[18]
33	4	2	[6, 7, 10]+[8]+[9]+[11]+[12]+[14]+[16]+[18]+[19]($\frac{1}{2}$)
34	4	2	[6, 7, 11]+[8, 9, 10]+[10]+[13]+[14]+[16]+[18]
35	4	2	[6, 8, 11]+[10, 10, 10]+[7]+[9]+[12]+[13]+[16]+[17]+[18]($\frac{1}{2}$)
36	4	2	[6, 7, 14]+[9, 10, 12]+[8]+[15]+[16]+[17]+[18]($\frac{1}{2}$)
37	4	2	[6, 7, 14]+[8, 9, 11]+[10]+[12]+[18]+[19]
38	4	2	[11, 11, 11] ($\frac{1}{4}$)+[6, 8, 7]+[9]+[10]+[12]+[13]+[16]+[17]+[18]+[19]+[20]
39	4	2	[6, 7, 8]+[9, 10, 12]+[11]+[16]+[17]+[18]+[20]
40	4	2	[7, 8, 9]+[6, 10, 13]+[11]+[12]+[14]+[18]+[19]+[20]+[23]($\frac{1}{2}$)
41	4	2	[6, 9, 8]+[7, 12, 10]+[11]+[13]+[14]+[16]+[20]+[21]
42	4	2	[6, 7, 8]+[9, 10, 12]+[11]+[14]+[16]+[18]+[20]+[23]+[24]($\frac{1}{2}$)
43	4	2	[6, 7, 8]+[9, 10, 12]+[11]+[14]+[16]+[17]+[20]+[23]+[24]
44	4	2	[8, 9, 10]+[6, 7, 11]+[12]+[14]+[15]+[16]+[20]+[21]+[22]+[24]+[25]($\frac{1}{2}$)
45	4	2	[6, 14, 9]+[7, 10, 13]+[8]+[11]+[12]+[18]+[19]
46	4	2	[5, 6, 8]+[7, 9, 12]+[10]+[13]+[15]+[17]+[20]+[22]+[23] ($\frac{1}{2}$)
47	4	2	[5, 7, 8]+[6, 10, 9]+[11]+[13]+[14]+[17]+[18]+[21]+[23]
48	4	2	[12, 12, 12] ($\frac{1}{4}$)+[5, 6, 8]+[7, 9, 12]+[10]+[13]+[15]+[17]+[18]+[22]+[23] ($\frac{1}{2}$)
49	4	2	[5, 7, 11]+[6, 13, 8]+[9]+[10]+[14]+[15]+[16]+[17]+[20]+[24]
50	4	2	[4, 9, 15]+[5, 6, 8]+[7]+[10]+[12]+[16]+[17]+[18]+[20]+[21]+[23]+[24]+[25]($\frac{1}{2}$)

Table 6: Catalogue of CPDs in Blocks of Sizes $k_1 = 4$ & $k_2 = 2$ with $\alpha = 6$.

v	k_1	k_2	Sets of shifts
28	4	2	[7, 7, 7]($\frac{1}{4}$)+[8]+[9]+[10]+[11]+[12]+[13]
29	4	2	[8, 8, 8]($\frac{1}{4}$) +[7]+[9]+[10]+[11]+[12]+[13]+[14]+[15]
30	4	2	[7, 8, 9]+[10]+[12]+[13]+[14]+[16]
31	4	2	[7, 8, 9]+[10]+[11]+[13]+[14]+[16]+[18]($\frac{1}{2}$)
32	4	2	[7, 8, 9]+[10]+[11]+[12]+[14]+[16]+[18]
33	4	2	[8, 9, 10]+[7]+[12]+[13]+[14]+[15]+[16]+[18]($\frac{1}{2}$)
34	4	2	[7, 8, 10]+[9]+[11]+[12]+[13]+[16]+[17]+[19]
35	4	2	[10, 10, 10]+[8, 9, 10]+[7]+[11]+[12]+[14]+[15]+[16]+[18]($\frac{1}{2}$)
36	4	2	[7, 8, 10]+[9]+[11]+[12]+[13]+[14]+[17]+[19]+[20]

Table 6: Continued

37	4	2	[7, 10, 11]+[8, 9, 13]+[16]+[18]+[19]($\frac{1}{2}$)
38	4	2	[10, 7, 11]+[8, 12, 9]+[13]+[16]+[19]
39	4	2	[7, 11, 10]+[8, 9, 13]+[12]+[15]+[19]+[20]($\frac{1}{2}$)
40	4	2	[7, 8, 10]+[9, 12, 11]+[14]+[16]+[17]+[19]
41	4	2	[7, 8, 10]+[9, 11, 12]+[13]+[16]+[17]+[19]+[22]($\frac{1}{2}$)
42	4	2	[7, 8, 9]+[10, 12, 14]+[13]+[16]+[18]+[19]+[20]
43	4	2	[7, 8, 10]+[13]+[14]+[17]+[19]+[22]+[24]($\frac{1}{2}$)
44	4	2	[7, 8, 10]+[11, 9, 12]+[13]+[14]+[16]+[19]+[22]+[23]
45	4	2	[7, 8, 9]+[11, 10, 13]+[12]+[14]+[15]+[18]+[19]+[20]+[22]+[25]($\frac{1}{2}$)
46	4	2	[5, 6, 8]+[7, 9, 12]+[10]+[13]+[15]+[17]+[20]+[22]+[23] ($\frac{1}{2}$)
47	4	2	[5, 7, 8]+[6, 10, 9]+[11]+[13]+[14]+[17]+[18]+[21]+[23]
48	4	2	[12, 12, 12] ($\frac{1}{4}$)+[5, 6, 8]+[7, 9, 12]+[10]+[13]+[15]+[17]+[18]+[22]+[23] ($\frac{1}{2}$)
49	4	2	[5, 7, 11]+[6, 13, 8]+[9]+[10]+[14]+[15]+[16]+[17]+[20]+[24]
50	4	2	[4, 9, 15]+[5, 6, 8]+[7]+[10]+[12]+[16]+[17]+[18]+[20]+[21]+[23]+[24]+[25]($\frac{1}{2}$)

Table 7: Catalogue of CPDs in Blocks of Sizes $k_1 = 4$ & $k_2 = 2$ with $\alpha = 7$.

v	k_1	k_2	Sets of shifts
40	4	2	[8, 9, 10]+[11]+[12]+[14]+[15]+[16]+[20]($\frac{1}{2}$)
41	4	2	[8, 9, 10]+[11]+[12]+[13]+[15]+[16]+[20]
42	4	2	[8, 10, 9]+[11]+[12]+[13]+[14]+[16]+[17]+[20]+[21]($\frac{1}{2}$)
43	4	2	[8, 9, 10]+[11]+[12]+[13]+[15]+[16]+[18]+[20]+[21]
44	4	2	[9, 10, 11]+[8]+[12]+[13]+[15]+[16]+[17]+[18]+[20]+[22]($\frac{1}{2}$)
45	4	2	[8, 10, 11]+[9]+[12]+[13]+[14]+[15]+[17]+[19]+[20]+[22]
46	4	2	[8, 9, 11]+[10]+[12]+[13]+[14]+[15]+[16]+[19]+[21]+[22]+[23]($\frac{1}{2}$)
47	4	2	[9, 10, 11]+[8, 12, 13]+[15]+[16]+[18]+[23]
48	4	2	[8, 12, 13]+[9, 10, 11]+[14]+[16]+[17]+[22]+[24]($\frac{1}{2}$)
49	4	2	[9, 13, 12]+[8, 10, 11]+[14]+[16]+[17]+[19]+[23]
50	4	2	[8, 11, 13]+[9, 12, 15]+[10]+[16]+[17]+[20]+[22]+[25]($\frac{1}{2}$)

Table 8: Catalogue of CPDs in Blocks of Sizes $k_1 = 4$ & $k_2 = 2$ with $\alpha = 8$.

v	k_1	k_2	Sets of shifts
45	4	2	[9, 10, 11]+[12]+[13]+[14]+[16]+[17]+[18]+[20]+[22]
46	4	2	[9, 11, 10]+[12]+[13]+[14]+[15]+[17]+[18]+[19]+[22]+[23]($\frac{1}{2}$)
47	4	2	[10, 11, 12]+[9]+[13]+[15]+[16]+[17]+[18]+[19]+[20]+[22]
48	4	2	[9, 11, 12]+[10]+[13]+[14]+[15]+[17]+[18]+[19]+[21]+[22]+[24]($\frac{1}{2}$)
49	4	2	[9, 10, 12]+[11]+[13]+[14]+[15]+[16]+[17]+[20]+[21]+[23]+[24]
50	4	2	[9, 11, 12]+[10]+[13]+[14]+[15]+[16]+[17]+[19]+[21]+[22]+[24]+[25]($\frac{1}{2}$)
51	4	2	[9, 11, 13]+[10, 12, 14]+[16]+[17]+[19]+[21]+[23]
52	4	2	[9, 13, 14]+[10, 11, 12]+[15]+[17]+[18]+[20]+[24]+[26]($\frac{1}{2}$)
53	4	2	[9, 13, 14]+[10, 11, 12]+[15]+[16]+[18]+[19]+[24]+[26]
54	4	2	[9, 10, 13]+[11, 14, 12]+[15]+[16]+[18]+[20]+[21]+[24]+[27]($\frac{1}{2}$)
55	4	2	[9, 10, 14]+[11, 12, 15]+[13]+[16]+[18]+[20]+[21]+[25]+[26]
56	4	2	[9, 10, 14]+[11, 15, 12]+[13]+[16]+[17]+[20]+[21]+[22]+[25]+[28]($\frac{1}{2}$)
57	4	2	[9, 12, 10]+[11, 13, 14]+[15]+[16]+[17]+[18]+[20]+[23]+[25]+[28]
58	4	2	[9, 10, 13]+[11, 14, 16]+[12]+[15]+[18]+[20]+[21]+[22]+[24]+[27]+[29]($\frac{1}{2}$)
59	4	2	[9, 10, 13]+[11, 14, 12]+[15]+[16]+[17]+[18]+[20]+[21]+[24]+[28]+[29]
60	4	2	[9, 10, 14]+[11, 12, 16]+[13]+[15]+[17]+[18]+[20]+[22]+[24]+[25]+[26]+[29]+[30]($\frac{1}{2}$)

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