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# On the Nishimura's Identity Involving Stirling Numbers

[1]*J. Yaljá Montiel-Pérez and* [2]*J. López-Bonilla*

[1]Centro De Investigación En Computación, Instituto Politécnico Nacional,
[2]ESIME-Zacatenco, Instituto Politécnico Nacional,
Edif. 4, 1er. Piso, Col. Lindavista CP 07738, CDMX, México

**Abstract:** We employ the Nishimura's formula to obtain values of Stirling numbers of the second kind.

**Key words:** Stirling numbers - Nishimura's identity - Combinatorial expressions

## INTRODUCTION

Nishimura [1] deduced the following identity for $n$ odd and $k$ even, $n > k$:

$$2\,k!\;S_n^{[k]} = \sum_{r=0}^{\frac{n-k-1}{2}} R(r)\binom{k+2r}{k-1}(k+2r+1)!\;S_n^{[k+2r+1]}, R(r) = \frac{2}{r+1}(2^{2r+2}-1)B_{2r+2}\,, \tag{1}$$

Involving Bernoulli and Stirling numbers of the second kind [2-7]:

$$B_0 = 1, B_1 = -\frac{1}{2}\,, B_2 = \frac{1}{6}\,, B_4 = B_8 = -\frac{1}{30}\,, B_6 = \frac{1}{42}\,, \dots, \tag{2}$$

Therefore:

$$R(0) = 1, R(1) = -\frac{1}{2}\,, R(2) = 1, R(3) = -\frac{17}{4}\,, R(4) = 31, \dots \tag{3}$$

Hence from (1) for $k = n-1$ and (3):

$$2\,(n-1)!\;S_n^{[n-1]} = R(0)\binom{n-1}{n-2}\,n!\;S_n^{[n]} \;\therefore\; S_n^{[n-1]} = \frac{n}{2}\,(n-1) = \binom{n}{2}, \tag{4}$$

In agreement with [2, 5, 8]. We know the relation:

$$S_n^{[n-2]} = \frac{n}{24}\,(n-1)(n-2)(3n-5) = \frac{1}{4}\binom{n}{3}\,(3n-5) = \binom{n}{3} + 3\binom{n}{4}, \tag{5}$$

Then we use (1) with $k = n-3$ and the values (3) and (5) to obtain:

$$2\,(n-3)!\;S_n^{[n-3]} = R(0)\binom{n-3}{n-4}(n-2)!\;S_n^{[n-2]} + R(1)\binom{n-1}{n-4}\,n!\;S_n^{[n]}\,,$$

That is [2, 5, 8]:

$$S_n^{[n-3]} = \frac{n}{48}\,(n-1)(n-2)^2\,(n-3)^2 = \frac{1}{2}\binom{n}{4}(n-2)(n-3). \tag{6}$$

Similarly, we have the property:

$$S_n^{[n-4]} = \frac{1}{48}\binom{n}{5}(15\,n^3 - 150\,n^2 + 485\,n - 502), \tag{7}$$

**Corresponding Author:** J. López-Bonilla, ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso, Col. Lindavista CP 07738, CDMX, México. E-mail: jlopezb@ipn.mx

Then from (1) for $k = n - 5$ and the expressions (3) and (7):

$$2(n-5)!\,S_n^{[n-5]} = R(0)\binom{n-5}{n-6}(n-4)!\,S_n^{[n-4]} + R(1)\binom{n-3}{n-6}(n-2)!\,S_n^{[n-2]} + R(2)\binom{n-1}{n-6}n!\,S_n^{[n]},$$

Therefore [2, 5, 8]:

$$S_n^{[n-5]} = \frac{1}{16}\binom{n}{6}(n-4)(n-5)(3n^2 - 23n + 38). \qquad (8)$$

Thus with this process we can calculate $S_n^{[n-7]}, S_n^{[n-9]}, \ldots$, if we know $S_n^{[n-6]}, S_n^{[n-8]}, \ldots$

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