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Derivation of Cast Iron Welding-Current Resistance (WCR) and Its Evaluation Using General Quadratic Equation Formular

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Abstract: This paper presents a derivation of cast iron welding-current resistance (WCR) based on WCR and weldment HAZ hardness of aluminium, mild steel and cast iron similarly cooled in water and its evaluation using General Quadratic Equation Formular (GQEF). The weldment HAZ hardness for aluminium, mild steel and cast iron were 458, 560 and 1010 VPN respectively. A quadratic model was derived from first principle and cast iron WCR evaluated using GQEF. The validity of the model referred to as Cast Iron Welding-Current Resistance Model (C-WCR Model) was found to be rooted in the core model expression; $\vartheta/\gamma + \gamma/\varepsilon + \vartheta/\varepsilon = (\mathfrak{S}/\ell + \ell/\mathfrak{h} + \mathfrak{S}/\mathfrak{h})^N$ where both sides of the core model expression are equal. The model was validated using computational and deviational analysis. Computational analysis of experimental and model predicted results indicate that cast iron WCR per unit weldment HAZ hardness are 0.0012 Ω/VHN and 0.0012 Ω /VHN respectively, indicating zero deviation from experimental result. This invariably implies 100% confidence level for the derived models as well as unity (1.0) Reliability Dependence Coefficients of cast iron WCR on the WCR and weldment HAZ hardness of aluminium, cast iron and mild steel similarly cooled in water.

Key words: Evaluation · Cast iron · Welding-current resistance · General quadratic equation formular

INTRODUCTION

Structural failures in fabricated steel and allied alloys put to service have raised the need for intensive studies on the material weldability and durability. During welding, significant changes in the microstructure of the weldments depend directly on the welding process and techniques employed. And so durability of welded joints can only be guaranteed by improvement of the microstructure of the heat affected zone (HAZ).

Research [1] has revealed mathematical models developed to study the effects of process variables and heat input on various metallurgical aspects, namely, the widths of the HAZ, weld interface and grain growth and grain refinement regions of the HAZ. The response surface methodology and color metallography technique were also used.

These models also help to improve understanding of the effect of process parameters on bead quality, for quantitative evaluation of the interaction effects of process variables on HAZ characteristics and to optimize the size of the weld bead's HAZ in order to obtain a better quality welded joint with desirable properties at a relatively low cost.

Predictive analysis of weldment HAZ hardness of selected engineering materials such as mild steel, cast iron and aluminum has been successfully carried out using some empirical models [2-7]. The materials used were

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welded using Shielded Metal Are Welding (SMAW) technique and similarly cooled (for each research) in palm oil, air, water and groundnut oil. Each of these models recorded a maximum deviation less than 0.5%. These are deviations of model-predicted weldment HAZ hardness values from the corresponding experimental values.

Expressions and models existing in quadratic form, being derived from through computational analysis of experimental results could be solved as quadratic equation using different methods.

The earliest methods for solving quadratic equations were geometric [8]. Quadratic equations have also been solved with a method more recognizably algebraic than the geometric algebra used in the work [8]. In this method [9], the solution to the equation gives only one root, even when both roots are positive.

It has been shown [10] that quadratic equations can be solved algebraically.

Mathematicians have revealed several methods with which quadratic equations can be solved. These includes: by substitution [11], by algebraic identities [12], by Lagrange resolvents [13] which is an early part of Galois Theory [14]. This method can be generalized to give the roots of cubic polynomials and quadratic polynomials and leads to Galois Theory, which allows one to understand the solution of algebraic equations of any degree in terms of the symmetry group of their roots, the Galois group.

Investigations [15] have shown that the quadratic formular is the most general method of solving quadratic equations and is derived from another general method which is ‘‘completing the square’’

The present work is to derive cast iron welding-current resistance (WCR) based on WCR and weldment HAZ hardness of similarly water cooled aluminium, cast iron & mild steel and evaluate it using General Quadratic Equation Formular.

MATERIALS AND METHODS

Clean samples of aluminum, cast iron and mild steel obtained from First Aluminum Company Ltd. Port Harcourt were used for the welding operations. Prior to welding, two parts of each standard sample of these materials were butt welded end to end at the interface of separation. The joints were prepared by chamfering the edges to be joined to create a “double V” kind of groove. The welding operation was carried out using the Shielded Metal Arc Welding (SMAW) process. This technique was considered because of its versatility and ability to give moderately sized heat affected zone. Furthermore, the technique was employed because it offers protection to the molten metal (during welding) against atmospheric gas interference. Water was selected as the cooling medium because it confers greater hardness than air [16]. Consumable electrodes of length 230-240 mm were used. These electrodes were coated with SiO_2. The welded samples were similarly cooled in water (maintained at room temperature) and the HAZ hardness of their respective weldments determined using Vickers hardness testing machine. Ten samples from each of the three materials were welded, similarly cooled in water and their respective weldment HAZ hardness tested. The average HAZ hardness for the weldments of each of the three materials investigated were evaluated and are as presented in Table 1. Table 2 shows the welding current and voltage used, as well as welding current resistance.

Table 1: Hardness of HAZ in weldments

Material	HAZ Hardness (VHN)
Aluminium	458
Cast Iron	1010
Mild Steel	560

Table 2: Variation of materials with welding voltages, currents and resistances

Material	C/Type	WV	WC	WCR
Aluminium	D.C	280	120	2.333
Cast Iron	A.C	220	180	1.222
Mild Steel	A.C	220	180	1.222

RESULTS AND DISCUSSIONS

Table 2 shows the variation of materials with the input welding current type (C/Type), welding current (WC), voltage (WV) and current resistance (WCR). The result of hardness of the HAZ obtained from aluminium, cast iron and mild steel weldments similarly cooled in water (as presented in Table 1) shows that the weldment HAZ hardness for aluminium, mild steel and cast iron were 458, 560 and 1010 VHN respectively.

Computational analysis of results in the 3rd and 4th column of Table 2 as well as 2nd column of Table 1 gave rise to Table 3. It is strongly believed that the precision and validity of the model is rooted on this table (Table 3).

Deriving and Evaluating Cast Iron Welding-Current Resistance (WCR) Using General Quadratic Equation Rule: Electrical resistance to the welding current flow is given by a relationship:

$$R = \left(\frac{V}{I}\right) \tag{1}$$

Experimental data generated from the highlighted research work were used for the derivation. Computational analysis of these data shown in Tables 1 and 2, gave rise to Table 3 which indicate that;

$$\frac{\vartheta}{\text{ɤ}}+\frac{\text{ɤ}}{\varepsilon}+\frac{\vartheta}{\varepsilon}\approx\left[\frac{\text{₰}}{\ell}+\frac{\ell}{\text{ꜧ}}+\frac{\text{₰}}{\text{ꜧ}}\right]^{N} \tag{2}$$

where

(₰) = HAZ hardness of aluminium weldment cooled in water (VHN)

(ℓ) = HAZ hardness of cast iron weldment cooled in water (VHN)

(ꜧ) = HAZ hardness of mild steel weldment cooled in water (VHN)

(ϑ) = Welding current resistance during welding of aluminium (Ω)

(ɤ) = Welding current resistance during welding of cast iron (Ω)

(ε) = Welding current resistance during welding of mild steel (Ω)

V = Welding voltage (V)

I = Welding current (A)

N = 1.3998. This is equalizing constant (determined using C-NIKBRAN [7]).

Substituting the values of N, ₰, ℓ and ꜧ (from Table 1) into equation (2), reduces it to;

$$\frac{\vartheta}{\text{ɤ}}+\frac{\text{ɤ}}{\varepsilon}+\frac{\vartheta}{\varepsilon} = (3.075)^{1.3998} \tag{3}$$

$$\frac{\vartheta}{\text{ɤ}}+\frac{\text{ɤ}}{\varepsilon}+\frac{\vartheta}{\varepsilon} = 4.8182 \tag{4}$$

$$\frac{\vartheta\varepsilon + \text{ɤ}^2 + \vartheta\text{ɤ}}{\text{ɤ}\varepsilon} = 4.8182 \tag{5}$$

$$\vartheta\varepsilon + \text{ɤ}^2 + \vartheta\text{ɤ} = 4.8182\ \text{ɤ}\varepsilon \tag{6}$$

Rearranging equation (6) gives

$$\text{ɤ}^2 + \vartheta\text{ɤ} + \vartheta\varepsilon = 4.8182\ \text{ɤ}\varepsilon \tag{7}$$

Equation (7) reduces to

$$\text{ɤ}^2 + \vartheta\text{ɤ} + \vartheta\varepsilon - 4.8182\,\text{ɤ}\varepsilon = 0 \tag{8}$$

Equation (8) is further rearranged as;

$$\text{ɤ}^2 + \vartheta\text{ɤ} - 4.8182\,\text{ɤ}\varepsilon + \vartheta\varepsilon = 0 \tag{9}$$

$$\text{ɤ}^2 + (\vartheta - 4.8182\varepsilon)\,\text{ɤ} + \vartheta\varepsilon = 0 \tag{10}$$

This is similar to the general quadratic equation rule in which

$$ax^2 + bx + c = 0\ [17] \tag{11}$$

Research has shown [17] that evaluation of equation (11), using completing the square method, gives the general formular;

$$x = -\frac{b \pm \sqrt{(b^2 - 4ac)}}{2a} \tag{12}$$

Based on the foregoing, since equation (10) matches with the general quadratic equation in equations (11) and both are equal, it implies that a = 1; x = ɤ; b = (ϑ -4.8182 ε) and c = ϑε. Therefore, on substituting these values into equation (12), the general quadratic equation formular evaluates the value of ɤ instead of x as;

$$x = -\frac{(\vartheta - 4.8182\varepsilon) \pm \sqrt{((\vartheta - 4.8182\varepsilon)^2 - 4\vartheta\varepsilon)}}{2} \tag{13}$$

Substituting the values of ϑ and ε (from Table 2) into equation (13) gives the welding current resistance as ɤ = 2.333 or 1.222. Substituting each of the evaluated values of ɤ (as well as values of ϑ and ε) into the first principle (equation (2) or (4)) gives 4.8183 on the LHS which approximately equals 4.8182 on the RHS in each case. Furthermore, substitution of each of the evaluated values of ɤ (as well as values of ϑ and ε) into in equation (10) gives zero in accordance with the general quadratic equation rule. Based on the foregoing, equation (10) is referred to as the Cast Iron Welding-Current Resistance Model (C-WCR Model). This is because it can predict the encountered resistance to current flow during welding of cast iron based on the WCR and HAZ hardness for aluminium and mild steel similarly cooled.

Model Validation: The validity of the derived model was tested through computational and deviational analysis of both the experimental and model-predicted results. This will be done by comparing the cast iron WCR per unit weldment HAZ hardness as evaluated from experiment and derived model. The validity of the model was found to be rooted in equations (2) or (4) (core of the model) where both sides of the equation are equal. Table 3 also agrees with equations (2) and (4) following the values of $\vartheta/\text{ɤ} + \text{ɤ}/\varepsilon + \vartheta/\varepsilon = (\text{₰}/\ell + \ell/\text{ꜧ} + \text{₰}/\text{ꜧ})^N$ evaluated from the experimental results in Tables 1 and 2.

Table 3: Variation of $\vartheta/\text{ɤ} + \text{ɤ}/\varepsilon + \vartheta/\varepsilon$ with $(\mathfrak{S}/\ell + \ell/\text{Ђ} + \mathfrak{S}/\text{Ђ})^N$

$\vartheta/\text{ɤ} + \text{ɤ}/\varepsilon + \vartheta/\varepsilon$	$(\mathfrak{S}/\ell + \ell/\text{Ђ} + \mathfrak{S}/\text{Ђ})^N$
4.8183	4.8182

Following welding of aluminium, cast iron as well as mild steel and similarly, cooling their respective weldments in water, the WCR encountered during cast iron welding could be evaluated using C-WCR Model ($\text{ɤ}^2 + (\vartheta - 4.8182\,\varepsilon)\text{ɤ} + \vartheta\varepsilon = 0$) which is quadratic in nature.

Computational Analysis: Computational analysis of the experimental and model-predicted cast iron WCR per unit weldment HAZ hardness was carried out to ascertain the degree of validity of the derived model. The evaluation was from calculations involving experimental results and derived model.

Cast iron WCR per Unit Weldment HAZ Hardness: Cast iron WCR per unit weldment HAZ hardness ɤ_H; was calculated from the equation;

$$\text{ɤ}_H = \text{ɤ}/\ell \tag{14}$$

Substituting the values of WCR and HAZ hardness of cast iron weldment (as shown in Tables 1 and 2) into equation (14) gives 0.0012 Ω/ VHN as the WCR per unit HAZ hardness of the weldment. This is the experimentally obtained cast iron WCR per unit weldment HAZ hardness.

Similarly, substituting the model-predicted cast iron WCR (1.222 Ω) and the weldment HAZ hardness (in Table 1) into equation (14) also gives 0.0012 Ω/ VHN as the model-predicted WCR per unit HAZ hardness of the weldment.

The two evaluated values of cast iron WCR as obtained from experiment and derived model show equality, indicating a high validity level for the derived model.

Deviational Analysis: Deviation (Dv) of model-predicted cast iron WCR from that of the experiment is given by;

$$Dv = \left(\frac{\text{ɤ}_p - \text{ɤ}_{ex}}{\text{ɤ}_{ex}}\right) \times 100 \tag{15}$$

Correction factor (Cr) is the negative of the deviation i.e;

Table 4: Variation of model predicted WCR per unit weldment HAZ hardness with associated deviations and correction factors

Material	ExD	MoD	Dv (%)	Cr(%)
Cast Iron	0.0012	0.0012	0	0

$$Cr = -Dv \tag{16}$$

Therefore,

$$Cr = -\left(\frac{\text{ɤ}_p - \text{ɤ}_{ex}}{\text{ɤ}_{ex}}\right) \times 100 \tag{17}$$

where,

Dv = Deviation (%)

ɤ_p = Model-predicted WCR (Ω)

ɤ_e = WCR from experiment (Ω)

Cr = Correction factor (%)

Comparative analysis of cast iron WCR from the experiment and derived model show zero deviation of the model-predicted value from the experiment result. Since deviation is zero, it implies from equations (16) and (17) that correction factor will also be zero.

Deviational analysis of Table 4 indicates clearly, complete model acceptability and reliability. This invariably implies 100 % confidence level for the derived models as well as unity (1.0) Reliability Dependence Coefficients of cast iron WCR on the WCR and weldment HAZ hardness of aluminium, cast iron and mild steel similarly cooled in water.

CONCLUSIONS

Cast iron welding-current resistance (WCR) has been evaluated based on WCR and weldment HAZ hardness of aluminium, mild steel and cast iron similarly cooled in water, using General Quadratic Equation Formular (GQEF). The weldment HAZ hardness for aluminium, mild steel and cast iron were 458, 560 and 1010 VPN respectively. The validity of the derived model (quadratic in nature) referred to as Cast Iron Welding-Current Resistance Model (C-WCR Model) was rooted in the core model expression; $\vartheta/\text{ɤ} + \text{ɤ}/\varepsilon + \vartheta/\varepsilon = (\mathfrak{S}/\ell + \ell/\text{Ђ} + \mathfrak{S}/\text{Ђ})^N$ where both sides of the core model expression are equal. The model could be validated using computational and deviational analysis. Computational analysis of experimental and model predicted results gave cast iron WCR per unit weldment HAZ hardness as 0.0012 Ω/VHN and 0.0012 Ω/VHN respectively, indicating zero deviation from experimental result. This invariably implies 100% confidence level for the derived models as well as unity (1.0) Reliability Dependence Coefficients of cast iron WCR on the WCR and weldment HAZ hardness of aluminium, cast iron and mild steel similarly cooled in water.

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