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A Comment on the Lanczos Conformal Lagrangian

[1]J. Morales, [1]G. Ovando and [2]J. López-Bonilla

[1]CB1-Área de Física, AMA, Universidad Autónoma Metropolitana-Azcapotzalco,
Av. San Pablo 180, Col. Reynosa-Tamaulipas CP 02200, CDMX, México
[2]ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso,
Col. Lindavista CP 07738, CDMX, México

Abstract: We show that, in four dimensions, the Lagrangian $\sqrt{-g}\ C_{\mu\nu\alpha\beta}\ C^{\mu\nu\alpha\beta}$ gives the same field equations as $\sqrt{-g}\ K^{\mu\beta\alpha}{}_{;\beta}\ R_{\mu\alpha}$, in terms of the Lanczos spintensor.

Key words: Lanczos potential・Weyl tensor・Lanczos scalar

INTRODUCTION

In [1, 2] was showed that the Lagrangian $L_1 = \sqrt{-g}\ C_{\mu\nu\alpha\beta}\ C^{\mu\nu\alpha\beta}$, involving the Weyl tensor [3, 4] in four dimensions, gives the same field equations as $L_2 = \sqrt{-g}\left(R_{\mu\nu}\ R^{\mu\nu} - \frac{1}{3}R^2\right)$, in terms of the Ricci tensor and the scalar curvature.

Here we consider the Lanczos Lagrangian [5-7]:

$$L_3 \equiv \sqrt{-g}\ R^{\mu\nu\alpha\beta} * R^*_{\mu\nu\alpha\beta} = 2L_2 - L_1, \tag{1}$$

with the participation of the double dual of Riemann tensor $*R^*_{\mu\nu\alpha\beta}$ [3, 4] such that:

$*R^{*\beta}{}_{\mu\beta\alpha} = G_{\mu\alpha} \equiv R_{\mu\alpha} - \frac{R}{2} g_{\mu\alpha}$ [Einstein tensor], $*R^*_{\mu\nu\alpha\beta};\beta = 0$ [Bianchi identities] [6],

$$R_{\mu\nu\alpha\beta} = C_{\mu\nu\alpha\beta} + \frac{1}{2}(R_{\mu\alpha}\ g_{\nu\beta} + R_{\nu\beta}\ g_{\mu\alpha} - R_{\mu\beta}\ g_{\nu\alpha} - R_{\nu\alpha}\ g_{\mu\beta}) + \frac{R}{6}\left(g_{\mu\beta}\ g_{\nu\alpha} - g_{\mu\alpha}\ g_{\nu\beta}\right), \tag{2}$$

$$*R^*{}_{\mu\nu\alpha\beta} = C_{\mu\nu\alpha\beta} + \frac{1}{2}(R_{\mu\alpha}\ g_{\nu\beta} + R_{\nu\beta}\ g_{\mu\alpha} - R_{\mu\beta}\ g_{\nu\alpha} - R_{\nu\alpha}\ g_{\mu\beta}) + \frac{R}{3}\left(g_{\mu\beta}\ g_{\nu\alpha} - g_{\mu\alpha}\ g_{\nu\beta}\right).$$

On the other hand, we have the Lanczos potential $K_{\mu\nu\alpha}$ for the conformal tensor [6, 8-10]:

$$C_{\mu\nu\alpha\beta} = K_{\mu\nu\alpha;\beta} - K_{\mu\nu\beta;\alpha} + K_{\alpha\beta\mu;\nu} - K_{\alpha\beta\nu;\mu} + K_{\mu\beta}\ g_{\nu\alpha} - K_{\mu\alpha}\ g_{\nu\beta} + K_{\nu\alpha}\ g_{\mu\beta} - K_{\nu\beta}\ g\mu\alpha,$$

$$K_{\mu\nu\alpha} + K_{\nu\alpha\mu} + K_{\alpha\mu\nu} = 0,\ \ K_{\mu\nu\alpha} = -K_{\nu\mu\alpha},\ \ K^{\nu}_{\mu\nu} = 0,\ \ K_{\mu\nu\alpha}{}^{;\alpha} = 0,\ \ K_{\mu\nu} \equiv K_{\mu\beta\nu}{}^{;\beta} = K_{\nu\mu}. \tag{3}$$

The application of (2) and (3) into (1) allows obtain the relation:

Corresponding Author: J. López-Bonilla, ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso, Col. Lindavista CP 07738, CDMX, México.

$$L_1 = 4\sqrt{-g}\ K_{\mu\nu}\ R^{\mu\nu} - (4\sqrt{-g} K_{\mu\nu\alpha} * R^{*\mu\nu\alpha\beta}), \beta, \quad (4)$$

Hence L_1 gives the same field equations as the Lagrangian

$$\sqrt{-g}\ K_{\alpha\beta}\ R_{\alpha\beta} = \sqrt{-g}\ K_{\alpha\mu\beta}{}^{;\mu}\ R^{\alpha\beta},\ \text{q.e.d.}$$

The expression (4) is equivalent to:

$$L_1 = -4\sqrt{-g}\ K_{\mu\alpha\beta}\ R^{\mu\beta;\alpha} + (4\sqrt{-g}\ K_{\mu\nu\alpha}\ C^{\mu\nu\alpha\beta})_{,\beta}. \quad (5)$$

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