

Relopt: Model Application for Reliability-based Optimization of Water Distribution Networks

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Abstract: Applying mathematical models to obtain an optimum solution or evaluate reliability of water distribution networks has been of major concern during the last decade. Optimization problem for water distribution networks is considered a non-linear hard problem during search process to identify the decision variables (i.e. pipe diameters). Concluding reliability in optimization problem makes it more complicated and increases the required run time. Reliability-based optimization for water distribution networks requires four steps those are defining risk components; reliability analysis and evaluation; treating of reliability and selection of the reliability-based optimum solution using an automatic search method. In this research the reliability-based optimization model RELOPT has been applied to existing cases studies with different scales and skeletonization. RELOPT is a standalone reliability-based optimization model consists of four sub-models those are; Reliability model RELWNET; hydraulic model EPANET; Pre-estimator optimization model AGM; and optimization model OPTWNET. The model RELWNET is a probabilistic model based on Generic Expectation Functions while the model OPTWNET is an optimization search engine based on Linear Adaptive Genetic Algorithm (LAGA). Application of RELOPT to the selected cases studies proves its ability to handle reliability-based optimization problem for water distribution networks having unlimited configuration.

Key words: Optimization • Reliability • Genetic algorithm • Hydraulic modeling • probabilistic models

INTRODUCTION

Water distribution system is a hydraulic infrastructure consisting of elements such as pipes, tanks, reservoirs, pumps and valves. It is essential to investigate and establish a reliable network ensuring adequate demands and pressure heads. However, the optimal network design is quite complicated due to nonlinear relationships between flow, head loss and the presence of discrete variables, such as commercial pipe sizes according to Kessler and Shamir [1], Eiger *et al.* [2] and Dandy *et al.* [3]. In addition, the objective function, which represents the cost of the network, is also nonlinear and causes great difficulty for design-based optimization of the network. Researchers during recent years have focused on probabilistic approach to overcome these difficulties considering a combination of random and deterministic steps as been stated by Savic and Walters [4]; Abebe and Solomatine [5]; and Eusuff and Lansey [6].

Genetic Algorithms (GA), Simulated Annealing (SA), GLOBE and Shuffled Frog Leaping Algorithms (SFLA), are few widely used algorithms in this field of study. Alperovits and Shamir [7] presented a Linear Programming Gradient (LPG) in optimizing water distribution networks. Segmental length of a pipe with differential diameter was used as decision making variable. The LPG method was later further improved by Kessler and Shamir [1]. A nonlinear programming (NLP) converges to local minima due to their reliance on the initial solution and derivatives of the unconstrained objective function as per Gupta *et al.* [8]. Su *et al.* [9] proposed an innovative iterative method using three combined models to define the reliability for water distribution network. Simpson *et al.* [10] applied simple GA in which each individual population is represented in a string of bits with identical length that encodes one possible solution. Savic and Walters [4] also used simple GA in conjunction with EPANET [11] network solver instead of using a

single optimization algorithm. Eusuff and Lansey [6] proposed SFLA, a new meta-heuristic algorithm based on memetic evolution (transformation of frogs) and information exchange among the population. Chengchao and Goulter [12] developed a new approach for reliability-based optimization of water distribution networks. The approach was capable of recognizing the uncertainty in nodal demands and pipe capacity as well as the effects of mechanical failure of system components. Avi [13] stated that it is a straight forward reply to the question "Is the system reliable?" and the most difficult reply is to the question "What is the reliability system level?" The author defined the reliability of water distribution networks as its ability to deliver the required quantity of water demand with a certain defined minimum pressure during specified period of time. Avi demonstrated that reliability is an integral part of all decisions regarding water distribution system layout, design, operation and maintenance. Avi highlighted that evaluation of reliability for water distribution systems is complicated task due to many factors that affecting reliability, inherent non-linear behavior of the system and its consumers and due to the different conflicting objectives facing a water distribution system utility. Muhammad and Juned [14] developed hydraulic reliability model to evaluate the water distribution network reliability. The authors used random generation numbers for pipe roughness, diameters and demands for both pipes and nodes. Certain probability distribution has assigned to each variable and Generic Expectation Function (GEF) as introduced by Tyagi and Haan [15] has been applied to calculate the system reliability for both nodes and whole system. Moneim *et al.* [16] have developed Linear Adaptive Genetic Algorithm (LAGA) model to define the optimum solution for water distribution networks. LAGA has been developed recently by Attia and Horacek [17] to solve optimization problem of complicated mathematical functions. Moneim [18] has developed the model RELOPT that is acting as a standalone reliability-based optimization model. RELOPT consists of four sub-models those are: pre-estimator optimization model, optimization model based on LAGA technique, reliability model based on load-resistance method and hydraulic model based on dynamic link library for EPANET [11] developed by Rossman. In this research the model RELOPT is applied to two existing water networks cases studies to demonstrate its power to define the optimum least cost or the reliability-based optimization design.

Theoretical Background

Objective Function: Simulation of hydraulic behavior of a pressurized pipe network is a complex task requires solving system of non-linear equations. The most effective way to define the optimum solution among specific number of population is to assign penalty term that represents weight factor for each individual solution during the automatic search process that is carried out by an optimization algorithm. In this research, Linear Adaptive Genetic Algorithm (LAGA) has been used as the optimization search engine. The objective function is a function in pipe diameter (d), pipe length (l) and penalty term (P) and takes the following form:

$$F = \sum_i^n f(d,l) + P \quad (1)$$

The objective function (F) subjects to the following constraints:

d and $l > 0$; system reliability, nodal reliability and residual pressure $>$ or $=$ specific defined values.

Optimization Technique: Average Gradient Method (AGM) has been developed and applied by Moneim [18] to reduce the search space that is used by LAGA to define the optimum solution. AGM is a method which defines the critical hydraulic grade along the water network by defining the junction of minimum residual pressure. AGM starts by assigning pipe diameters equal to the diameter of the main source pipe. Using a basic cost function for water network, trial and errors are applied until reaching to certain accuracy between the current and previous network cost. AGM then passes the obtained pipe diameters to LAGA which represents feasible solutions of population. LAGA is a form of Genetic Algorithms (GA) models depends on applying dynamic concept for GA' parameters those are cross over probability (P_c), mutation probability (P_m) and population (L). The use of variable parameters through GA achieves flexibility during the run and accelerates the search process by reducing the search-time. As the optimum solution tends to validate the pre-defined constraints such as reliability and residual pressure levels, LAGA updates the correspondence probabilities and population accordingly. This process is applied during each generation of the search process. The following equation describes how LAGA updates GA' parameters:

$$P_c = \frac{-0.5}{M-1} (g-1) + 1 \quad (2)$$

$$P_m = \frac{0.005}{M-1} (g-1) \quad (3)$$

$$L_m = L * 0.8 \quad (4)$$

LAGA in conjunction with AGM reduce the required number of generations and population which act effectively to reduce the total run time for optimization process.

Reliability Concept: Load-Resistance reliability concept is applied to calculate nodal and system reliability. The safety margin (SM) for any hydraulic structure is the difference between the resistance (R) and the load (W) which can be expressed by the following equation:

$$SM=R-W \quad (5)$$

As long as the safety margin SM is greater than zero, the system deems to be reliable. The failure probability (FP) can be expressed by equation (6) as follows:

$$FP=P(FP<0)=P(R-W)<0 \quad (6)$$

Due to uncertainty of the hydraulic behavior for both the resistance R and the load W within a water network, R and W should be considered as random variable and their probability distributions are essentially needed to develop a reliability model. Equation (6) can then be expressed as follows:

$$FP = \int_0^{\infty} f_R(R) \left[\int_0^R f_W(W) dW \right] dR = \int_0^{\infty} f_R(R) F_W(R) dR \quad (7)$$

Equation (7) represents how the failure probability can be calculated using the double integration method. Another solution method is to define the probability distribution for FP and the area under the probability curve from (0) to (∞) can be calculated which represents the failure probability. Subtracting the failure probability from 1 gives the system reliability RS.

Generic Expectation Functions: According to Tyagi and Haan [16], the following kth-central moment of Z where k is the moment order and μ is the mean value:

$$\mu_k = E[(Z - \mu_k)^k] = \sum_{s=0}^k (-1)^k \binom{k}{s} \mu_s^k \mu_{k-s} \quad (8)$$

Obtaining measures of Skewness and Kurtosis is essential to define the symmetry and peak for a specific

probability distribution relative to normal distribution. To obtain the higher order of expectations for a random variable Z (i.e. RS) has a relation to other two random variables (W1 and W2) of the same form of equation (5), Tyagi and Haan stated the following expectation formulas:

$$E[Z] = E[W1-W2] \quad (9)$$

$$E[Z^2] = E[W_1^2] - 2E[W_1]E[W_2] + E[W_2^2] \quad (10)$$

$$E[Z^3] = E[W_1^3] - 3E[W_1^2]E[W_2] + 3E[W_1]E[W_2^2] - E[W_2^3] \quad (11)$$

$$E[Z^4] = E[W_1^4] - 4E[W_1^3]E[W_2] + 6E[W_1^2]E[W_2^2] - 4E[W_1]E[W_2^3] + E[W_2^4] \quad (12)$$

Hydraulic Reliability: Reliability concept is applied to define the hydraulic reliability for pipe networks. Hydraulic reliability can be expressed by the flow capacity that a pipe could carry within a water network (i.e. resistance) to satisfy pre-defined water demand at the junction node that is served by this pipe (i.e. load). The resistance is expressed by the flow rate that a pipe could carry out through its cross sectional area with non-corrosional velocity (i.e. 2m/s). Equation (13) represents Hazen-William equation that is used to calculate the flow capacity (i.e. resistance) for a pipe under specific load conditions such as water demands (Q_j), hydraulic grade (S), pipe roughness (C_{HW}) and pipe diameter (d).

$$Q_p = 0.27842 C_{HW} d^{2.63} S^{0.54} \quad (13)$$

The load is expressed by the actual required flow that a pipe should carry to satisfy the nodal demand. Equation (14) represents the hydraulic load (Q_d) for a pipe that can be expressed as a percentage (D_p) of the total demand at specific junction crossing that pipe:

$$Q_d = D_p * Q_j \quad (14)$$

Once Q_p and Q_d have been defined, the hydraulic safety margine (SH) for the pipe flow capacity can be represented by equation of the same form of equation (5) as follows:

$$SH = Q_p - Q_d \quad (15)$$

The probability distribution for both Q_p and Q_d can be obtained by considering their variables in equations (13) and (14) as random variables. In this research MATLAB random functions have been used to generate historical

random values. Equation (15) can then be used to define the failure probability distribution for (SH = 0) from which, the hydraulic failure probability (FP_h) can be calculated according to equation (16) as follows:

$$FP_h = \frac{(0) - \mu}{\sigma} \quad (16)$$

Mechanical Reliability: Mechanical reliability can be expressed similarly as hydraulic reliability. The mechanical reliability safety margin (SM) is the difference between the number of real breaks and the expected number of breaks according to Muhammad and Juned [15]. The real number or the real rate of break could be represented using data analysis for historical records of the pipe network in concern. Equation (17) demonstrates application of the mechanical safety margin (SM) as follows:

$$SM = \frac{\ln(1 + R)F_n}{C_{n+1}} - N(t_0)e^{A(t-t_0)} \quad (17)$$

The first term on the right hand of equation (17) represents the critical break rate in terms of discount rate (R), replacement cost (Fn) and the repair cost (C) at year (n+1). The second term represents the expected rate of breaks in terms of breaks (N) at the base time (t_0), growth rate coefficient (A) and the targeted time t. Applying the same concept in equation (16) gives the mechanical failure probability (FP_m).

System Reliability: To obtain the system reliability for a water network components, the failure components need to be defined. One of available methods to define the system's failure component is the minimum cut-set method. The minimum cut-set is defined as a set of system component which when fails causes failure of the whole system. Once the failure components has been defined (set of pipes of total number n), the overall failure probability (FP) can be calculated by equation (18) as follows:

$$FP = \prod_{i=1}^n FP_{m_i} * FP_{h_i} \quad (18)$$

The system reliability (RS) then can be calculated using equation (19) as follows:

$$RS = 1 - FP \quad (19)$$

Hydraulic Availability: Hydraulic availability for a water network is expressed by the minimum residual pressure

that is available at its junction nodes to meet consumer needs. For each minimum cut-set (MC^i), the hydraulic availability (HA_{MC^i}) is the multiplication of the ratios between the actual and required residual pressure at each junction node. It should be noticed that in case the actual pressure is greater than the required, the ration should be set to 1. Equation (20) represents the system reliability in terms of hydraulic/mechanical reliability and hydraulic availability as follows:

$$RS = 1 - FP = 1 - \sum_{i=1}^n [(1 - HA_{MC^i}) * FP_{MC^i}] \quad (20)$$

Main Model: The main model RELOPT is a Meta-Heuristic reliability-based optimization model utilizes Linear Adaptive Genetic Algorithm (LAGA) as an optimization search engine and the Load-Resistance concept for reliability evaluation. RELOPT has been consists of four linked models those are: Hydraulic solver EPANET; Pre-estimation model based on Average Gradient Method (AGM); OPTimization model for Water NETwork (OPTWNET); RELiability model for Water NETwork (RELWNET). RELOPT is coded in MATABL language and plays a managing role between the different models by calling, retrieving and passing inputs/outputs of the sub-models EPANET, AGM, OPTWNET and RELWNET. RELOPT methodology depends on registering the water network within EPANET environment and developing input file for the entire network that would be utilized by RELOPT.

Model Applications: The model RELOPT could be utilized for three cases of applications those are; aquainting optimum design least cost for new water network; aquainting reliability-based optimization for new water network reliability; or reliability evaluation for an existing water network. At the start of each run, the model RELOPT requests user defined input parameter either 0, 1 or 2 which correspondes to only one application as stated above. In this paper two existing cases studies have been selected to demonstrate the use of RELOPT.

CASE STUDY (1): CITY OF 15th. MAY: The City of 15th May is located to the west of Cairo city and accommodates for 102,000 capita. The total length of the water distribution network is around 52 kms and consists of 115 pipe links, 82 junctions, 1 ground reservoir, 1 pumping station and 2 elevated water tanks. The total population demand that is served by the City water

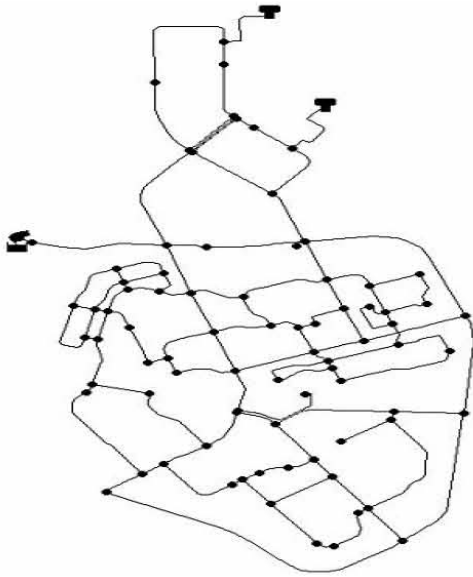


Fig. 1: City of 15th. May Water Network

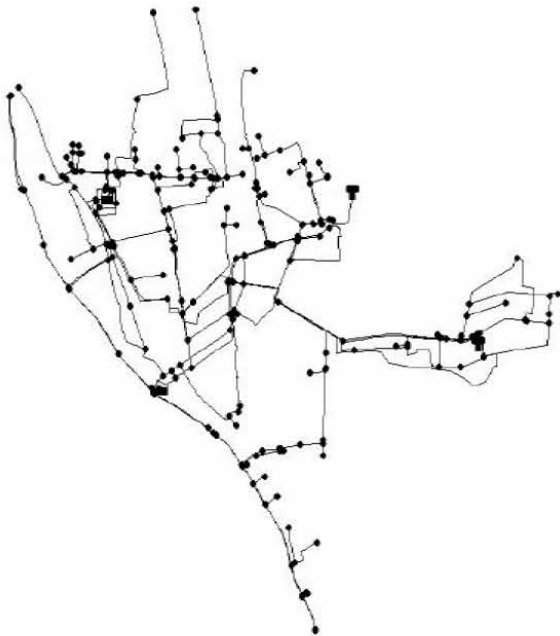


Fig. 2: Maadii City Water Network

network is around 65,000 cubic meters per day. Figure 1 illustrates the layout of the water network. Reservoir head is 78.5m, tanks elevations are 171 and 187 with 3 m water depth. Pump duty point is 832 l/s and 120 m head. The cost function shall be according to equation (21).

$$\text{Minimize}(f) = \sum_{i=1}^n 1.1 * d_i^{2.4} * l_i \quad (21)$$

Where: d_i is the pipe diameter (inches), l_i is the pipe length (feet).

The following constraints are applied to the problem:

- Pressure for all demand nodes should be greater than or equal (27.57 m);
- Minimum system reliability should be greater than or equal (0.8);
- Minimum nodal reliability should be greater than or equal (0.95);
- All pipe diameters should be greater than 0.

The mechanical failure probability for pipe networks is expressed using a regression equation obtained using failure data from the City of St. Louis (1985) according to Su *et al.* [9]. The equation computes the average break rate μ (break/mile/year) as follows:

$$\mu = 0.819 * e^{-0.1363d} \quad (22)$$

Case Study (2): Maadii City: Maadii city is located to west of Cairo city and accommodates for 587,000 capita. The total length of the water distribution network is around 152 km and consists of 306 pipe links, 277 junctions, 2 ground reservoir, 1 pumping station and 2 elevated water tanks. The total demand that is served by this water network is 295,971m³/day. Figure 2 illustrates the layout of the water network.

- Reservoir heads are 97 and 77m, tanks elevations are 130 and 55 with 5m water depth.
- Pump duty point is 111 l/s and 85 m head.
- The cost function is based on equation (21).

Where:

d_i = pipe diameter in inches for the pipe number i

l_i = pipe length in meters and n is the total number of pipes in the network.

RESULTS

Results of Case Study (1): The model RELOPT has been run to define the optimum design-based cost and Reliability-based optimization design. Ten population and Two generations have been selected among the feasible solutions specified by the Pre-estimator model AGM. Only 18 seconds were required to define the optimum design-based cost that is 762,900 while 1801 seconds needed to define the Reliability-based optimization cost that is 1,343,000. The relation between the obtained residual pressure and reliability versus optimization cost are demonstrated in Figures 3 and 4 respectively.

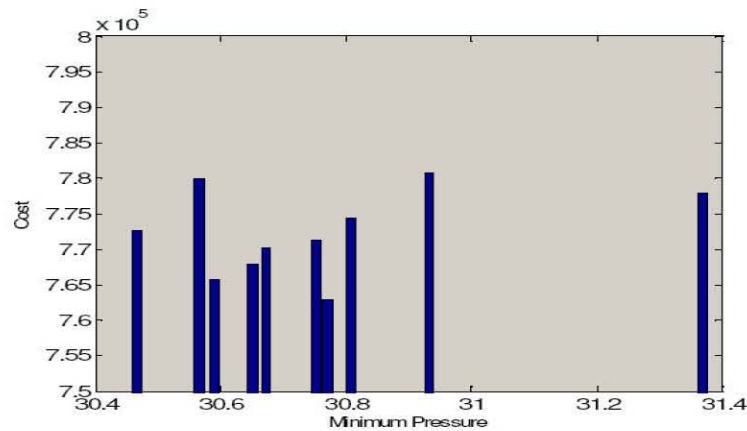


Fig. 3: Optimization cost versus residual pressure for City of 15th. May Water Network

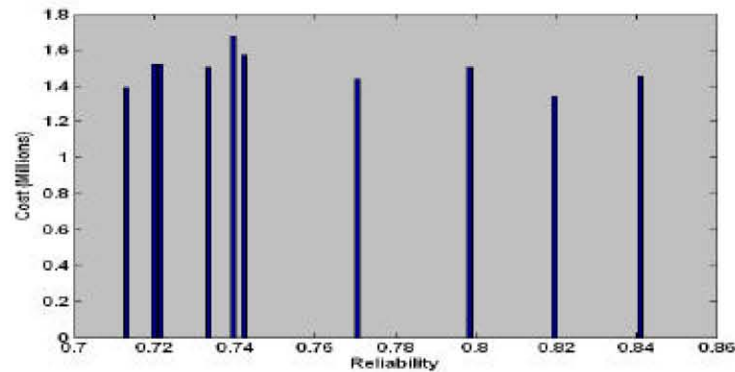


Fig. 4: Optimization cost versus reliability for City of 15th. May Water Network

- The optimum reliability-based optimization solution is represented by the population index (9) as illustrated in Figure (4) and its optimum cost has found 1,343,400 with reliability 0.8195 while the optimum design least cost 762,900.
- Minimum cut-sets of the network are represented by only (14) pipes out of (115) which represent almost 12% of the total number of pipes.
- Minimum nodal reliability is 0.9413.
- Minimum residual pressure is 32.42 m.
- It should be noticed that to ensure 82% reliability based cost design for the City of 15th. May, the optimum design based cost has been increased by 72%.
- The best solution is represented by the population index 59 (see Figure 6)
- Optimum cost for the optimum design based cost is 6,531,000 of unit cost while the cost of the original design is 10,258,000 using the future water demands.
- The minimum residual pressure is 30.25 m.
- Figure (5) demonstrates the minimum pressure versus 100 population's indices.
- Figure (6) demonstrates the optimization cost versus 100 population's indices.

The model RELOPT has been run to obtain the reliability-based optimization solution for water distribution networks and results are listed as follows:

Results of Case Study (2): The model RELOPT has been run to obtain the optimal design cost and results have generated in 202.14 seconds (i.e. 3.3 minutes) using 100 populations and 2 generations. Summary of the results are listed below:

- 10 population and 1 generation have been used to reduce the run time.
- Run time was 2891.57 seconds (i.e. 48.18 minutes).
- The optimum reliability-based optimization solution has found costing 23,512,000 with reliability 0.7832.

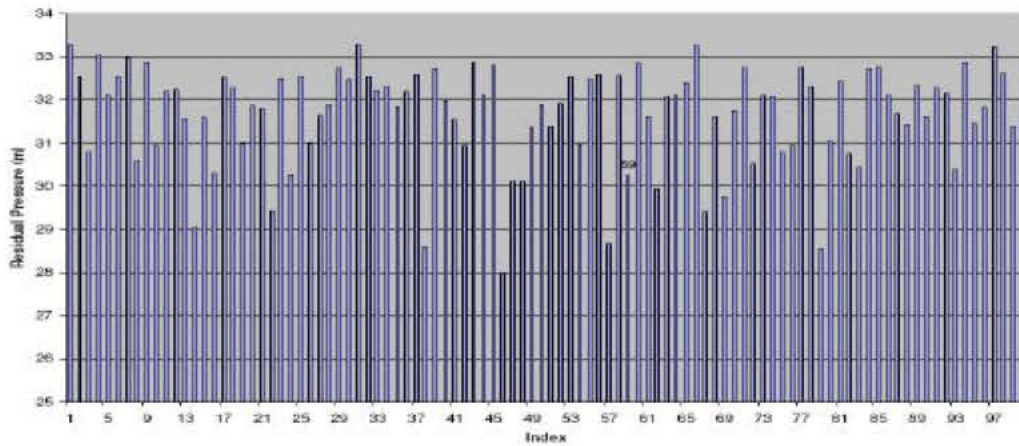


Fig. 5: Population versus obtained residual pressure for Maadii City

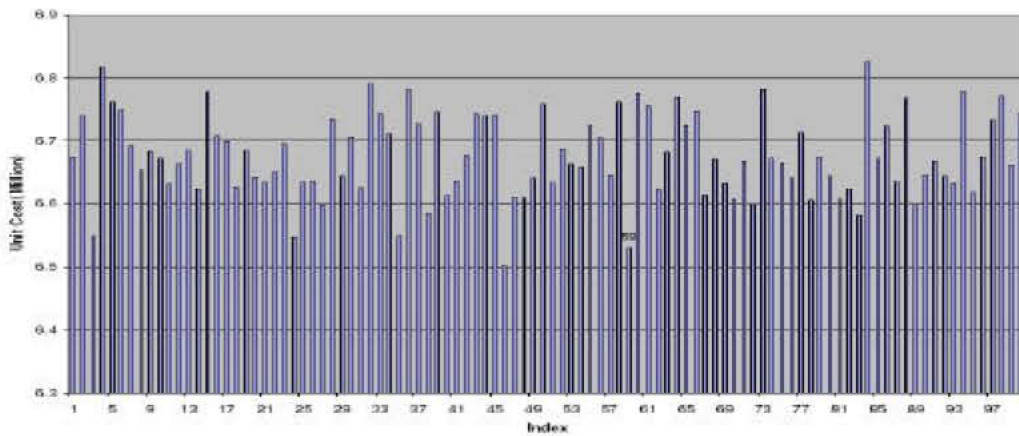


Fig. 6: Population versus obtained optimization cost for Maadii City

- Cut-sets for Maadii water network consist of (137) pipes out of (306) which represent 45%. The fact behind this high percentage is due to Maadii City's water network is a tree system compared to the City of 15th. May's water network.
- It should be noticed that to ensure 78% reliability based cost design for Maadii City, the optimum design based cost should be increased by almost 300%. This fact due to the nature of the water network of Maadii city which is tree system comparing to City of 15th. May network which is closed loop.

DISCUSSION

In this research a new reliability-based optimization model RELOPT has been applied using 2 cases studies

of existing water networks in Cairo, Egypt to demonstrate the ability and the power of the model to handle complex water networks. RELOPT can be used as standalone optimization search model to define least cost design through 3 minutes for a complex water network consists of 306 pipes and 277 as illustrated in case study number 2. In the mean time, RELOPT can be used as a reliability-based optimization model to evaluate an existing water network or design a new reliable and optimized water network through approximately 50 minutes. Another advantage of the of the model RELOPT is that it uses a pre-estimation model which define specific number of feasible solutions act as population basket for the optimization search engine. This process reduces the search space that optimizes the run time required for optimization search process.

CONCLUSION

The model RELOPT has demonstrated its power to undertake the optimization and reliability evaluation problems for water distribution networks as a standalone model. RELOPT provides high efficient reliability-based optimization model that deals with large water networks through an optimum run time comparing to other available software in the market.

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