A Suggested Simple Model for Evaluating Maximum Crosswind Concentration of Air Pollutants

Khaled S.M. Essa, Fawzia Mubarak and Asmaa Abo Bakr

Abstract: A suggested simple mathematical model describing the crosswind-integrated concentrations is presented for dispersion of pollutants in the atmospheric boundary layer. The mathematical analysis is based on Gaussian plume model after parameterization of meteorological conditions. Maximum crosswind concentration was determined and explicit approximate expression is provided for it, allowing an analytically simple expression for its position and value. The predictive concentration due to the proposed model is evaluated with the observations obtained from Copenhagen diffusion experiments in unstable and neutral conditions. Predicted values of crosswind concentration by the proposed model and that computed by Gaussian formula are compared with the observed data. It is concluded that the present model is performing well with the observations than Gaussian formula and can be used to predict the short-range dispersion from a continuous release.

Key words: Crosswind concentration · Gaussian model · Long-range dispersion · Continuous source

INTRODUCTION

Air pollution is the top environmental cause of premature death in the world [1]. It is also, increases incidence of a wide range of diseases with both long- and short-term health effects. WHO’s International Agency for Research on Cancer (IARC) concluded in 2013 that outdoor air pollution is carcinogenic to humans. In addition to the impacts on human health, air pollution also has several environmental impacts affecting the quality of water, soil and the ecosystem services they support. These effects have considerable economic impacts. Radioactive pollutants are one of the most dangerous pollutants. The concentrations at which air pollutants reach the population depend on the degree of dispersion of the pollutants in air [2]. Modeling is necessary to imagine the dispersion scenarios of pollutants. The purpose of air dispersion modeling is to predict pollutant concentrations resulting from a source or group of sources under various meteorological conditions. These models are especially useful to engineers and environmental scientists who study pollutant transport, since they allow parameter sensitivity and source estimation studies to be performed [3]. In order to evaluate such scenarios one needs efficient procedures, which yield immediate results, for instance evaluating the ground level concentration of pollutants and especially the maximum concentration and its position [4]. From the operational point of view, in estimating of crosswind concentration there are used models with semi-empirical or analytical approach (ex. Gaussian plume or puff) that require as input meteorological data coming from local surface measurements [5-7]. The crosswind integrated concentration, $C_\tau$, is a function of the release height, plume rise, vertical dispersion and the reflection with the surface and any interactions at the top of the convective mixed layer [8]. The classical approach based on conventional models, such as the Gaussian puff/plume or the K-theory with suitable assumptions, are known to work reasonably well during most meteorological regimes, except for weak and variable wind conditions[9]. This is because (i) down-wind diffusion is neglected with respect to advection, (ii) the concentration is inversely proportional to wind speed, (iii) the average conditions are stationary and (iv) there is a lack of appropriate
estimates of dispersion parameters in low wind condition [10]. In practice, most of the estimates of dispersion are based on the Gaussian plume model, which assumes the constant wind speed and turbulent eddies with height, while a non-Gaussian model assumes the non constant wind speed and turbulence with height [11]. The Gaussian plume model is an atmospheric dispersion model that most widely used for estimating airborne radionuclide exposures within 10 km of the release point [12].

The primary objective of this study is to develop and evaluate a simple analytical model describing the crosswind integrated concentrations, for dispersion of air pollutants released from a continuous source in the atmospheric boundary layer.

**Theoretical Aspects**

**Gaussian Plume Model:** The Gaussian plume model is used in more regulatory application than any other model. It has been used by most of the countries participating in the NATO plume modeling in Federal Republic of Germany. It is widely used and has been evaluated with many data sets. For a continuous point source, the basis of the model is a single simple formula which assumes constant wind speed “u” and complete reflection from the ground surface [2, 13]. The concentration associated from point source of strength Q, is expressed as:

\[
C = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp \left( \frac{-y^2}{2\sigma_y^2} \right) \exp \left( \frac{-H^2}{2\sigma_z^2} \right) + \exp \left( \frac{-(z-H)^2}{2\sigma_z^2} \right)
\]  (1)

where the parameters are defined by the following descriptions:

- \( C \ (g \ m^{-3}) \) = Concentration of air pollutant;
- \( Q \ (g \ s^{-1}) \) = Continuous point source strength;
- \( u \ (m \ s^{-1}) \) = Wind speed at height H;
- \( \sigma_y(m) \) = Lateral dispersion parameter;
- \( \sigma_z(m) \) = Vertical dispersion parameter;
- \( x(m) \) = Horizontal distance in the direction of downwind;
- \( y(m) \) = Lateral distance from plume centerline,
- \( z(m) \) = Height above ground;
- \( H(m) \) = effective height of plume above ground;
- \( H=h+\Delta h\); where his the stack height and \( \Delta h \) is the plume rise equals 3(wD/u); D is the internal stack diameter (m) and w is the exit velocity of the pollutants (m/s) [14 & 15].

Although the downwind distance x does not appear explicitly in the equation, it is implicitly included through \( \alpha_x \), \( \alpha_y \), which are function of x as well as the Pasquill atmospheric stability categories and can be calculated from the formula [16]:

\[
\alpha_y=ax, \alpha_z= cx^d
\]  (2)

where a, b, c, d are constants values depending on stability classes (Table 1) [17].

By substituting from (2) in (4) one gets:

\[
C(x, y, O, H) = \frac{Q}{\pi u ac x^{b+d}} \exp \left( \frac{-y^2}{2a^2x^{2b}} \right) \exp \left( \frac{H^2}{2c^2z^{2d}} \right)
\]  (5)

Table 1: Constants for calculating lateral (\( \alpha_y \)) and vertical dispersion parameter (\( \alpha_z \)).

<table>
<thead>
<tr>
<th>Stability</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very unstable</td>
<td>0.4</td>
<td>0.91</td>
<td>0.41</td>
<td>0.91</td>
</tr>
<tr>
<td>Unstable</td>
<td>0.36</td>
<td>0.86</td>
<td>0.33</td>
<td>0.86</td>
</tr>
<tr>
<td>Neutral</td>
<td>0.32</td>
<td>0.78</td>
<td>0.22</td>
<td>0.78</td>
</tr>
<tr>
<td>Stable</td>
<td>0.31</td>
<td>0.71</td>
<td>0.06</td>
<td>0.71</td>
</tr>
</tbody>
</table>

At ground level z=0, hence:

\[
C(x, y, O, H) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp \left( \frac{-y^2}{2\sigma_y^2} \right) \exp \left( \frac{-H^2}{2\sigma_z^2} \right)
\]  (4)

From the power law, by substituting from (2) in (4) one gets:

\[
C(x, y, O, H) = \frac{Q}{\pi u ac x^{b+d}} \exp \left( \frac{-y^2}{2a^2x^{2b}} \right) \exp \left( \frac{H^2}{2c^2z^{2d}} \right)
\]  (5)
To get on the crosswind integrating concentration, equation (5) is integrated with respect to (y) as follow:

\[ C_y (x, H) = \frac{Q}{\pi u acx^{b+d}} \left[ \int_{-\infty}^{\infty} e^{-\frac{y^2}{2a^2x^2}} dy \left[ \exp \left( \frac{H^2}{2c^2x^{2d}} \right) \right] \right] \]

(6)

\[ C_y (x, H) = \frac{Q}{\pi u acx^{b+d}} \left[ \frac{\sqrt{\pi}}{2a^2x^{2d}} \left[ -\frac{H^2}{e^{2c^2x^{2d}}} \right] \right] \]

(7)

\[ = \frac{Q}{2\pi uacx^{b+d}} e^{-\frac{H^2}{2c^2x^{2d}}} \]

(8)

The maximum crosswind concentration value occurs when \( \Delta C_y/\Delta x = 0 \), then

\[ \frac{\partial C_y}{\partial x} = \frac{(-3b - d)Q}{2\sqrt{\pi}uacx^{3b+d+1}} e^{-\frac{H^2}{2c^2x^{2d}}} + \frac{Q}{2\sqrt{\pi}uacx^{b+d}} e^{-\frac{H^2}{2c^2x^{2d}}} \left( -\frac{2H^2}{2c^2x^{2d}} \frac{\partial H}{\partial x} + \frac{2H^2d}{2c^2x^{2d+1}} \right) \]

(9)

when \( \frac{\partial C_y}{\partial x} = 0 \)

\[ -(3 + b + d) + \frac{H^2d}{c^2x^{2d-1}} - \frac{H}{c^2x^{2d-1}} \frac{\partial H}{\partial x} = 0 \]

(10)

Multiplying by \( C^2x^{2d-1} \)

\[ -(3 + b + d)c^2x^{2d-1} + \frac{H^2d}{x} - H \frac{\partial H}{\partial x} = 0 \]

(11)

Let \( H = (v(x)x)^d \)

\[ -(3d + d)c^2x^{2d-1} + d(v(x)x)^d - d(v(x)x)^d (v(x)x)^d \left( v + x \frac{\partial v}{\partial x} \right) = 0 \]

(12)

\[ -(3b + d)c^2x^{2d-1} + dv^2x^{2d-1} - dv^2x^{2d-1} - dv^2x^{2d-1} x^{2d-1} \frac{\partial v}{\partial x} = 0 \]

(13)

\[ -x + \frac{d}{c^2}v^{2d-1}v^{2d-1} \frac{\partial v}{\partial x} = -3b + d \]

(14)
\[ \therefore v^{2d-1}dv = -\frac{c^2}{d} = -(3b + d) \frac{dx}{x} \]  

(15) \[ \therefore x^{2d} = \frac{H^2 d}{c^2 (3b + d)} \]  

(26) \[ \therefore x_{\text{max}} = \left( \frac{dH^2}{c^2 (3b + d)} \right)^{1/2d} \]  

(27) \[ \text{Then, the point of maximum ground level concentration line on x-direction is,} \]  

\[ (x_{\text{max}}, y_{\text{max}}) = \left( \frac{dH^2}{c^2 (3b + d)} \right)^{1/2d}, 0 \]  

(28) \[ \text{Substituting from (28) in (8) one gets,} \]  

\[ (c_y)_{\text{max}} = \frac{Q}{2\sqrt{\pi} ua^3 c x^{3b+d}} e^{-\frac{-H^2 c(3b+d)}{2c^2 H^d}} \]  

(29) \[ (c_y)_{\text{max}} = \frac{Q}{2\sqrt{\pi} ua^3 c x^{3b+d}} e^{-\frac{(3b+d)}{2d}} \]  

(30) \[ \text{Parameterization: In atmospheric diffusion modeling, the turbulent parameterization represents a fundamental aspect of the contaminant dispersion. The reliability of each model strongly depends on the way turbulent parameters are calculated [18]. The choice of the turbulent parameterization is set to account for the dynamic processes occurring in the ABL. In this study, a simple vertical profiles of wind and eddy diffusivity for unstable and neutral conditions were calculated according to equation (31). Normally U(z) is parameterized in the dispersion model using an empirical formulation in term of power law function of z [19].} \]  

\[ \frac{U(z)}{U_1} = \left( \frac{z}{z_i} \right)^{\alpha} \]  

(31) \[ \frac{z}{z_i} \]  

where \( u_i \) is the mean wind velocity at the height \( z_i \), while \( \alpha \) is an exponent related to the turbulence intensity[4]. Dispersion parameters \( \alpha \& \sigma_z \) are parameterized using equation (2) and Table (1). The minimum distances \( x_o \) was 8-13 m for unstable case and 15-22 m in neutral case.

**RESULTS AND DISCUSSION**

Turbulence parameterizations are shown in Table (2). Crosswind concentrations were calculated using the suggested model and Gaussian model as shown in Table (3).
The predictive concentration due to the present model was evaluated with Gaussian model and the observation obtained from the Copenhagen (in Denmark) diffusion experiment in unstable and neutral conditions [20, 21] as shown in Fig. (1). It can be deduced that the suggested model is able to more accurately reproduce the concentrations at farther distances from the source (lower concentrations), where the plume penetrates the free convection layer, rapidly decreasing the concentration [18].

**CONCLUSION**

On the basis of a mathematical analysis, the appropriate functional forms of effective height and maximum ground level concentration were derived to get the maximum crosswind concentration. A special case of this model for constant effective height is deduced. The present model was evaluated with the observation obtained from the Copenhagen diffusion experiment in unstable and neutral conditions and it performs well with the observation.
In addition, it can be deduced that the suggested Convective Boundary Layers, Atmospheric model is able to more accurately reproduce the concentrations at farther distances from the source (lower concentrations), where the plume penetrates the free convection layer, rapidly decreasing the concentration.

Generally, the present model can be used for predicting the value and position of the crosswind integrated normalized concentration of pollutants released from a continuous source, which is very useful for environmental and health impacts assessment.

REFERENCES

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