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Computer Algebra Systems as the Mathematics Teaching Tool

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Abstract: In this study, feasibility of computer algebra systems, which are more commonly used in science and engineering fields especially in mathematics and physics, is examined in terms of teaching mathematics. Primarily common features of general purpose computer algebra systems and the symbolic computation theory forming the basis of these systems are briefly mentioned. Under the light of various researches stressing the importance of computers in the education process of mathematics, as well as computer algebra systems, various software whose main function was not teaching mathematics was evaluated according to their usage in terms of teaching mathematics and as a result, past usages of computer algebra systems in the field of mathematics teaching, their present situation and their future positions were discussed.

Key words: Computer aided mathematics education . mathematics software . computer algebra system . symbolic computation

INTRODUCTION

Even though they have a mathematical theoretical infrastructure, computers were used only to solve problems faced in areas which were suitable to mathematical modeling such as in engineering, science and economy via numerical computation methods. Start of computers' usage as an educational material efficiently, when we consider the time scale of the developments in science, coincides a quite recent time. The only reason of that is not the improvements in hardware of computers or new approaches emerged in the theories that form the basic logic of the computers because computers are machines that are built according to binary digit system developed by George Boole in 1854 and consisting of binary digits 0 and 1, that are designed based on the algebraic structure called Boolean Algebra and current computers operate under basically same principles. But the biggest advantage of the computers in our age is that they have been able to operate more sophisticated and complex software thanks to the innovations in microprocessor technologies as a result of the recent developments in related disciplines. All these developments, surely, have led some new approaches in mathematics programs to appear.

It is definite that one of these approaches is symbolic computation theory which is developed against the hypothesis which argues that the computers can only be used to operate with numbers. The aim in this theory is to evolve computers in a way that it can operate with symbols as well as numbers as a human being endeavoring with mathematics will also operate with symbols. As a result, the more success is obtained, the more "human-like" attitude the computer, which is dealing with the solution of a mathematical problem, will display. It is crucial for a computer, whose aim is to do math or teach math, to be in a human-style approach by means of teaching math in a computer-aided way in every level. Otherwise, the computer may only be used as a demonstration tool and this situation will render computer inefficient in terms of human-computer interaction. In other words, the interaction's being mutual will be an argumentative fact. The one that is learning will expect from the machine against him to understand the responses he gave and to guide him towards the learning level; nevertheless, it is impossible to realize this by classical style software.

COMPUTER ALGEBRA SYSTEMS

The computations used in numerical methods, as well as basic arithmetic operations, include more complex operations such as computing numerical values of mathematical functions, finding roots of polynomials, solving initial-value and boundary-value problems, numerical integration and computing eigenvalues and eigenvectors of matrices. All of these

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operations have one thing in common: numbers. Computations are just realized over numbers. Moreover, these calculations are mostly "uncertain" since the data include mostly floating-point numbers and the operations will bring a portion of error in direct proportion with the increase in number of steps. Meanwhile, thanks to the error estimation formulas obtained as a result of error analysis, keeping errors within acceptable limits becomes possible.

Another research field of mathematical computation includes methods which are called as symbolic and algebraic computation or computer algebra and briefly defined as "making operations over symbols used in displaying mathematical objects". These symbols may be the ones that show numbers as integers, rational numbers, real numbers or complex numbers; algebraic objects such as polynomials, rational functions, equation systems or that show more concrete algebraic structures such as groups, rings or fields [1].

The word "symbolic" stresses that the last point to reach in mathematics is usually in a closed and symbolic formula. In other words, the result to be reached should be expressed analytically. By the word "algebraic", it is meant that the computations are based on absolute result steps rather than floating-point arithmetic. For instance, the symbol $\sqrt{2}$ shows the irrational number of 1,4142135623730... whose decimal part is infinite. But one can multiply it with 2 without using its decimal value; consequently, $2\sqrt{2}$, which is also an irrational number may be obtained and shows the number of 2,82842712474619.... As it is seen, a computation was made here without using any numerical values, directly by symbols.

The biggest problem of symbolic computation is designing algorithms or optimizing present algorithms that operates on symbols and that will realize operations such as computation, decision making, analyzing. The input data for these algorithms are the data called as string (character arrays) in programming languages. Therefore, to evolve these data into the "information" that can be given to symbolic algorithm, first they have to be brought about into a meaningful mathematical structure. The data brought about into meaningful structures will be input data for the computer algebra systems. Computer algebra systems generally are developed in a way that they will also realize the process of evolving the data into input data. In fact, the function of the computer algebra systems starts with that process. The process after this point, also depending on the features of the computer algebra system in use, will be directed by user's interaction with computer.

COMPUTER-AIDED MATHEMATICS TEACHING

The role of computers in mathematics teaching, in 90s, has increased after the studies that emphasize importance of visuality in mathematics teaching. One of the studies which played a key role in this matter was performed by Dörfler [2]. Dörfler, who said that the symbolic and algebraic structures could easily be evolved into geometric structures by computers, stressed that visual fields could not substitute for formal definitions but they could greatly increase their "intelligibility".

An important study in favor of this is the one that was made by Dreyfus [3] and which emphasized computer as a cognitive tool. Dreyfus made it clear in a striking way using fractal geometry that theoretical knowledge could be visualized with computers, thus the existing knowledge could promote to "learned knowledge" from "memorized knowledge". In addition, visualizing iterative functions is almost impossible in the absence of computer.

The usability and reliability of computer algebra systems which is one of the most important phases of computer-aided mathematics has been at the top of the most argued issues in this field since the early years of 90s. The concept of usability is a concept that was brought forward in relation with the content of assistance documents and its being easily learned and it has been used as an important scale in terms of determining "how good" software or a web site is. When we examine computer algebra systems from this angle, we observe that their cards are not so bright. Except that the MathCAD which is more user-friendly than the others, computer algebra systems have not heeded this feature much. Maple and Mathematica, which are two leading computer algebra systems, insisted on an interface that compels their users to master at least in one programming language to use the program for long. However, they changed their conception a little bit in their new versions and they have evolved their program in a way that they can be used by a larger user group.

It can be accepted as natural for these software, which aim scientists and engineers in the beginning as the traditional user group, not to heed being used easily and by everyone. Because these systems, until recently, were being used by the users who already had a certain knowledge and skill in numerical analysis, that is, in a programming language such as Basic, Fortran, Pascal or C as of the user groups they address. Nevertheless, computer-aided mathematics education, whose popularity has been increasing recently, rendered the usage of these systems in education inevitable. For this reason, almost all computer algebra systems added "education" category to their web pages and developed application libraries and components in cooperation with scientists both by using the way of sharing with their users and by benefiting from their own software teams. Most of them are provided to the users free of charge in a way that it is now possible to coincide worksheets and activities in secondary education, even in elementary education which we were not used to see in the web pages of these software.

Kajler and Soiffer [4], in their studies in which they examined Maple, Mathematica, Macsyma and MathCAD software in terms of "human-computer interaction", they made a lot of statements related to the feasibility of computer algebra systems in teaching. According to Kajler and Soiffer, the most important pedagogic advantages of these systems are that they can give the outputs very close to textbook-style, that they prefer symbolic illustrations of rational numbers or irrational numbers such as π and e.

However, they couldn't achieve the success in graphics output that they achieved in mathematical statements of computer algebra systems as soon as it was expected. Schoenfeld [5], in his study dealing with this subject draw attention to the fact that it may have some results such as confusing students or leading them to misconceptions. Crowe and Zand [6] gave the drawing of two parabolas in which one was the translation of the other in two units and which resulted like in Fig. 1 as an example to all these problems such as dependence to features of resolution, size etc., misscaling:

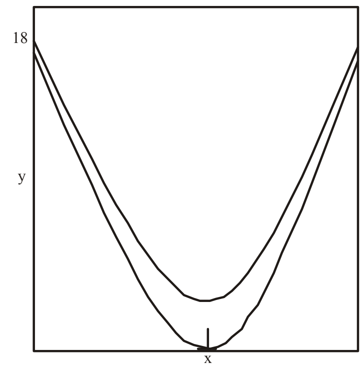

Fig. 1: f(x) and f(x)+2 parabolas

Even though the difference between the f(x) and f(x)+2 parabolas are 2 units, it doesn't seem so because of the scaling problem in the output graphic.

This problem which has been especially asserted in recent years has been overcome by many computer algebra systems in a certain measure. For example, the sphere drawings realized by using the implicitploted function in plots component of Maple looks more like an ellipsoid as the scaling was made by taking the rectangular structure of the screen as basis until the ninth version of the software. The sphere drawing in Fig. 2 was formed by using Maple 10 and it may be accepted as a quite good graphics output in terms of scaling:

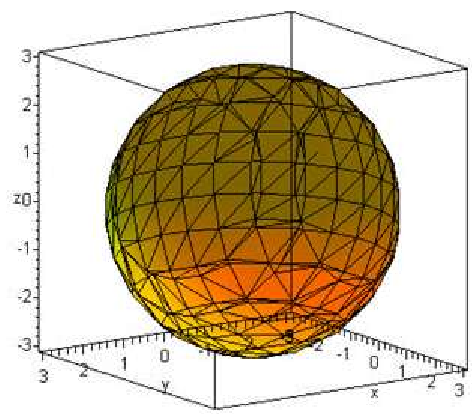

Fig. 2: Maple 10's sphere

Crowe and Zand, in the first [6] of their series of articles they published pertaining to the role of computers in university level mathematics teaching, to trigger a discussion about the issue, they made a classification (Fig. 3) consisting of mathematics software, information tools and external software for computer-aided mathematics teaching.

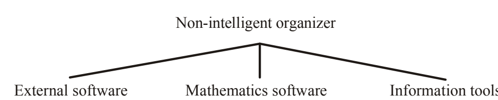

Fig. 3: Crowe and Zand's classification of the non-intelligent organiser

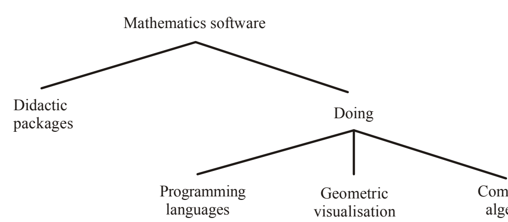

Fig. 4: Crowe and Zand's classification of the mathematics software

Crowe and Zand in their same studies, in accordance with the ideas of Tall [7] which clarifies the distinction even more between the didactic software and mathematics software, they categorized (Fig. 4) mathematics software as "doing" or "didactic packages" which render the user passive. By "the other

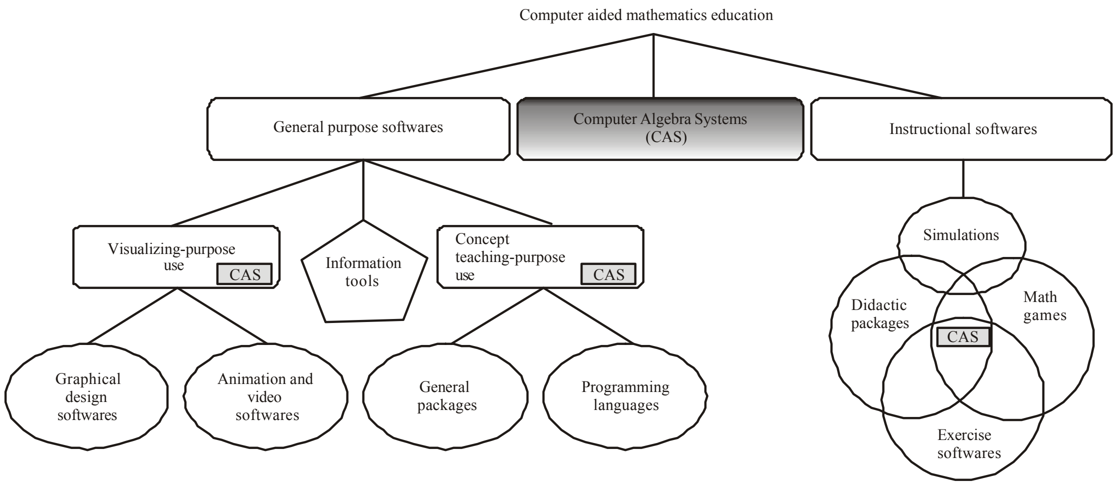

Fig. 5: A general taxonomy of the computer aided mathematics education

software" the feasibility of mostly general-purpose software, whose aim was not teaching mathematics, was explained. As an example to this, Sutherland's [8] using Excel, an electronic tabulation program, to teach his students the fundamentals of algebra may be given. Sutherland, in his same study, made use of the software Logo to teach the "variable" concept in mathematics; however, this software is in the category of "mathematics software" of the classification.

Departing from these approaches and ideas, classification of computer-aided mathematics education and the place of computer algebra systems within this classification may be made as it was made in Fig. 5. In the first level of this, computer algebra systems', which took their place in three main categories, visual purpose usages was the usage type that started the increase of popularity in the field of education. These software, at the same time, may be used in teaching formal definitions of mathematical concepts. In addition to this, didactic packages, mathematical computer games and exercise software intersect with computer algebra systems in a common intersection. Once the instructional applications of computer algebra systems are examined, one can coincide to examples that fit in all three of these categories.

The electronic tabulation software, which we can put into general packages group, thanks to their formula features, can efficiently be used in teaching basic concepts such as decimal rounding or proving basic algebraic equities, or in showing the relation between the formal definitions of concepts and their visual components. Applications which enable drawing graphics based on data and which is also one of the general features of these software will be an efficient tool in teaching basic function graphics. This education will be beneficial for students in two different ways. Firstly, a more motivating medium will have been provided for students to learn basic graphics. The second benefit will be obtained thanks to the development of "drawing graphics of function" activity by the computer in its most primitive form. Electronic tabulation software, in a certain interval, computes the y values, which correspond to x values that take value with a certain number of steps, using the definition of the function and than turn these values into graphic utilizing the graphic application in its body. Student's consciousness of how the process is fulfilled will be very beneficial for the student to establish the logical relation between the function and its graphic. The same operation may also be realized using a programming language, however, the student is expected to master in at least one computer programming language. Nevertheless, to realize the aforementioned activity, having a general knowledge in any electronic tabulation software will be enough.

An activity realized by the popular electronic tabulation software, Microsoft Excel, was given step by step. The graphic of sinus function which is among the basic trigonometric functions is drawn using the "line graphics" feature of the software.

Step 1: The x values forming the definition set determined for the function are entered into a column (Fig. 6, A column).

Step 2: Values in degrees are turned into radian just as the sinus function of the software requires, thus the students are made to see the radian values which correspond to degrees. The new values obtained are formed as a new data column (Fig. 6, B column).

Step 3: The sinus formula is applied to the cells in the columns that consist of radian values and the results obtained forms a third column (Fig. 6, C column).

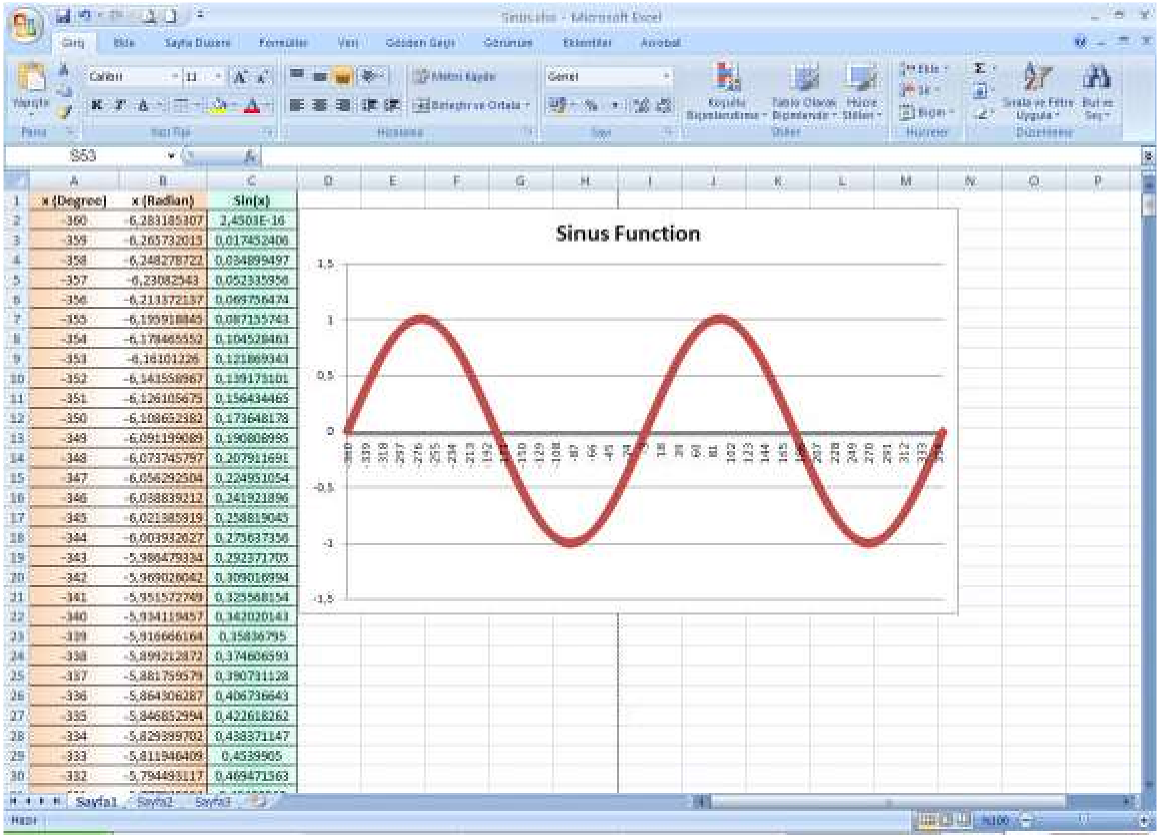

Fig. 6: An excel application for the graphics of the functions

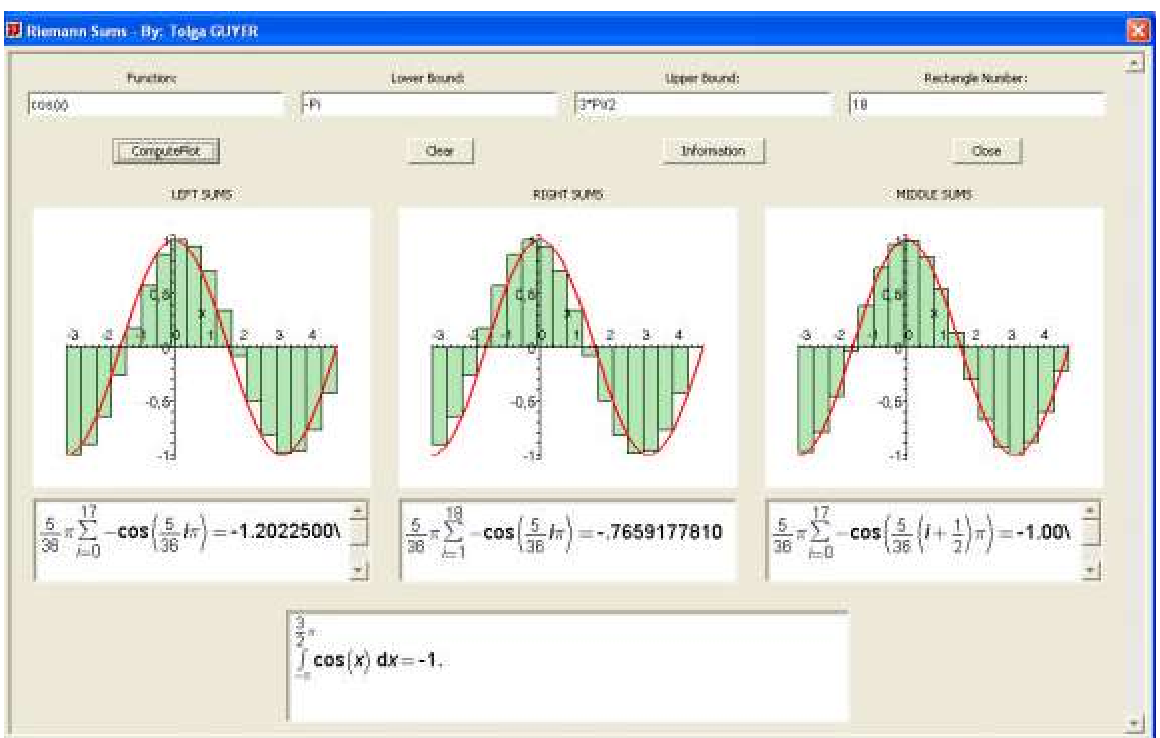

Fig. 7: The maplet of the Riemann sums

Step 4: A graphic within the "line graphics" category is drawn in the graphic drawing application of Excel using the f(x) values obtained (Fig. 6, C column).

The aim of the mathematics games within the category of educational software is carrying the student to aimed level in the handled subject as well as entertaining the student. For this reason, educational games, with assistance of artificial intelligence, may evaluate student by observing him/her during the game and direct the game in accordance with the student's lack in certain subjects, private to the student.

Information tools include mostly internet-based technologies. Web sites whose content is mathematics education, mathematical dictionary software, knowledge sharing mediums on the web such as "Ask Dr.Math" or the electronic books published on internet are within this concept.

We can assume computer algebra systems' educational purpose usage in two different ways. First of them are "tutors" that are developed using software and that don't require many skills to use the software. An important part of the tutors consist of special interfaces prepared for users. This user-friendly interface will incite the student to focus on the subject rather than dealing with the complicated system of the software, hence the software continuing to perform at the background will realize all these complicated operations, results and input will be made via this private interface.

On the other hand, direct usage of the software will surely provide a larger horizon. However, it will be inevitable that the user should have some knowledge in some subjects. These may be listed as an advanced computer usage skill, geometrics knowledge for geometric visualizations, being accustomed to the input and output formation of the computer algebra system in question. If it is to be handled in a more advanced way, basic knowledge level in computer algebra to understand recently developed algorithms and to enter in the system and coding some procedures that will realize some complicated operations may be listed among the features that have to be possessed.

Tall, in his study, defined mathematics teachers as the ones that didn't need to have a skill of using any computer algebra systems, instead, he stressed that it would be enough to be able to use the learning mediums prepared for them. Despite the absolute distinction that Tall made in mathematics software as "didactic packages" and "learning by doing math", Crowe and Zand believe that computer algebra systems has an important role in learning and teaching in all aspects. As a support to their argument, they offer the article of Stoutmeyer [9] which emphasizes the pedagogic benefits of computer algebra systems written in a time when the personal computers did not become commonplace and the studies about computer algebra systems were at the start point as a proof. Stoutmeyer's argument, which was "a radical proposal" at that time according to his own statement, is in fact among the inevitable applications in mathematics education today.

Crowe and Zand, in the second [10] of their article series, draw attention to the fact that both mathematicians and mathematics teachers tend to use computers today more than ever. They put forward two facts for this. The first one is stressed as the computers' increasing importance today at the education of almost any subject and the second one is emphasized as the desire of mathematics teachers (from teachers in primary education level to professors in universities) to have more "tools" to concretize the abstract concepts. One of the most important tools in this subject is certainly computer algebra software. For this reason, within the groups working for developing the software, instruction technologists begin to exist as well as mathematicians as these software are not just "scientific programming" tools anymore; they are also "instruction materials" that has become one of the most important tools of mathematics teachers.

Crowe and Zand, in their same article, reemphasize the usages of electronic tabulation software in mathematics education and they draw attention to the examples developed by Golshan [11] using Excel to be used in teaching some topics of introduction to analysis. Golshan, in his study, performed many examples including the topics of determining the maximums and minimums of functions, finding approximated roots of equations, approximated solutions of equation systems, polynomials, probability problems and Binomial expansions using Excel. Likewise, there have been some studies in which electronic tabulation software is used in combination with computer algebra systems. As examples to them, Shelton's [12] study in which he developed a content similar to the one worked by Golshan by using the Excel and Mathematical software together and Swartz's [13] study in which he displayed the results of the Maple codes he wrote to teach "complex analysis" topics such as Laurent series and residues in Excel may be given. Swartz, in his study, used Excel in a sense of an interface for Maple; however, present computer algebra software tends to solve this need internally and added various tools in their systems for the user-friendly interface design.

SOME INSTRUCTION TOOL SAMPLES IN COMPUTER ALGEBRA SYSTEMS

Maple, which is one of the most preferred computer algebra systems all over the world, by its concept of Maplet it developed in its 8th version, has enabled designing user interfaces using Maple engine at

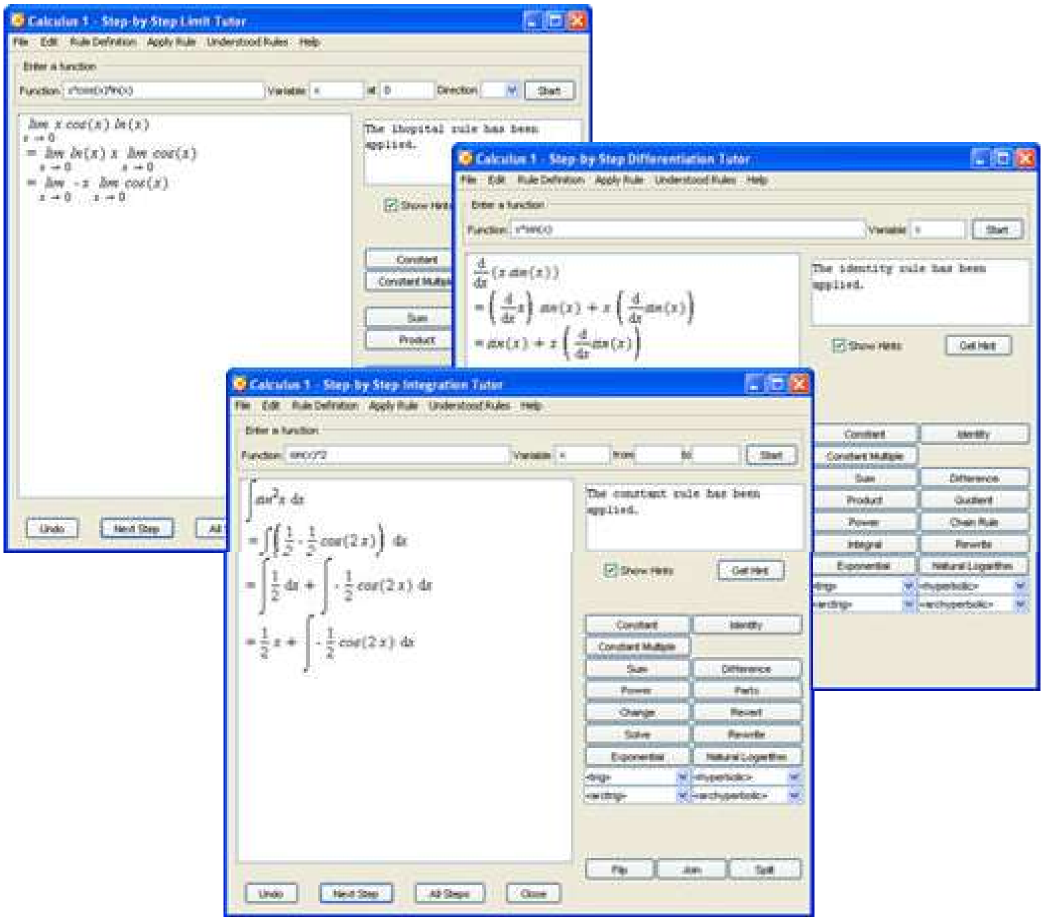

Fig. 8: Step-by-step tutors

the background. While these applications developed using a script-type programming language are started within the software until the ninth version, it has been enabled to start each maplet out of the program, but as the computations made at the background are made by the software itself, the software has to be installed at the same computer. That is, users will be able to use the powerful computation skills of the software without the requirement of learning the complicated usage that requires an advanced level of programming logic. Moreover, the software will not need an interface program (like Excel) except itself.

This technology has especially increased the pace of computer algebra systems usage in the mathematics teaching field because teachers don't have to know maplet developing language to use such a design. The teachers that will use the applications developed by professional programmers (and naturally mathematicians) can provide computer support while teaching almost any subject. The fact that these user interfaces can also easily be used by students will make a contribution mathematics education. Maplet samples which have been developed towards education of some subjects are given below.

RIEMANN SUMS MAPLET

The Riemann sums method, which is of great importance to understand the concept of determined integral, is a subject very suitable to simulation technique. The maplet whose screenshot was given below both shows the right, left and middle Riemann sums for the given function and interval according to the rectangle number both by graphic and finds its total formula and computes its numerical result. The user can visually and numerically observe the way of approaching to the field which is below the curve by increasing the rectangle number for the same function.

STEP-BY-STEP TUTORS

Step-by-step tutors have been developed by Maple software group and presented to the users in the applications site (http://www.mapleapps.com) of the software under education category as free from charge. These applications decreased the prejudiced approach against generally computer algebra systems in a measure because of thier being “inscrutable things”. Even though it is not too important for the mathematicians and engineers whose aim is only getting result, this situation differs a little bit when education comes into the question as how the software finds the solution gains importance as well as what it finds as a result.

This problem, which has been known for long by the computer algebra theorists but the solution of which is not easy at all, seems to be solved, at least partially because the solution steps which are very meaningful for the theorists are not easy ones for a mathematician who is not interested in computer algebra systems directly beyond a user who is in a state of learning mathematics. Consequently, the process of translating every step into a language that a student learning mathematics can also understand has been the work of the computer algebra specialists.

In Fig. 8, screenshots of three different didactic maplets which are developed for the three basic subjects of the analysis were given; limit, derivative and integral. In every application, the data required for function and application (such as point or interval) are entered by the user, the user can watch the solution step-by-step; he/she can make a solution suggestion in accordance with each step if is he/she wants. In the integration application for instance, he/she can make suggestion about the method such as changing the variable or partial integration. In exchange for this, he/she can receive feedback from the system about the accuracy of his/her suggestion.

With its current state, such as in the cases of the applications whose examples were given below, the maplets to be developed for symbolic computation transactions under certain rules are quite important as for gaining practicality in operations although it does not contribute much in teaching the theoretical dimension of the subject.

DISCUSSION AND CONCLUSION

Consequently, the computer algebra systems today have become an important step of computer support which is to be applied especially to undergraduate level mathematics education and it seems that their roles in this area will gain more importance soon. In addition to this, as a result of the increase in their feasibility, they have reached to a spectrum consisting of a wider age group, thus they have started to appear as an educational tool in primary and secondary education. These systems’ skills in computation are evident; furthermore, thanks to its features of designing interfaces, they will be inevitable materials for mathematics educators as well as mathematicians.

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