World Applied Sciences Journal 28 (12): 1879-1887, 2013
ISSN 1818-4952

DOI: 10.5829/idosi.wasj.2013.28.12.13902

Formalization and Solving the Problem of the Fuzzy Mathematical Programming to Optimize the Operation at Oil Equipment Plant

[1]*Batyr Orazbayev,* [1]*Kulman Orazbayeva,*
[2]*Lyailya Kurmangaziyeva,* [1]*Balbupe Utenova and* [1]*Ainur Mukhanbetkaliyeva*

[1]Atyrau Institute of Oil and Gas, Azattyk ave, 1, 060002, Atyrau, Kazakhstan
[2]Kh. Dosmukhamedov Atyrau State University, Atyrau, Kazakhstan

Abstract: This works researches the problems of Fuzzy Mathematical Programming (FMP) and methods of their solution; it is proposed a new approach to the formulation FMP problems and their effective solutions. Originality and novelty of the proposed formalization methods and solving FMP problems from known methods are defined so that problems can be posed and solved without preliminary transformation them to equivalent deterministic options that reduce the loss of the original fuzzy information (knowledge, experience, intuition of experts). Modification idea, Pareto optimality and principal criterion is proposed to solve the multi-criteria problem and fuzziness of the production tasks in this work. A practical use of the proposed approach to formalization and solving FMP problem as the example to optimize the production on oil equipment plant is provided as the example. It is showed that a better solution than the solution for the deterministic option of the original fuzzy problem can be achieved when solving the problem with the lack of fuzziness.

Key words: Fuzzy mathematical programming . fuzzy restraint . fuzzy sets theory . membership function . decision-maker

INTRODUCTION

Mathematical programming problems that arise in the production facilities, as a rule are multi-criteria as the work of such facilities is described by some local criteria.

Assume $f_1(x),\ldots,f_m(x)$ criteria, (objective function) is for evaluating the efficiency of the facility. Each of m criteria depends on the vector of n parameters (input stimulus) $X = (x_1,\ldots,x_n)$ and mutual importance of the criteria is described by the coefficients of the relative importance (weights) $\gamma_1,\ldots, \gamma_m$. Criteria $f_i(x), i=\overline{1.m}$ forms $f(x) = (f_1(x),\ldots,f_m(x))$ criteria vectors and coefficients $\gamma_1,\ldots,\gamma_m$ -weight vector $\gamma = (\gamma_1,\ldots, \gamma_m)$.

Criteria $f_i(x)$ included the vector criterion is called local. Each alternative is characterized by its inherent vector estimate (value of the vector criterion at the points x) $f(x) = (f_1(x),\ldots,f_m(x))$, where $f_i(x), i=\overline{1.m}$ -criteria value $f_i(x)$ at points x (in the values of the manipulated value).

At given values, x functions $f_i(x), i=\overline{1.m}$ accept certain values. One of the problems of the mathematical programming is to choose such values of x vector, which allocate Pareto set (set of efficient solutions), where improvement of any criteria $f_i(x)\in f(x)$, $i\in I$ is possible only at the expense of others-$f_i(x)\in$, $j\in I$, $i\neq j$, where $I = \{1,\ldots,m\}$-set of indices. Each $f_i(x)$ local criteria is associated with the values of the input actions, this relationship is described by the object model system.

Tasks of the mathematical programming in fuzzy environment can be characterized by the following elements, which arise from a combination of specific objectives:

1. Fuzzy measures $\tilde{f}_i(x), i=\overline{1,m}$;
2. Instructions such as: «it is to be wished that the values-$\tilde{f}_i(x), i=\overline{1,m}$, were more»-maximization fuzzy $m\tilde{a}x\tilde{f}_i(x)$;
3. Fuzzy criteria constraints such as: «it is to be wished that criteria values [phi] $\varphi_q(x)\tilde{\le} b_q$, $q=\overline{1,L}$ were no more (not less-$\tilde{\ge}$, equal-$\tilde{=}$ than b_q);
4. Information about approximate importance of the criteria (weight vector, number of priorities);
5. Fuzzy constraints for vector of argument of type: $x\in\tilde{C}$, where $\tilde{C}$ -fuzzy set;
6. Deterministic constraint on the argument: $x\in\Omega$.

Corresponding Author: Orazbayev, Atyrau Institute of Oil and Gas, Azattyk ave, 1, 060002, Atyrau, Kazakhstan

Given the above information, the task of the Fuzzy Mathematical Programming (FMP) in the management of the production facilities and processes in general can be formalized as follows:

Find the control vector $x^* = (x_1^*,...,x_m^*)$, providing such values of the local criteria that satisfy the Decision Maker (DM):

$$\max_{x \in X} \tilde{f}_i(x), i = \overline{1,m} \quad (1)$$

$$X = \left\{x : x \in \Omega, \varphi_q(x) \tilde{>} b_q, q = \overline{1,L}\right\} \quad (2)$$

where $\tilde{f}_i(x)$ -fuzzy local criteria, values of which are calculated by the models (all or some of them are fuzzy); $\varphi(x), q = \overline{1,L}$ fuzzy constraints that define the accessible region Ω [omega] of multicriteria task (1)-(2); b_q-predetermined numbers that can be fuzzy.

Information about the criteria importance can be presented: priority (I_c) and weight vector (γ [gamma]), which may not be fuzzy.

Combination of the different sources of the fuzziness (in criteria, limitations, requirements to them) will lead to various FMP problems.

Thus, under the objective of the fuzzy mathematical programming-FMP we understand the problem containing the objective function or a vector of objective functions (criteria, local criteria) that must be optimized and the system of inequalities or equations that describe the conditions-constraints and part or all elements of the task (criteria limitations, information about their importance, etc.) are described clearly.

Let's consider the results of the analysis of approaches to the optimization and management of the production facilities in the context of uncertainty [1, 2]. One approach that gave considerably success in solving optimization problems rather complex processes under conditions of uncertainty, was developed in the mid 50's by Dantzig in the work on analysis of the solution in linear programming.

Another approach to solve the problems of the lack of reliable information about the object is considered in the stochastic programming [3], which discusses the elaboration of the efficient algorithms for optimization in noisy information and problems to analyze the accuracy as per known probability characteristics of the information used.

It should be noted that accurate information is used in these approaches to analyze the status of the object and to develop the control and the impact of the blurring the information has been taken into account explicitly in the models. However, the need to incorporate the fuzzy information presented by the experts judgment (managers, production personnel cash) about the functioning of the facility and reflecting their preference in the selection of solutions to manage the production in the mathematical models and in control, has stimulated the development of new approaches [4, 5].

Importance of these approaches is explained by the fact that many production facilities operate under the uncertainty and the primary or sole source of the information is a person who expresses his views, usually qualitatively, by means of the fuzzy statements. Methods for direct analysis of the impact of information blurring on the nature of modeling solution and control use either tool of probability and mathematical statistics [6], or the theory of fuzzy sets [7-11].

Work [12] have shown that a number of simulation and optimization under uncertainty cannot be formalized in the framework of the probability theory with the classical definition of the objective probabilities. This primarily relates to the synthesis models for fuzzy initial information and tasks for accounting the subjective information such expert certainty in more or less judgment or preference of the decision-makers when choosing a solution.

Thus, the optimization problem and making the best decisions during management the production facilities can be reduced to problems of mathematical programing for which there are well-studied and effective approaches to the solution [13].

Generalization of the mathematical programming to the class of the fuzzy numbers and some approaches to solve such problems of the fuzzy mathematical programming (FNMP) are discussed in the works [14-16].

Generally in these works FMP problems reduce to problems of achieving the fuzzy definite purpose, which can be solved using the approach Bellman-Zadeh, i.e. intersection of the membership functions to achieve the fuzzy target ($\mu_G(x)$) and performing the constraints ([mu] $\mu_R(x)$) is considered:

$$\mu_D(x) = \min\{\mu_G(x), \mu_R(x)\}$$

where $\mu_D(x)$-decisions membership function.

In such way there is a problem with the way how to choose alternative. One of the best known ways is to choose the alternative that has the maximum degree of fuzzy decision D:

$$\max_{x \in X} \mu_D(x) = \max_{x \in X} \min\left\{\mu_G(x), \mu_K(x)\right\}$$

Another well-known approach to solve FMP problems is that the initial task at the setting stage is replaced by the deterministic equivalent.

Application of these approaches to solve production problems that characterized, as a rule, by multiple criteria and large sizes is difficult, as it requires large computational problems, there are disadvantages of their use by the users-production staff (they need to know the theory basis). Moreover, these approaches lost the part of the initial fuzzy information (descriptions).

In a practical side the essential subclass of FMP problems are problems of fuzzy linear programming (FLP), which are discussed in [17]. The main approach to solve the Fuzzy Linear Programing is to replace the fuzzy task by clear one, for example, based on the level set α:

$$A_\alpha : \forall \alpha \in [0,1], A_\alpha = \{x : x \in X, \mu_A(x) \geq \alpha\}$$

Work [18] provide for the classification of fuzzy mathematical programming and consider the general case of FMP problems based on the mini-max principle of the generalization, issues for setting the fuzzy preference relations were discussed, a number of theorems to reduce FMP tasks to a set of common problems of the mathematical programming has been proved.

Work [18] has considered the linear mathematical programming problem with fuzzy objective functions: Z = cx, where c = $[c_1,\ldots,c_n]$-undetermined coefficients with the known intervals from change $[c_1^l, c_1^u],\ldots,[c_n^l, c_n^u]$. Approaches to determine a compromise solution of this problem, which is to choose different coefficients of the objective function from a given interval have been discussed: class mark; on the basis of information about the best chance of the appearance of some representatives (if the decision-maker has this information); by sequential reduction of the infinite set of the entire function.

Both a stochastic programming and fuzzy mathematical one provides for two fundamentally different approaches to solve the problems. First implies that the membership functions of fuzzy parameters of optimization model are known. But in many case we can replace the initial fuzzy problem by equivalent clear (indirect methods). The second approach assumes that the decision-maker can evaluate the membership function of the fuzzy variable at any point. Decision methods to solve FMP problems using this estimate, are called the direct methods.

Some approaches using direct and indirect methods to solve FMP tasks are considered in. All these approaches are based on reducing the initial fuzzy problem to a sequence of clear objectives.

Main part. FMP problems new setting and solution methods. In comparison of the approaches of other authors discussed above, this paper proposes a more efficient method to solve the formalizing and FMP problems solution, which retains the fuzzy initial information provided in the description of the criteria and constraints and the multi-criteria issue is resolved based on the various compromise schemes of problem solving in an easy to the decision-maker form.

This allows a more adequate to describe the production situations in fuzzy environment and get effective methods of FMP problem solution on the selection of optimal work regime of the production facility in fuzzy environment. Using various optimality principles in a statement of the problem, we thus give rise to different production methods and solutions of the initial multi-criteria problems. This allows the decision maker without thinking just compare the various solutions to choose the best.

Let's consider the formalization option and setting the problems of the mathematical programming with fuzzy elements. Let's pay attention to the situation when FMP problem is for one criteria (in case of multicriteria we can get the convolution of the local criteria, i.e. multicriteria problem is converted to one-criterion) and a few limitations.

Assume there is one normalized criteria $\mu_0(x)$ (or convolution of the local criteria $\mu_0(x) = \varphi(\mu_0^i), i = \overline{1,m})$) and L constraints of type $f_q(x) \tilde{\geq} b_q, q = \overline{1,L}$ with fuzzy instructions- $f_q(x) \tilde{\geq} b_q, q = \overline{1,L}$. Suppose that the membership function of each performance constraints $\mu_q(x), q = \overline{1,L}$ is set in the result of the dialogue with the decision-maker, experts. Let the priority number I = {1,…,L} is known or weight vector β = $(\beta_1,\ldots,\beta_L)$ for constraints reflecting the essence of the mutual restrictions as of the time of setting the optimization task.

Then, general, FMP problem:

$$\max_{x \in X} \mu_0(x)$$

under conditions

$$\varphi_q(x) \tilde{\geq} b_q,\ q = \overline{1,L}$$

may be written as:

$$\max_{x \in X} \mu_0(x), X = \left\{x : \arg\max_{x \in \Omega} \mu_q(x), q = \overline{1,L}\right\}$$

This formulation of FMP problem with a clear objective function and fuzzy constraints with fuzzy instruction reflects a convergence to maximize the objective function has fully satisfied constraints. If we assume that all membership functions are normal, then the setting FMP problem becomes as follows:

$$\max_{x\in X} \mu_0(x) \quad (3)$$

$$X = \{x : x \in \Omega \wedge \mu_q(x) = 1, q = \overline{1,L}\} \quad (4)$$

We have got a clear mathematical programming problem with maximization of the objective function on the clear set of X. Further we assume the concavity of the objective function $\mu_0(x)$, constraints $\mu_q(x), q = \overline{1,L}$ and convexity of the feasible set X. This problem is solved by the usual methods of the mathematical programming.

In practice, it is possible that the set X is empty because of the absence of alternative x that satisfies all the constraints and, therefore, there is no solution. In this case, it is needed to refuse from clear decision and using the fuzziness of the constraints, to set tasks of the mathematical programming that take into account the fuzziness.

In this case due to impossibility to satisfy all criteria constraints simultaneously we have to use compromise schemes of the requirements traceability of the various criteria limitations. We will use the ideas and schemes of the compromise enshrined in the deterministic methods of multi-criteria evaluation of alternatives to set FMP problems and determine solutions to these problems.

In the beginning we reduce the initial task to maximize the objective function on Pareto set points [3] formed by the restrictions:

$$\max_{x\in X} \mu_0(x) \quad (5)$$

$$X = \left\{x : \arg\max_{x\in\Omega} \sum_{q=1}^{L} \beta_q \mu_q(x) \wedge \sum_{q=1}^{L} \beta_q = 1 \wedge \beta_q \geq 0, q = \overline{1,L}\right\} \quad (6)$$

Solution of this problem depends on the weight vector [beta] ß and consists of control vector (independent variables), objective function and a set of values of constraints:

$$x^*(\beta), \mu_0(x^*(\beta)), \mu_1(x^*(\beta)), ..., \mu_L(x^*(\beta))$$

We propose the following algorithm to find solutions to the given task:

Algorithm FMP-1.

1. Set $p_q, q = \overline{1,L}$-number of steps on each q coordinate.
2. It is defined $h_q = \frac{1}{p_q}, q = \overline{1,L}$-steps sizes to measure the coordinate of the weight vector ß [beta].
3. Draw the set of the weight vectors

$$\beta^1, \beta^2, ..., \beta^N, N = (p_1+1)\cdot(p_2+1)...(p_L+1)$$

by variation the coordinates at sections [0, 1] with step h_q.
4. On the basis of the information received from the decision-maker, experts it is defined the set of the fuzzy parameters and constraints are constructed for each membership function to perform the constraints [mu] $\mu_q, q = \overline{1,L}$.
5. Tasks N (5)-(6) are solved at $\beta^t, t = \overline{1,N}$ and the current decision is defined: $x(\beta^t), \mu_0(x(\beta^t)), \mu_1(x(\beta^t)), ..., \mu_L(x(\beta^t))$.
6. Found current decision is proposed to the decision-maker to select the best of the final decision. The best solution is selected using the preferences of the decision-maker.
7. If the current solution does not meet the decision-makers, they are assigned new values of a set of weight vectors (corrected) $\beta^t, t = \overline{1,N}$ and returns to step 4. Otherwise, go to the point 8.
8. Finding the solution stops, final results by the decision-maker are shown: optimal value of the regime, control parameters-$x^*(\beta^t)$; providing the best value of the criterion- $\mu_0(x^*(\beta^t))$ and extent to perform fuzzy limits-$\mu_1(x^*(\beta^t)), ..., \mu_L(x^*(\beta^t))$.

In this algorithm the initial Pareto solution set is approximated by N points for which solutions are sought. The decision-maker is responsible for the choice of the best solution in this algorithm. There is a special method of the search dialog of the best Pareto solution.

We have considered FMP with one objective function (criterion) and several criteria constraints. In the case of multi-criteria of the facility it is suggested to select one of the criteria for objective function (or make a convolution of the local criteria) and rest are considered as constraints. In the presented form and proposed statement of FMP problems, fuzzy or clear could be as objective function and individual constraints. In practice, due to various physical criteria, fuzziness of their description because of non-programming of the decision-makers' preferences in the process of describing and decision-making to reduce the problem to one-criteria often fails, i.e. there is a situation when you have to put FMP problem with multiple objective functions and constraint vector. We proceed to formalize and FMP task solution in these conditions.

Let $\mu_0(x) = (\mu_0^1(x),...,\mu_0^m(x))$ -normalized vector of the criteria assessing the quality of the facilities. Suppose that on the basis of the expert procedures, each constraint $\varphi_q(x), q = \overline{1,L}$, is constructed the membership function to perform constraints-$\mu_q(x), q = \overline{1,L}$.

Let the range of priorities for local criteria I_C = {1,…,m} and limitations I_R = {1,…,L} or weight vector reflecting the mutual importance of the criteria $\gamma = (\gamma_1,..., \gamma_m)$ and $\beta = (\beta_1,...,\beta_L)$ constraints. Then on the basis of the above compromise schemes we can formalize various multi-criteria FMP with several limitations and to propose algorithms to solve them.

For example, based on the principle of Pareto-optimal, the general FMP problem with several criteria and constraints:

$$\max_{x\in X} \mu_O^i(x), i = \overline{1,m}$$

$$X = \left\{x : \arg\max_{x\in\Omega} \mu_q(x), q = \overline{1,L}\right\}$$

may be written as follows:

$$\max_{x\in X}\left(\sum_{i=1}^{m} \gamma_i \mu_O^i(x)\right), \quad i = \overline{1,m} \tag{7}$$

$$X = \left\{x : \arg\max_{x\in\Omega} \sum_{q=1}^{L} \beta_q \mu_q(x) \wedge \sum_{q=1}^{L} \beta_q = 1 \wedge \beta_q \geq 0, q = \overline{1,L}\right\} \tag{8}$$

where the effective set of solutions at Pareto's set of points formed by the constraints is sought.

Search for solutions for task (7)-(8) can be performed using the following algorithm.
Algorithm FMP-2.

1. By expert estimate the values of the weight vector γ are defined assessing the mutual importance of the local criteria: $\gamma = (\gamma_1,..., \gamma_m)$.
2. Number of sets $p_q, q = \overline{1,L}$ are set on each q coordinate.
3. It is defined $h_q = \frac{1}{p_q}, q = \overline{1,L}$ -steps sizes to measure the coordinate of the weight vector ß.
3. Draw the set of the weight vectors $\beta^1, \beta^2,..., \beta^N$, $N = (p_1+1)\cdot(p_2+1)...(p_L+1)$ by variation the coordinates at sections [0, 1] with step h_q.
4. On the basis of the information received from the decision-maker, experts it is defined the set of the fuzzy parameters and constraints are constructed for each membership function to perform the constraints: $\mu_q(x), q = \overline{1,L}$.
5. Task (13)-(14) for set of the weight vectors and the current decision is defined: $x(\gamma,\beta^t); \mu_0^1(x(\gamma,\beta^t)),...,\mu_0^m(x(\gamma,\beta^t)); \mu_1(x(\gamma,\beta^t)),...,\mu_L(x(\gamma,\beta^t))$
6. Found current decision is proposed to the decision-maker to select the best of the final decision. The best solution is selected using the preferences of the decision-maker.
7. If the current solution does not meet the decision-makers, they are assigned new values of a set of weight vectors (corrected) γ and (or) $\beta^t, t = \overline{1,N}$ and returns to step 5. Otherwise, go to the point 9.
8. Finding the solution stops, final results by the decision-maker are shown: optimal value of the regime, control parameters-$x^*(\gamma,\beta^t)$; providing the best value of the criterion-$\mu_0^1(x^*(\gamma,\beta^t)),...,\mu_0^m(x^*(\gamma,\beta^t))$ and extent to perform fuzzy limits-$\mu_1(x^*(\gamma,\beta^t)),...,\mu_L(x^*(\gamma,\beta^t))$.

Production optimization on the oil equipment plant using FMP.

Let the oil equipment plant produces two types of drill bits for oil and mining industry-D_1 ? D_2 and. Their production is not clearly limited by the availability of the raw materials (synthetic diamonds) and processing time on the machines.

For each D_1 product requires 4 kg of the artificial diamond and for D_2 product-5 kg. The plant can get from their suppliers up to 100 kg of the artificial diamonds per week. To produce each D_1 product it is required 12,5 hours of operating time on the machines and for D_2 product-10 hours. You can use up to 240 hours of work on the machines per week. You can purchase additional raw materials and get more time, but at a higher price.

How many products of each type can the plant produce per week if each product of D_1 type brings 5 dollars of profit and each product of D_2 type-7 dollars?

To formalize this problem, we denote via x_1 and x_2-number of the products manufactured per week, respectively, D_1 and D_2 types. The task is to find the best values x_1 and x_2 that maximize the weekly profit f (x) ($x = (x_1, x_2)$), defined by the following:

$$f(x) = 5x_1 + 7x_2 \tag{9}$$

Expression (9) is the objective function (criterion), which is necessary to maximize. As can be seen from the structure of the criteria to increase f (x) it is needed

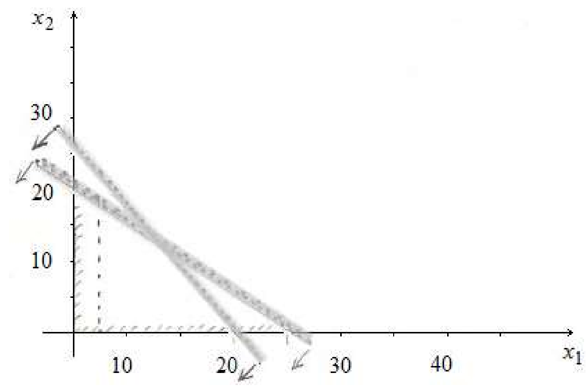

Fig. 1: Feasibility region

to increase x_1 and x_2, but (and this is the problem), the values of these variables cannot be increased partially because they are limited for raw materials and processing time. As noted above, these constraints may not be clear.

Since both x_1 and x_2 express the weekly volume of the products, they cannot be negative, that is:

$$x_1 \geq 0, x_2 \geq 0 \tag{10}$$

Restrictions on the availability of the artificial diamond and processing time can be written mathematically as the following fuzzy inequalities;

$$4x_1 + 5x_2 \lesssim 100 \text{ (for raw material)} \tag{11}$$

$$12.5x_1 + 10x_2 \lesssim 250 \text{ (for processing time)} \tag{12}$$

Therefore, formulated the task is to find the amount of the product produced of each type (values x_1 and x_2), that satisfy the conditions of the non-negativity (10) and maximizing the criteria f (x).

Task (9)-(12) will be re-written in the standard type of the mathematical programming:

$$f(x) = 5x_1 + 7x_2 \rightarrow \max \tag{13}$$

at

$$4x_1 + 5x_2 \lesssim 100 \tag{14}$$

$$12.5x_1 - 10x_2 \lesssim 250 \tag{15}$$

$$x_1 \geq 0, x_2 \geq 0 \tag{16}$$

For clarity, we construct fields defined by constraints (fuzzy inequality $4x_1 + 5x_2 \lesssim 100$, $12.5x_1 + 10x_2 \lesssim 240$ (Fig. 1).

First construct the line $4x_1+5x_2<100$ by two points with coordinates $x_1 = 0$, $x_2 = 20$ and at $x_1 = 25, x_2 = 0$.

Table 1:

0	1	- x_1	-x_2
x_3	100	4	5
x_4	250	12,5	10
f	0	-5	-7

$$\frac{100}{5} = \underline{20}$$
$$\frac{250}{10} = 25$$

Plot the points with coordinates (0, 20), (25, 0) on the graph and draw a straight line $f_1(x)$ (unclear). To determine which part of the plane defined by the inequality $4x_1 + 5x_2 \lesssim 100$, substitute a random point coordinate, for example (30, 30), we obtain a contradiction to (120 150> 100), i.e. inequality defines a half-plane, which does not contain the point (30, 30). Similarly we construct the line $f_2(12.5x_1+10x_2 = 250)$ ((0.25), (20, 0)) and set the direction of the admissible plane.

Constraints (14) indicate that the decision is in the first square $x_1 \geq 0$, $x_2 \geq 0$.

To solve the given problem, we use different approaches. First, by setting strict limits, the problem is transformed to normal (clear) problem. Let's consider the following mathematical programming problem:

$$f(x) = 5x_1 + 7x_2 \rightarrow \max \quad 4x_1 + 5x_2 \leq 100$$

$$12.5x_1 - 10x_2 < 250$$

$$x_1 \geq 0, x_2 \geq 0 \text{ (entire)}$$

To solve this problem based on the simplex method, we introduce new variables x_3 and x_4:

$$x_3 = 100 - 4x_1 - 5x_2$$

$$x_4 = 250 - 12.5x_1 - 10x_2$$

$$x_3 \geq 9, x_4 \geq 0$$

and draw the following simplex tables.

A basic plan $x_1 = 0$, $x_2 = 0$, $x_3 = 100$, $x_4 = 250$ meets this table. The value of the objective function f for this plan is zero. This plan is not optimal, because the line of the objective function «f» has negative elements (-5), (-7).

We choose one of these numbers the most negative (-7), while column (-x_2) will be allowed. According to the algorithm the figures being a permitted column "1" is divided by the number found in permissive column,

Table 2:

0	1	$-x_1$	$-x_3$
x_2	20	4/5	1/5
x_4	50	6.5	-2
f	140	3/5	7/5

i.e. in column (x_2). The smallest positive attitude is achieved in the line x_3, which is taken as resolving. Then the resolving element will be 5.

Further construct a new Table 2. In it x_3 and x_2 change places.

In this table elements resolving line and column are defined by dividing the old values for resolving element, that is, to "5". Rest of the elements is defined by the rectangle rule:

$$250 \Rightarrow (250 \cdot 5 - 100 \cdot 10)/5 = 50$$

$$12.5 \Rightarrow (12.5 \cdot 5 - 4 \cdot 10)/5 = 6.5$$

$$0 \Rightarrow (0 \cdot 5 - 100 \cdot (-7)/5 = 140$$

$$-5 \Rightarrow ((-5) \cdot 5 - 4 \cdot (-4)/5 = 3/5$$

Appropriate plan to the Table 2: $x_1=0$, $x_2=20$; $x_3=0$; $x_4=50$; $f = 140$.

This plan is optimal, since the line «f» is composed of non-negative elements, so f_{max} = 140. Thus, if the plant will produce D_2 20 pieces of products per week (D_1 product is not produced, $x_1=0$), it has 140 dollars profit a week. If any other number of production output for given stringent restrictions the plant will have a relatively low income.

Now let's consider a fuzzy approach to solve this problem based on fuzzy mathematical programming methods. Since the constraints are not clear, they are approximate and their breaking up to a certain extent is allowed.

Having normalized the criteria f(x): $\mu_0(x) = \varphi(f(x)) \in [0,1]$, task (13)-(16) with taking account fuzzy limits we will put FMP in general type:

$$\max_{x \in X} \mu_0(x) \quad (17)$$

at

$$f_q(x) \tilde{<} b_q,\ q = 1,2 \quad (18)$$

$$x_i \geq 0,\ i = 1,2 \quad (19)$$

Assume that each constraints are constructed the membership function to perform limitations $\mu_q(x)$, q = 1,2 and a range of the priority of the constraints I = {1,2} are identified. Then, based on the idea of the main criteria method, the task (17)-(19) takes the following formulation:

$$\max_{x \in X} \mu_0(x)$$

$$X = \{x : x \in \Omega \wedge \arg(\mu_q(x) \geq \mu_q^R),\ q = 1,2\}$$

Normalization of the objective function we do based on the following expression:

$$\mu_0(x) = \frac{f(x) - \inf_{x \in X} f(x)}{\sup_{x \in X} f(x) - \inf_{x \in X} f(x)} = \frac{f(x) - 0}{155 - 0}$$

where 155-max possible profit.

To solve this task we use the algorithm FMP-2 based on the modification of the idea of the main criteria method (constraints).

1. Let more important limitation is the restriction on processing time. We define the following set of priority I_R = {1,2}, where 1-limitation on processing time (10), 2-restriction on raw materials (9).
2. With the views of the decision-maker and production staff we define the set and construct the membership function to perform each limitations $\mu_q(x)$, q = 1,2:

$$\mu_1(x) = \begin{cases} 1, \text{if } 12{,}5x_1 + 10x_2 \leq 245 \\ 1 - \dfrac{255\text{-}(12.5x_1 + 10x_2)}{5}, \\ 0, \text{if } 12{,}5x_1 + 10x_2 \geq 255 \end{cases}$$
$$\text{if } 245 < 12.5x_1 + 10x_2 < 255$$

$$\mu_2(x) = \begin{cases} 1, \text{if } 4x_1 + 5x_2 \leq 85 \\ 1 - \dfrac{155\text{-}(4x_1 + 5x_2)}{15}, \text{if } 85 < 4x_1 + 5x_2 < 155 \\ 0, \text{if } 4x_1 + 5x_2 \geq 255 \end{cases}$$

   where d = 5, d = 15-feasible threshold.
3. Decision-maker assigns the initial boundary value constraints $\mu_q^{R(l)}$, $q = 1,2, l = 1$. Suppose the following boundary values on degree of fulfillment fuzzy constraints are given: $\mu_1^{R(1)} = 0{,}90$; $\mu_2^{R(1)} = 0{,}65$.
4. Solve the task of the maximization of the objective function $\mu_0(x)$ at the feasible set X and define the set of solutions:

$$x\left(\mu_1^{R(1)}\right), x\left(\mu_2^{R(1)}\right), \mu_0\left(x\left(\mu_1^{R(1)}, \mu_0^{R(1)}\right),\right),$$
$$\mu_1\left(x\left(\mu_1^{R(1)}\right)\right), \mu_2\left(x\left(\mu_2^{R(1)}\right)\right)$$

In the result of the modeling and search of optimal values x_1, x_2 taking into account the fuzziness constraints, the objective function value greater than 140 dollars resulting in a clear solution of the problem has been identified.

At $x_1=5$; $x_2=18$ (Fig. 1) we will get:

$$f(x) = 5x_1 + 7x_2 = 5\cdot 5 + 7\cdot 18 = 25126 = 151 \text{ dollars}$$

$$\mu_1(x) = 1 \text{ , as } 12.5x_1 - 10x_2 \le 250$$

$$\mu_2(x) = 1 - \frac{115 - (4\cdot x_1 + 5\cdot x_2)}{15} = 1 - \frac{5}{15} = 0{,}67$$

Then the task is as follows:

$$\max_{x\in X} \mu_0(x) = \frac{\max_{x\in X}(f(x))}{155}$$

$$X = \{x : x \in \Omega = [x_i \ge 0, i = 1{,}2] \wedge \arg(\mu_1(x) \ge 0{,}90) \wedge \arg(\mu_2(x) \ge 0.65)\}$$

We get the following optimal decision:

$$x^*\left(\mu_q^{R(1)}\right) = (5{,}8)$$

number of manufactured products of each type ($x_1=5$, $x_2=8$),

$$\mu_0\left(x^*\left(\mu_q^{R(1)}\right)\right) = 0.97 \Rightarrow f(x^*) = 151$$

optimal value of the criteria

$$\mu_1\left(x^*\left(\mu_q^{R(1)}\right)\right) = 1;\ \mu_2\left(x^*\left(\mu_q^{R(1)}\right)\right) = 0{,}67$$

degree of fulfillment of fuzzy constraints.

CONCLUSION

As can be seen, the resulting solution meets the specified requirements to constraints and we obtained relatively (compared results with a clear solution of the problem) greater value of the objective function. Constraints with the priority 1 (processing time) is not violated, i.e. the membership function of constraints is performed in full (membership functions to perform this constraint $\mu_1(x)=1$), membership function with the priority 2 (for raw material) is $\mu_2(x)=0{,}67$ (set limit value of this function equals to 0,65 i.e. $\mu_2(x)\ge 0{,}65$). Thus, we have reached the improvement the value of the objective function taking into account fuzzy constraints.

Findings: A scientific paper based on the modification of various principles of the optimality has proposed new setting of FMP problems to optimize the production processes (production plan), algorithms for interactive tasks have been developed. Algorithms developed are based on the ideas of various compromise schemes to make decision modified to work in fuzzy environment.

Originality and novelty of the proposed in this paper the formalization method and solving FMP tasks in comparison with known methods, is to achieve more adequate and optimal solution of the initial production problem in fuzzy environment based on maximum use of the fuzzy information.

In addition, the use of different compromise schemes to make decisions of decision depending on the available information allows the decision-maker to solve the problem of a multi-criteria production problems easily.

Results of the execution of the proposed approach in practice to optimize the production plan of the oil equipment plant with fuzzy constraints are presented. In this case, the more optimal solution for FMP task, which is better than the results in the solution of the initial fuzzy problem at deterministic option, has been defined.

Thus the novelty and originality of the work is defined that the task are posed and solved in fuzzy environment without preliminary conversion to deterministic tasks. This ensures the maximum use of the information collected and getting more adequate solution of the complex production problem at the fuzziness of the initial information.

REFERENCES

1. Orazbayev, B.B., 1990. Modelling process using the theory of fuzzy sets. Proceedings of young scientists and specialists of the Moscow Physical-Technical Institute. Moscow, pp: 44-49.
2. Rykov, A.S., B.B. Orazbaev and A.G. Kuznetsov, 1999. Fuzzy sets application for modeling and control of rectification technology. Preprints IFAC. International Symposium ADCHEM 91. Advanced Control of Chemical Process. Toulouse: France, pp: 95-99.
3. Mausumi, S. and P. Debnath, 2011. Lacunary statistical convergence in intuitionistic fuzzy n-normed linear spaces. Mathematical and Computer Modelling, 54: 2978-2985.
4. Dubois, D., 2011. The role of fuzzy sets indecision sciences: Old techniques and new directions. Fuzzy Sets and Systems, 184: 3-28.

5. Quanxin, Zh. and X. Li, 2012. Exponential and almost sure exponential stability of stochastic fuzzy delayed Cohen-Grossberg neural networks. Fuzzy Sets and Systems, 203: 74-94.
6. Zhao, Zh.W., D.H. 2012. Wang Statistical inference for generalized random coefficient autoregressive model. Mathematical and Computer Modelling. 56: 152-166.
7. Ghodousian, A. and E. Khorram, 2012. Linear optimization with an arbitrary fuzzy relational inequality. Fuzzy Sets and Systems, 206: 89-102.
8. Dubois, D. and H. Prade, 1980. Fuzzy Sets and Systems. Theory and Application. Acad. Press. N-York.
9. Sakawa, M. and H. Katagiri, 2011. Takeshi Matsui. Interactive fuzzy random two-level linear programming through fractile criterion optimization. Mathematical and Computer Modelling, 54: 3153-3163.
10. Sakawa, M., 1983. interactive computer programs for fuzzy linear programming with multiple objective. Int. J. Man-machine Studies, 18: 483-503.
11. Antczak, T., 2011. A new characterization of (weak) Pareto optimality for differentiable vector optimization problems with G-invex functions. Mathematical and Computer Modelling, 54: 59-68.
12. Orazbayev, B.B., 1994. Methods of mathematical modeling of technological systems with fuzzy initial information. Automation, Telemetry and Communications in the Oil Industry. Moscow, 4: 11-13.
13. Minoux, M., 1979. Programmation mathematique. Nheorie et algorithmes. Bordas et G.N.E.T.-E.N.S.T/ Paris.
14. Rykov, A.S. and B.B. Orazbayev, 1995. Multi criterial fuzzy choice.-Moscow: Moscow Institute of Steel and Alloys, Metallurgy, pp: 124.
15. Yazenin, A.V., 1991. Linear programming with fuzzy random data. As Technical Cybernetics, 5: 52-58.
16. Dubey, D., S. Chandra and A. Mehra, 2012. Fuzzy linear programming under interval uncertainty based on IFS representation. Fuzzy Sets and Systems, 188: 68-87.
17. Wierzchon, S.T., 1987. Linear programming with fuzzy sets: A general approach. Math. Modelltng, 9 (6): 447-459.
18. Zaichenko, Y.P., 1991. Operations research: fuzzy optimization. Kiev: Vishcha School.