Procedure of Evaluation Development for Drilling-in and Well Completion

Saltanbek Talapedenovich Mukhambetzhanov and Zharasbek Duisembekovich Baishemirov

1Al-Farabi Kazakh National University, Almaty, Kazakhstan
2Abay Kazakh National Pedagogical University, Almaty, Kazakhstan

Abstract: The article deals for simplicity with the plane filtration problem of immiscible fluids in the area, limited by impenetrable bottom and top of the stratum and also by delivery and operating wells. A method of weak approximation is used when approaching the boundary layer. The obtained splitting can be taken as a basis for forming-up a row of differencing diagrams. As the methods of mathematic modeling of complex filtration processes in oil strata are now developing in two directions - the creation of strict models, fully accounting for the filtration laws of multiphase fluids in porous mediums and the creation of engineering models based on the simplified filtration diagrams. The first direction results in setting of complex spatial problems of multiphase (multicomponent) filtration in oil strata, which are further implemented by numerical methods. This direction also comprises solution methods of model problems of filter theory as applied to the oil strata. Mathematical models of the specified direction are intended, firstly, for the detailed analysis of expulsion mechanisms and are very useful for theoretical analysis of new technologies.

Key words: Fluid filtration • Weak approximation method • Porous medium • Masket-Leverett model

INTRODUCTION

In 1960 O.A. Oleynik [1] introduced a notion of generalized solution for multivariate Stefan problem. At that thermal conductivity equations for different phases are interpreted as one equation

\[
\frac{\partial U}{\partial t} = \text{div}(\chi \nabla \theta)
\]

where \(U(x, t)\) is the specific internal energy, \(c(x, t)\)- the temperature and specific internal energy are the familiar temperature functions and \(U = \Phi(c)\), which is discontinuous of the first kind at temperature values, equal to the fusion point and smooth enough everywhere beyond this point.

As each classical solution is the generalized one, then the topic about the uniqueness of the classical solution of the Stefan problem was fully solved. From this moment the study of structure of generalized solution became a central one, as well as the revealing of conditions, when the generalized solution of Stefan problem is a classical one. Naturally, it was expected, that if in the initial time moment the medium state in the area under consideration has two phases, there are only liquid and solid phases, separated by a smooth surface, then the generalized solution of Stefan problem coincides with the classical one. For homogenous thermal conductivity equation and one spatial variable it was proved by A. Freedman [2] and D. Cannon, D. Henry, D. Kotlov [3]. For many spatial variables, the existence of classical solution in little time was proved by A.M. Meirmanov [4].

Nonsteady fluid filtration in elastic-porous stratum taking into consideration the diffusion and mass exchange

An interacting process of drilling mud filtrates, containing solve, suspended, emulsified solid and liquid substances, is accompanied, as is known, by their diffusion with stratum fluids and mass exchange with two-phase (liquid and solid) components of rock (ground). General approach to description of mass exchange and diffusion process is given in the works [5, 6]. At that, the equations to determine the velocity of substance, associated with the liquid phase and the law of its saving are written as follows:

Corresponding Author: Baishemirov, Abay Kazakh National Pedagogical University, 050014, Almaty, Urban District “Áinabulak-4”, 173-87, Kazakhstan.
\[ \ddot{u} = c\ddot{u} - D grad c \]  
\[ \frac{\partial (mc)}{\partial t} + d m u + \frac{\partial N}{\partial t} = 0 \]  
(1.1) and the equation of mass balance of the substance at the constant pore volume is the following:

\[ \frac{1}{r} \left( r D \frac{\partial c}{\partial r} \right) - \frac{\partial c}{\partial r} = m_0 \frac{\partial c}{\partial t} \]

Further reduction of equation (1.7) depends on the relation of liquid filtration velocities and molecular diffusion of the substance, associated with it, in other words, on Péclet number:

\[ P_e = \frac{\nu \cdot d \cdot m_0}{D} \]

\( d \) is the characteristic length. Let us write over the equation (1.7) at \( D=\text{const} \) in dimensionless form:

\[ P_e^{-1} \frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \xi \frac{\partial c}{\partial \xi} \right) - \frac{1}{m_0} \frac{\partial c}{\partial \xi} - \frac{1}{m_0} \frac{\partial N}{\partial \tau} = \frac{\partial c}{\partial \tau} \]

At \( \xi = \frac{r}{d}, \tau = \frac{\nu t}{d} \) laminar flow it is possible to consider three characteristic intervals of the number \( P_e \).

- \( P_e \ll 1 \) - the diffusion has purely molecular character. Then the equation (1.7) in dimensional variables is written in the following way:

\[ \frac{\partial c}{\partial t} \left( \frac{\partial^2 c}{\partial r^2} + \frac{1}{2} \frac{\partial c}{\partial r} \right) = \frac{D}{m_0} \frac{\partial c}{\partial t} \]  

(1.9)

- \( P_e \gg 1 \) - filtration velocities are significant, then we have:

\[ \frac{\partial c}{\partial \xi} + \frac{\partial N}{\partial \tau} + m_0 \frac{\partial c}{\partial \tau} = 0 \]  

(1.10)

Let us consider a case \( P_e \gg 1 \). We will consider that the filtration process is a result of the filtrate constant volume incoming to the stratum for the time \( t = M \), after which the steady-state filtration conditions are set and the productive stratum is divided into two areas. In the first area \( R_1 < r < R \) (\( r \) is the radial coordinate with the origin in the well center, \( R_1 \) is the well radius) the diffusion and the intense mass exchange of the substance with rock (stratal medium) liquid and solid phases take place. We consider the boundary of this area to be constant and equal to \( r = R_1 \). The second area is not occupied with the filtrate, that is why filtrate penetration here is determined only by diffusion. The equation to determine the concentration of the diffusing substance is written in the following way:

\[ u = c \cdot v - D \frac{\partial c}{\partial r} \]
Thus, we have:

\[ p_1(r) = \frac{Q}{2\pi R_1} \ln \frac{r}{R_1}, \quad p_2(r) = \frac{Q}{2\pi R_2} \ln \frac{R_k}{r} \]

\[ Q = \frac{2\pi \rho}{\mu_1} \frac{1}{e_1^2} \ln \frac{R_1}{R_k} + \frac{1}{e_2^2} \ln \frac{R_k}{R_1}, \quad e_1 = \frac{k_1 h}{\mu_1}, \quad e_2 = \frac{k_2 h}{\mu_2} \]

Filtration velocities in each area will be equal to:

\[ u_i = \frac{q_i}{r}, \quad q_1 = \frac{Q}{2\pi R_1}, \quad q_2 = \frac{Q}{2\pi R_2}. \]

Substituting \( u_i \) and \( u_2 \), finally, we obtain:

\[ \frac{\partial c_i}{\partial t} + m_0 \frac{\partial c_i}{\partial r} + \gamma c_i = 0 \quad (\gamma_1 = \gamma, \quad \gamma_2 = 0) \quad (1.13) \]

The equations (1.13), meeting the condition \( c_i = c \) at \( r = R_c, c_i = c \) at \( r = R_k \), are solved by a Laplace transform method in time:

\[ c_1 = c_e e^{\frac{r^2 - R_c^2}{2\theta}} H[t - a_1(r)] \]

\[ c_2 = c_e e^{\frac{R_k^2 - r^2}{2\theta}} H[t - a_2(r)] \quad (1.14) \]

where \( H(\xi) \) is the Heaviside function,

\[ a_1(r) = \frac{r^2 - R_c^2}{2q_1} - m_0, \quad a_2(r) = \frac{r^2 - R_c^2}{2q_2} - \frac{R_k^2 - R_c^2}{2q_1} - m_0 \]

\[ 0 < t < t_*, \quad t_* = \frac{R_k^2 - R_c^2}{2q_2} - \frac{R_k^2 - R_c^2}{2q_1} - m_0 \]

Excluding the shift of the solution filtrate with stratum fluid (for instance, oil), the concentration of substance equals to zero respectively behind the displacement fronts \( t = a_1(r) \) and \( t = a_2(r) \) and is changed exponentially ahead of the front \( t = a_1(r) \) and is constant \( c_e e^{\frac{r^2 - R_c^2}{2\theta}} \) ahead of the front \( t = a_2(r) \).

The function \( N(r, t) \) -concentration of the substance on adsorbent (rock matrix) - is found as per the formula \( \frac{\partial N}{\partial t} = \gamma c \), which gives:

\[ N(r,t) = \frac{7m_0}{2q_1} (R_c^2 - r^2)c_1(r,t) \]

where \( R_a = \sqrt{\frac{r^2 + 2q(t)^2}{m_0}} \) on the well surface \( r = R_a \), we have:

\[ N(R_c,t) = \gamma c(R_c,t) = \gamma c_\ast \cdot t. \]

It follows that the obtained solution is fair only up to the beginning of the surface limitary saturation \( r = R_c \), i.e. at \( t \leq t_{eq} = \frac{N}{\gamma c} \) (\( N_{eq} \) is the value of limitary saturation) whilst \( t_{eq} < t_c \). At \( t \geq t_c \), the impact of the stratum boundaries on the diffusion process shall be taken into consideration.

The Study of the Process of Mud Filtrate Penetration to the Productive Stratum: When carrying out the technological measures, aimed at depth penetration reduction of the mud filtrate during drilling in, there emerges a problem to determine a displacement radius of stratum fluids [5-7]. To solve this problem we use main filtration equations, describing the patterns of fluid motion in stratum. Accepting, that a process of filtrate penetration takes place without the diffusion mixing, we write the continuity equation in case of radiosymmetrical motion of the filter (motion); the expression for displacement radius was obtained:

\[ 2\pi hm \frac{dR_p}{dt} = Q(t) \]  

(1.15)

If the fluid flow is a known function, then, despite the type of stratum and filtration character, we have:

\[ R_p = R_c \sqrt{1 + \frac{W(T)}{\pi h m R_c^2}} \]  

(1.16)

where \( h \) is the stratum depth, \( R_c \) is the radius of displacement front of the stratum fluid, \( m \) is the pore volume, \( Q(t) \) is the flow rate at \( r = R_p \) ( \( r \) is the current stratum radius, \( R_c \) is the well radius), \( W(T) = \int_{0}^{T} Q(t)dt \) is the fluid volume, coming to the stratum in the time \( T \).

Usually (for instance, at overburden on the stratum) the flow rate \( Q(t) \) is an unknown function, depending on the pressure drop \( \Delta P(t) \). In this case it is necessary to determine the flow rate \( Q(t) \) from the solution of the relevant nonsteady filtration problem:

\[ Q(t) = \frac{4\pi \epsilon \Delta P}{\ln \left( \frac{2.25\epsilon t}{R_0^2} + 2S \right)} \]

\[ \Delta P = \text{const} \]

where \( \epsilon = \frac{k}{\mu} \) \( \chi = \frac{k}{2m\mu} \) \( k \) is the permeability, \( \mu \) is the fluid viscosity, \( S \) is the degree of growth of surface resistance at \( S > 0 \) or its reduction at \( S < 0 \).

If \( \Delta P \) is the variable value in time, it is possible to use the approximate formula

\[ Q_n = \frac{4\pi \epsilon \Delta P_n}{\ln \left( \frac{2.25\epsilon (t_n - t_{n-1})}{R_0^2} + 2S \right)} \]

(\( n \geq 2; \ j = 1,2,...,n-1 \)),

where time intervals \( \Delta t_j = t_j - t_{j-1} \) shall meet the condition \( \Delta t_j > R_p^2/4\chi \). In case \( \Delta P = \Delta P_0 = \text{const} \) the form is the following:

\[ R_p = R_c \sqrt{1 + \frac{a\tau}{\tau_x} L_j(\tau_x)}, \]

where \( a = \frac{1.84\Delta P_0}{m\mu}, \tau = \frac{2.25\chi t}{R_0^2}, \tau_x = \tau e^{2S} \).

\[ L_j(\tau_x) = \left( \int_{0}^{\ln \tau_x} \right) \]

is an integral logarithm, tabulated function.

At \( \tau >> 1 \) using the asymptote \( L_j(\tau_x) = \frac{\tau_x}{\ln \tau_x} \) we obtain the function to find the radius of calculation front in time:

\[ R_p = R_c \sqrt{1 + \frac{a\tau}{\ln \tau + 2S}} \]

If the flow rate is calculated as per the formula (1.16), then we do the following.

By value of the parameters \( R_p/4\chi \) choose the time step \( \Delta t >> \frac{R_p^2}{4\chi} \), meeting the condition, compile a table of values \( \Delta t_j = t_{j+1} - t_j \) for the relative time moments \( t_n = n\Delta t_0 \) and at set value \( \delta \), we sequentially calculate \( Q_n = Q(t_n) \).

Then time interval \( M \) is divided by \( n \) equal intervals, where it is determined from the condition \( n >> \frac{4M \Delta P}{R_0} \) (\( E \) is the integral part of a number E). Then the function \( W(t) \) is approximately calculated using a formula:
The formula (1.19) can be also used for evaluation of penetration radius of cementing slurry, when mounting the lost circulation horizon, or acid solutions, when processing the bottom-hole area, if the colmatage area is destroyed. Simple calculation by formula (1.19) shows that if the clay coating is removed or destroyed during well cementing, then the filtrate penetration radius of the component solution can be quite considerable.

Evaluation Criteria of Quality of Drilling-in and Well Completion: The quality of drilling-in and well completion is the variation degree of productivity (water permeability) of the stratum. Quality factors are the parameters of OP or their derivatives. It is recommended to determine these factors based on the data of direct full-scale pressure transient analysis or the stratum testing by traditional methods [7-9]. Consideration of evaluation criteria of quality is based on methods of mathematical statistic.

Data interpretation of well survey and stratum tests is recommended to be carried out by two - tree familiar theoretically substantiated methods, one of which is the operation method, characterized by universality and interpretative advantages [10].

Conventionally, it shall be accepted that a completion technique includes the following controlled elements: drilling method, drilling practices, flushing regime, type and blend composition of drilling agent, time period from the full drilling-in to the moment of beginning of cementing.

Similarly, it shall be accepted that a technology of well completion includes the following elements: mounting method, perforation method, method of fluid stimulation from the stratum.

Quality evaluation of well completion shall be preceded by the quality evaluation of drilling-in. The technology quality factor in whole or of its separate elements is the value of relation of factual water permeability $E$ (productivity) of the stratum to potential $E_{\pi}$[11]:

$$O\Pi = E_{\phi}/E_{\pi}$$

or the skin effect factor $S$, characterizing additional filtration resistance (or conductivity) of bottom-hole area in case of its contaminating (cleaning).

In the formulas below for definiteness it is expected that the OP factor is distributed based on normal law [12]. Otherwise, it shall be replaced with $O\Pi = \varepsilon_{\phi}/\varepsilon_{\pi}$, where $\varepsilon_{\phi}$ is a value of relation of factual water permeability...
(productivity) of the stratum to the potential $e$. Besides, it is accepted that the validity of all estimates is not less than 90%.

To evaluate the quality of technology in one well, it is necessary:

- To determine the average value $O\bar{\Pi} = \frac{1}{m} \sum_{i=1}^{m} O\Pi_i$.
- Error mean square of measurements $\sigma = \frac{1}{m} \sum_{i=1}^{m} (O\Pi_i - O\bar{\Pi})^2$.
- To test a hypothesis of variation of stratum water permeability based on Student $t$-criterion

$$\frac{|O\Pi_i - O\bar{\Pi}| \sqrt{m}}{\sigma} > t$$

(1.20)

where $m$ is the number of measurements; $t$ is determined in dependence on the number of degrees of freedom $q = m - 1$.

If inequation (1.20) is fulfilled, then there is the following conclusion: water permeability of the stratum is changed [13], the factor $O\bar{\Pi}_{<1}$ (or $O\bar{\Pi}_{>1}$) characterizes the quality of the appropriate technology in relation to the stratum set point. If inequation (1.20) is not fulfilled, then there is the following conclusion: water permeability of the stratum is not changed; quality of the technology is characterized by OP factor, close to one. If modified factors $ln O\Pi$, $S$ or $\sqrt{S}$ are used, then in the criterion (1.20) 1 shall be replaced with 0.

To evaluate the quality of technology based on group from $n$ wells, it is necessary to determine:

- Average value and error mean square based on the measurement results in each $j$-well

$$O\bar{\Pi}_j = \frac{1}{m} \sum_{i=1}^{m} O\Pi_{ji}, \quad \sigma_j = \frac{1}{m-1} \sum_{i=1}^{m} (O\Pi_{ji} - O\bar{\Pi}_j)^2;$$

- Average value of error mean square

$$\sigma_{BOC} = \frac{1}{n} \sum_{j=1}^{n} \sigma_j;$$

- Average value and dispersion of OP factor of the object

$$O\bar{\Pi} = \frac{1}{n} \sum_{j=1}^{n} O\Pi_j, \quad \sigma = \frac{1}{n-1} \sum_{j=1}^{n} (O\Pi_j - O\Pi)^2;$$

- To test a hypothesis of validity of OP factor of the object

$$t\sqrt{\sigma^2/n} \leq \sigma_{BOC}$$

(1.21)

where $t$ is the value of Student criterion in dependence on number of degrees of freedom $q = n - 1$ [14].

If inequation (1.21) is fulfilled, then there is a conclusion that, the quantity of wells is sufficient to evaluate the quality of technology with the accuracy, not exceeding $\sigma_{BOC}$. The quality of technology shall be evaluated based on the Student $t$-criterion (1.19), where $m$ shall be replaced with $n$.

If inequation (1.21) is not fulfilled, then it is necessary to complete the data on the OP factor by means of investigations in other wells, introducing them one by one, until the condition (1.21) is not fulfilled; or to distribute the data based on physical and (or) geological properties and to evaluate each group of well separately.

**REFERENCES**


