Some Examples for Self Developing Informal Knowledge Models in Realistic Mathematics Education

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Abstract: The aim of this paper is to determine “self-developing informal knowledge models” bridging informal mathematics knowledge and formal mathematics knowledge those of which are principles of Realistic Mathematics Education. With this in mind, this paper aims to find out and exemplify what kind of informal methods or formulae are generated by secondary school students in education of some issues of mathematic course and in solutions of the problems in the related issues.

Key words: Realistic mathematics education • Mathematics teaching • Mathematics learning

INTRODUCTION

Realistic Mathematics Education (RME) is a theory of mathematics teaching which was developed in 1971 at Utrecht University Freudental Institute in Netherland by a dutch mathematician and educationalist Hans Freudenthal [1-3]. This theory have been valued in many countries like England, Germany, Denmark, Spain, Portugal, South Africa, Brasil, USA, Japan and Malaysia [4]. According to RME mathematics is learned by doing or living as it is in real life. Freudenthal states that the process of mathematics learning starts with real life problems, mathematical notions and formulae are reached afterward.

Moreover, Freudenthal introduced mathematics learning as an interpretation process and expressed his view as “mathematics for a child starts with interpretation and in order to study mathematics interpretation should be grounded on at each new phase” [5].

In RME, mathematics should not be a closed system that is to be taught; instead it should be perceived as a human activity. It is not explored, it is invented. Altun [6] states that social facts and necessities made people study mathematics. Like other kinds of knowledge, mathematics is also a product of human discoveries and social activities. It does not have a static nature which does not change. It emerges from reality and develops continuously with individual and block learning processes [7].

In RME, abstract principles or rules do not serve as a start point to learn practice at concrete situations or mathematics knowledge is not focused as an aiding knowledge [8]. According to this approach, which is a challenge against conventional teaching approaches, mathematics teaching should start with real lifelike problems and the necessity to study mathematics should be the major principle of teaching [6]. In relation to conventional approach, mathematics is a system of rules and algorithms. Verification and practice of these rules with problems similar to those of which were solved beforehand aim to use standard problem solving methods which are taught generally with individual exercises.

RME approach advocates that mathematics is an organized and top-down system. Therefore, learning process of mathematics education should be adapted to this system. In RME, learning might be interpreted as a problem solving process. Freudenthal states that mathematics, in history, started with real lifelike problems and it reached to a formal system afterwards. He remarks that such a learning, which gives formal mathematical knowledge first and then makes practises, is not an educational (anti-didactic) system. Studying mathematics in RME means learning mathematics. Solving contextual problems is a crucial part in this approach [9]. Another important part is that students should be given opportunities in the formulation of mathematical notions and there should be given a high-quality interaction in teaching-learning process. In short RME might be defined with the following five features: [10, 11].

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The use of contextual knowledge.
The use of models.
The use of students’ knowledge and comments.
Features affecting each other in teaching process.
The emergence of diverse learning difficulties.

RME postulates that models in problem solving and interpretation of issues are developed by students informally. These models developed by students are invaluable for themselves. When these models are formulated they turn out to be suitable for mathematical thought and serve like a bridge between formal and informal knowledge [12-15].

Therefore this paper mainly concentrates on determining “self-developing informal knowledge models” bridging informal mathematics knowledge and formal mathematics knowledge those of which are principles of Realistic Mathematics Education.

MATERIAL AND METHODS

Ten voluntary high school final year students from a private educational institute at Kocaeli (Turkey) were asked to answer the following question so as to determine whether they develop a method or formula to solve problems in any topic in the courses of mathematics or geometry.

“Do you use an informal formula or method (rule) in the courses of mathematics or geometry while you are learning a topic or solving a problem with the related topic in terms of making it easier?”

The answers given by the students were evaluated by interview method.

Findings and Comments: Findings and comments regarding the research question were exemplified and explanations regarding how the formula (rule) works were made. Students stated that their informal methods are known by almost all of the students enrolled in secondary schools in Turkey and they even exist supplementary books.

Rule 1: In area measurement of integral practice, when a symmetrical curve to y-axis is given, if the area interior to curve is called 2A and the area exterior to curve is called A, the wanted region’s area is measured easily with the area formula of the formed rectangular.

Example 1: Find the area of the region formed by \( y = x^2 \) curve, x axis and x=1 line.

\[
\int_0^2 x^2 \, dx = \frac{1}{3} x^3 \bigg|_0^2 = \frac{1}{3} 8 - 0 = \frac{8}{3}
\]

Informal solution: \( 2A = 1 \Rightarrow A = \frac{1}{3} b^2 \).

Example 2: Find the area of the region between \( y = 2(x - 2) \) curve, x-axis and y-axis.

Informal solution:

\[
\frac{16}{3}
\]

Formal solution;

\[
A = \int_0^2 (2x^2 - 8x + 8) \, dx = \left[ \frac{2}{3} x^3 - 4x^2 + 8x \right]_0^2 = \frac{16}{3} b^2
\]

This method given by the students is consistent with these examples.

But this rule does not give the wanted result for the area of the region verged by \( y = x^2 + 1 \) function, x-axis and x=1 line. Because;

Informal solution: \( 2A = 2 \Rightarrow A = \frac{2}{3} b^2 \).
Formal solution: \[
A = \int_0^1 (x^2 + 1) \, dx = \left[ \frac{1}{3} x^3 + x \right]_0^1 = \frac{4}{3} + 2 = \frac{10}{3}.
\]

As it is obvious from the example, the given rule is informal.

**Rule 2. Lapte:** Formula in partial integration is almost known by every student graduated from secondary education. As it is known;

\[
\int (f(x)g(x) \, dx = \int u \, dv = \int udv - \int vdu, \quad g(x)dx = dv)
\]

In terms of calculating the integral easily, choosing which function will be \( u \) and which function will be \( dv \) is crucial. In order to use LAPTE formula, the functions in integral are checked whether they are Logarithmic, Arc, Polynomial, Trigonometric and Exponential or not.

If there are such functions in the integral, the first function is taken as \( u \) and the others are taken as \( dv \). The acronym formed with the first letters of these functions “LAPTE” facilitates the students’ work with ease.

**Example 3:** Since there are logarithmic and exponential functions in the measurement of \( \int x^2 \) integral, it can be easily calculated if \( \ln x = u \), \( x^2 \, dx = dv \) according to the formula. In the solution of such problems it is apparent that the given formula contributes to the solution to a great extent. Yet, the order in “LAPTE” formula is not important in the calculation of \( \int x \sin x \, dx \) integral.

Therefore this is also an informal knowledge.

**Rule 3:** An example regarding the method developed in controlling whether numbers (especially in multiple-digit numbers) are multiplied correctly or not in a multiplication:

\[
\begin{array}{c|c}
672 & 341 \\
\hline
672 & \\
2688 & + \\
2016 & \underline{229152}
\end{array}
\]

The following processes are done, respectively, in order to check whether the multiplication is done correctly or not:

- Remainder 6, which is formed from the division of sum of number values of multipliers’ numerals \( 6+7+2=15 \), written to top, 6

- Remainder 8, which is formed from the division of sum of number values of multiplicands’ numerals \( 3+4+1=8 \), written to bottom, 8

- These two numbers are multiplied \( 8\times 6=48 \) and divided into 9. The remainder 3 is written to right, \( \frac{6}{3} \)

- Sum of the number values of the product \( 2+2+9+1+5+2=21 \) is divided into 9 and the remainder 3 is written to the left, \( \frac{3}{8} \)

- If the numbers at the left and right of the checking process are the same, the multiplication is correct.

However, the method in Rule 3 may not be true all the time. For instance; If we write multiplication process as \( 672 \times 341 = 221952 \), the method will be true but the result will be false. Therefore, this method is informal.

**Rule 4:** The use of “Trigonometry Window” in transformation and inverse transformation formulae in Trigonometry

\[
\begin{array}{c}
C + C \\
\hline
2 & 1 \\
c & c \\
S + S & 2 \\
\hline
S & S \\
1 & 1 \\
2 & 2 \\
\end{array}
\]

Therefore, this method is informal.
In the above figure the transformation formulae are obtained from outside to inside and inverse transformation formulae are obtained from inside to outside. As it is known;

\[
\sin a + \sin b = 2 \sin \left( \frac{a + b}{2} \right) \cos \left( \frac{a - b}{2} \right), \quad \cos a - \cos b = -2 \sin \left( \frac{a + b}{2} \right) \sin \left( \frac{a - b}{2} \right)
\]

\[
\sin a - \sin b = 2 \cos \left( \frac{a + b}{2} \right) \sin \left( \frac{a - b}{2} \right), \quad \cos a + \cos b = 2 \cos \left( \frac{a + b}{2} \right) \cos \left( \frac{a - b}{2} \right)
\]

\[
\sin a \sin b = \frac{1}{2} \left[ \cos(a-b) - \cos(a+b) \right], \quad \cos a \cos b = \frac{1}{2} \left[ \cos(a+b) + \cos(a-b) \right]
\]

For example, while obtaining \( \sin a + \sin b \) formula one should go in to the window from S + S side. As it is seen, coefficient of the formula comes from \( 2 \div 1 \). Then, the first letter inside is written \( \frac{a + b}{2} \). The second letter is written \( \frac{a - b}{2} \) and the formula is obtained.

Contrarily, while obtaining inverse transformation formula one should go out from inside to outside. For example, in order to obtain \( \cos a. \cos b \) formula the coefficient of the formula comes from \( 1 \div 2 \). Later on, the first letter outside, from left to right, is written \( (a+b) \) and the second letter is written \( (a-b) \). So the formula is ready. The other formulae are also obtained likewise.

Since this kind of models are highly relative from one student to another, they are informal.

**Rule 5:** Some other students associate a geometrical shape to an object and learn the features of that geometrical shape in the name of that object.

![Image of a geometrical shape with m(BOC) = a + b + c and a rocket with m(BOC) = a + b + c]

Since the geometrical shape is associated with a rocket, the feature of this geometrical shape is known as rocket rule.

**Rule 6:**

If \( \frac{BD}{AD} \) is a bisector
\[ z = \frac{x + y}{2} \]

This geometrical shape, on the other hand, is associated with a kite and therefore this geometrical shape is known as kite rule.

As the associations in Rule 5 and 6 may change according to the perceptions of the students, they are informal knowledge.

**Rule 7:** If there is a bisector in any of the angles of a quadrilateral, there is an isosceles triangle, \( |AD| = |DE| \)

![Image of a quadrilateral with a bisector]

The given feature contributes only to the solutions related with parallelogram. The use of this feature in other geometrical shapes does not work. Therefore it is an informal knowledge.

**CONCLUSION**

Informal rules (knowledge), self-developed by the students in the process of mathematics teaching-learning, are very practical and provide great convenience. The emergence of such models is normal in every phase of education-teaching process. Since the learning process is subjective, students can develop both formal and informal knowledge depending on their styles of learning. As it is seen from the examples, reaching the correct results with the help of these informal knowledge and models is an enormous source of motivation in terms of studying mathematics.

**REFERENCES**