Acceptance Double Sampling Plan Using Fuzzy Poisson Distribution

Ezzatallah Baloui Jamkhaneh and Bahram Sadeghpour Gildeh

Department of Statistics, Qaemshahr Branch, Islamic Azad University, Qaemshahr, Iran
Department of Statistics, Faculty of Basic Science, University of Mazandaran, Babolsar, Iran

Abstract: All classical acceptance sampling plans were constructed with the proportion of defective that is crisp value. In the manufacturing processes, this parameter in a production lot may not be precisely known. Hence it is meaningful to treat such parameter as fuzzy quantities. In this paper we have argued for the acceptance double sampling plan when the proportion of defective items is a fuzzy number and it is being modeled based on the fuzzy Poisson distribution. The calculation of the operating characteristic (OC) curves, average sample number (ASN) curves, average outgoing quality (AOQ) curves and average total inspection (ATI) curves of the plan will be presented for double plan by using the concept of fuzzy probability. The results show that these four curves are like band having high and low bounds with their width depending on the ambiguity proportion parameter in the lot, when the size and acceptance numbers of the samples are fixed. We have also shown that in this plan, if the process quality is perfect or poor, then the ASN and AOQ bands will be of lower value.

Key words: Statistical quality control • Acceptance double sampling • Fuzzy numbers • Average sample number • Average outgoing quality • Average total inspection

INTRODUCTION

Acceptance sampling plan is one of the most important components of statistical quality control. In this field, acceptance double sampling plan is one of the sampling methods for acceptance or rejection of a lot with classical attribute quality characteristics.

In different acceptance sampling plans the proportion of defective items, is considered as a precise value, but sometimes we are not able to obtain exact numerical value and there also exists some uncertainty in the value obtained from experiments, personal judgment or estimation. However, the quality characteristics in a lot are not often exact and certain. The fuzzy set theory is a mathematical model of vague data or uncertain values that are frequently generated by experiments, estimation or means of natural language. This theory is a powerful and well-known tool to formulate and analyze the parameters that cannot be estimated accurately. In dealing with the above problem, we tried to determine the uncertainty existing in the problem by defining the imprecise parameter as a fuzzy number, to achieve a result with a higher certainty. In the acceptance sampling plan, specifically double sampling plan, it is well known that the probability distribution plays a crucial role. Since in the traditional probability distributions, the parameters are assumed to be precise values, difficulties arise when the parameters become imprecise. Classical acceptance sampling plans have been studied by many researchers. They are thoroughly elaborated by Schilling [1]. Single sampling by attributes with relaxed requirements was discussed by Ohta and Ichihashi [2], Kanagawa and Ohta [3], Tamaki, Kanagawa and Ohta [4] and Grzegorzewski [5]. A sampling plan by attributes for vague data was considered by Hrniewicz [6, 7]. Chakraborty [8, 9] addresses the problem of designing single stage Dodge Romig lot tolerance percent defective (LTPD) sampling plans when the lot tolerance percent defective, consumer's risk and incoming quality level are modeled using triangular fuzzy numbers.

Grzegrozewski [10] also considered sampling plans by variables with fuzzy requirements. Dodge and Romig [11] provided the rectifying single sampling plan and double sampling plans for attributes with the protection of the lots tolerance percent defective (LTPD) or of the average outgoing quality limit (AOQL). Determination of rectifying plans for single sampling by attributes was discussed by Guenther [12]. Suresh and Ramkumar [13] justified
the use of maximum allowable average outgoing quality for developing a sampling plan. Hrniewicz [14] has shown why in the case of imprecise input information optimal inspection intervals are usually determined using additional preference measures than strict optimization techniques. Hrniewicz [15] provided a short overview of basic problems of statistical quality control that have been solved by using of the probability theory and the fuzzy set theory. Finally the properties of sampling plan under situations involving both impreciseness and randomness was studied by Sampath [16] Baloui et al. [17-20] considered acceptance sampling plan under the conditions of the fuzzy parameter. Hence a new approach is necessary in designing of sampling plan. Here, the fuzzy probability theory attributed to Buckly [21] is used to explore the possibility of introducing a suitable sampling plan for a situation having imprecision and the main characteristics of a double sampling plan such as OC, ASN, AOQ and ATI calculating have been solved by using of the probability theory and the fuzzy set theory. Finally the properties of sampling plan under situations involving both impreciseness and non-fuzzy set defined as . Hence we have , where the base of the triangle is the interval and vertex is at .

**Definition 1:** [22] A triangular fuzzy number is fuzzy number that membership function defined by three number .

\[
N^i[\alpha] = \{x \in \mathbb{R} | \mu_X(x) \geq \alpha \},
\]

where \( N^i[\alpha] = \sup\{x \in \mathbb{R} | \mu_X(x) \geq \alpha \} \) and \( N^i[\alpha] = \inf\{x \in \mathbb{R} | \mu_X(x) \geq \alpha \} \).

**Definition 2:** [22] The \( \alpha \)-cut of a fuzzy number \( N \) is a non-fuzzy set defined as \( N^\alpha = \{x \in \mathbb{R} | \mu_X(x) \geq \alpha \} \). Hence we have \( N^\alpha = \sup\{x \in \mathbb{R} | \mu_X(x) \geq \alpha \} \).

**Definition 3:** [22] The \( \alpha \)-cut of a fuzzy number \( N \) is a non-fuzzy set defined as \( N^\alpha = \{x \in \mathbb{R} | \mu_X(x) \geq \alpha \} \). Hence we have \( N^\alpha = \sup\{x \in \mathbb{R} | \mu_X(x) \geq \alpha \} \).

**Definition 4:** [21] Let \( k_i \), \( i=1, \ldots, n \) are fuzzy numbers and \( \tilde{P} \) is a discrete fuzzy probability function. We write \( \tilde{P} \) for fuzzy \( P \) and \( \tilde{P}(x_i) = \tilde{k}_i \), where the reference of \( \tilde{k}_i \) is \([0,1] \). Let \( A = \{x_1, \ldots, x_n\} \) be a subset of \( X \) and then define:

\[
\tilde{P}(A)[\alpha] = \left\{ \sum_{i=1}^{n} \tilde{k}_i \mid S \right\}
\]

for \( 0 < \alpha < 1 \), where \( S \) is the statement “\( \sum_{i=1}^{n} \tilde{k}_i \in [\alpha] \), \( \sum_{i=1}^{n} \tilde{k}_i = 1 \)” this is our restricted fuzzy arithmetic. The fuzzy mean is defined by its \( \alpha \)-cuts:

\[
\tilde{\mu}[\alpha] = \left\{ \sum_{i=1}^{n} x_i \tilde{k}_i \mid S \right\}
\]

Where, \( S \) as is before.

**Definition 5:** [21] In m independent Bernoulli experiment let us assume that \( p \), probability of a “success” in each experiment is not known precisely and needs to be estimated, or obtained from expert opinion. So that \( p \) value is uncertain and we substitute \( \tilde{p} \) for \( p \) and \( \tilde{q} \) for \( q \) so that there is a \( p \in [\tilde{p}] \) and a \( q \in [\tilde{q}] \) with \( p + q = 1 \).

Now let \( \tilde{P}(r) \) be the fuzzy probability of \( r \) successes in \( m \) independent trials of the experiment. Under our restricted fuzzy algebra we obtain

\[
\tilde{P}(r)[\alpha] = \{C_\alpha p^r q^{m-r} \mid S\}
\]

for \( 0 < \alpha < 1 \), where now \( S \) is the statement, “\( r \in [\tilde{p}] \), \( q \in [\tilde{q}] \), \( p + q = 1 \) ”.

If \( \tilde{P}(r)[\alpha] = \{P^\alpha[\alpha], P^\beta[\alpha] \} \) then

\[
P^\alpha[\alpha] = \min\{C_\alpha p^r q^{m-r} \mid S\}
\]

\[
P^\beta[\alpha] = \max\{C_\alpha p^r q^{m-r} \mid S\}
\]
And if \( \tilde{P}_{[a,b]} \) be the fuzzy probability of \( x \) successes so that \( a < x < b \), then

\[
P((a,b])[\alpha] = \left\{ \sum_{x=a}^{b} C_{n}^{x} p^{x} (1-p)^{n-x} \mid S \right\}
\]

if \( \tilde{P}_{[a,b]} \) then:

\[
P^{*}((a,b])[\alpha] = \min \left\{ \sum_{x=a}^{b} C_{n}^{x} p^{x} (1-p)^{n-x} \mid S \right\}
\]

\[
P^{*}((a,b])[\alpha] = \max \left\{ \sum_{x=a}^{b} C_{n}^{x} p^{x} (1-p)^{n-x} \mid S \right\}
\]

Where \( S \) is the same with past case.

**Definition 6:** [21] Let \( X \) be a random variable having the Poisson mass function. If \( P(x) \) stands for the probability that \( X = x \), then

\[
P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!},
\]

for \( x = 0, 1, 2, \ldots \) and parameter \( \lambda > 0 \).

Now substitute fuzzy number \( \tilde{\lambda} > 0 \) for \( \lambda \) to produce the fuzzy Poisson mass function. Let \( \tilde{P}(x) \) to be the fuzzy probability that \( X = x \). Then we find \( \alpha \)-cut of this fuzzy number as

\[
\tilde{P}(x)[\alpha] = \frac{e^{-\tilde{\lambda}} \tilde{\lambda}^{x}}{x!} \left[ \lambda \in \tilde{\lambda}[\alpha] \right]
\]

for all \( \alpha \in [0, 1] \). Let \( X \) be a random variable having the fuzzy binomial distribution and \( \tilde{\lambda} \) in the definition 4 be small, which means that all \( p \in \tilde{\lambda}[\alpha] \) are sufficiently small, then we approximate the \( \alpha \)-cut of fuzzy probability, \( \tilde{P}(a \leq X \leq b)[\alpha] \) by using the fuzzy Poisson distribution:

\[
\tilde{P}(a \leq X \leq b)[\alpha] = \sum_{x=a}^{b} \frac{e^{-\tilde{\lambda}} \tilde{\lambda}^{x}}{x!} \left[ \lambda \in \tilde{\lambda}[\alpha] \right].
\]

0 \leq \alpha \leq 1

**ACCEPTANCE DOUBLE SAMPLING PLAN WITH FUZZY PARAMETER**

In this section, first we introduce the double sampling plan for classical attribute characteristics. Suppose that we want to inspect a lot with a size of \( N \). Then the operating procedure of the double sampling plan is given by the following steps:

**Step 1:** First draw a random sample of size \( n_{1} \), and observe the number of nonconforming items \( D_{1} \).

**Step 2:** If \( D_{1} \leq c_{1} \), the first stage acceptance number, accept the lot. If \( D_{1} > c_{1} \), second stage acceptance number, reject the lot. If \( c_{1} < D_{1} \leq c_{2} \), go to Step 3.

**Step 3:** Take a second random sample of size \( n_{2} \) and observe the number of nonconforming items \( D_{2} \). Cumulate \( D_{1} \) and \( D_{2} \), if \( D_{1} + D_{2} \leq c_{2} \), accept the lot. If \( D_{1} + D_{2} > c_{2} \), reject the lot.

If the size of the lot is very large, the random variables \( D_{1} \) and \( D_{2} \) have binomial distribution with parameters \( (n_{1}, p) \) and \( (n_{2}, p) \), in which \( p \) indicates the fraction of nonconforming items of the lot [23]. However, if the size of the sample be large and \( p \) is small then the random variable \( D_{1} \) and \( D_{2} \) has a Poisson approximation distribution with parameter \( \lambda_{1} = n_{1} p \) and \( \lambda_{2} = n_{2} p \). So, if we represent probability of acceptance on combined samples with \( p_{s} \) and also the probability of the lot’s acceptance in first and second samples by \( p_{s}', p_{s}'' \), respectively, then

\[
p_{s} = p_{s}' + p_{s}'' \quad \text{Where } p_{s}' \text{ indicates the probability of the event } D_{1} \leq c_{1}.
\]

Thus

\[
p_{s}' = \sum_{x=0}^{c_{1}} e^{-n_{1} p} \left( n_{1} p \right)^{x} / x!,
\]

and \( p_{s}'' \) according to the independence of two random variables and their distributions will be calculated as follows:

\[
p_{s}'' = P(D_{1} + D_{2} \leq c_{1}, c_{1} < D_{1} < c_{2})
\]

Suppose that we want to inspect a lot with the size of \( N \), in which the proportion of defective items or the probability of the defectiveness is not known precisely and it is an uncertain value. So we represent this parameter with a fuzzy number \( \tilde{p} \) which is:

\[
\tilde{p} = (a_{1}, a_{2}, a_{3}), p \in \tilde{p}[\alpha] \equiv [a_{1}, a_{2}], 0 < a_{1} < a_{2} < 1 \quad \text{A double sampling plan with a fuzzy parameter is defined by the first sample size } n_{1}, \text{ the acceptance number on the first stage } c_{1}, \text{ the second sample size } n_{2}, \text{ the acceptance number on the second stage } c_{2}.
\]

If the size of the lot is very great, the random variables \( D_{1} \) and \( D_{2} \) have a fuzzy binomial probability distributions with parameters \( (n_{1}, p) \) and \( (n_{2}, p) \), in which \( p \) indicates the fuzzy proportion of the defective items. And if the \( p \) is small, then the random variables \( D_{1} \) and \( D_{2} \) have a fuzzy Poisson distribution with parameters \( \lambda_{1} = n_{1} p \) and \( \lambda_{2} = n_{2} p \) [20]. According to this case, if we show the fuzzy probability of the acceptance of lot in the combined samples with \( \tilde{p} \), and also the fuzzy probability of acceptance of the lot in the first and second samples, \( \tilde{p}', \tilde{p}'' \), respectively, then

\[
\tilde{p}_{s} = \{ \tilde{p}_{s}', \tilde{p}_{s}'' \} \quad \text{S} \}
\]
Example 1: Suppose that $\hat{p}=(0.01,0.02,0.03)$, $\hat{q}=(0.97,0.98,0.99)$, $n_1=10$, $n_2=10$, $c_1=0$ and $c_1=1$, then fuzzy probability of acceptance of this lot is as follows:

$$\tilde{p}[\alpha] = [0.01 + 0.01\alpha, 0.03 - 0.01\alpha]$$

Since $\frac{\partial(e^{-0.1\alpha})}{\partial p} < 0$ on $\tilde{p}[\alpha] = [0.01 + 0.01\alpha, 0.03 - 0.01\alpha]$, we obtain

$$\tilde{p}_d^U[\alpha] = e^{-0.1\alpha} \cdot e^{-(0.1+0.1\alpha)}.$$ 

Under $\alpha = 0$ we gain

$$\tilde{p}_d^U[0] = [0.7408, 0.9048].$$

Since $\frac{\partial(10pe^{-20p})}{\partial p} > 0$ on $\tilde{p}[\alpha]$ therefore

$$\tilde{p}_u^U[\alpha] = [(0.1 + 0.1\alpha)e^{-(0.2+0.2\alpha)}, (0.3 - 0.1\alpha)e^{-(0.6-0.2\alpha)}],$$

under $\alpha = 0$ we obtain

$$\tilde{p}_u^U[0] = [0.0819, 0.1646].$$

then

$$\tilde{p}_u[0] = [0.9055, 0.9867]$$

OC BAND WITH FUZZY PARAMETER

In assessing a sampling plan, one of the important criteria is its operating characteristic (OC) curve. This curve plots the probability of accepting the lot versus the proportion of nonconforming items in the lot. The OC curves are the primary tool for displaying and investigating the properties of a lot acceptance sampling plan. A double sampling plan has a main OC curve that shows the probability of acceptance on combined samples. This plan also has another OC curve which indicates the probability of an acceptance based on the first sample.

In a double sampling plan with a fuzzy parameter, OC curves occur as a band with upward and downward bounds. This band indicates the fuzzy probability of acceptance of the lot in terms of different values of the fuzzy proportion of nonconforming items. The degree of the uncertainty of a proportion parameter is one of the factors that bandwidth depends on. The lesser uncertainty value results in less bandwidth and if the proportion parameter gets a crisp value, the lower and upper bounds will become equal, so that the OC curve will be in classic state. Knowing the degree of the uncertainty of proportion parameter (given as $a_1$, $a_2$, $a_3$) and the variation of proportion parameter position on a horizontal axis, we have a different fuzzy number $(\tilde{p})$ which the FOC bands are plotted in terms of it. To achieve this aim, we consider the structure of $\tilde{p}$ as follows:

$$\tilde{p} = (k, b_2 + k, b_3 + k)$$

$$p \in \tilde{p}[1], q \in \tilde{q}[1], p + q = 1,$$

$$b_1 = a_i - a_1, i = 2,3$$

$$\tilde{\lambda} = n_1 \tilde{p} = (n_1 k, n_1 b_2 + n_1 k, n_1 b_3 + n_1 k) \text{ and } i=1,2,$$

with this variation of $k$ in the domain of $[0,1-b_i]$, then we find the $\alpha$-cut of these fuzzy numbers as

$$\tilde{p}[\alpha] = [p^L[\alpha], p^U[\alpha]]$$

$$= [k + b_2\alpha, b_3 + k - (b_3 - b_2)\alpha],$$

$$\tilde{\lambda}[\alpha] = \left[\lambda^L[\alpha], \lambda^U[\alpha]\right]$$

$$= \left[n_1 k + n_1 b_2\alpha, n_1 k + n_1 b_3 - n_1 (b_3 - b_2)k\right],$$

$$i = 1,2$$

The main FOC band is plotted according to the calculation of the following fuzzy probability:
Fig 2: FOC band for a double sampling plan with fuzzy parameter of \( n_1 = 10, c_1 = 1, n_2 = 5, c_2 = 2 \),
\[
\tilde{p}_a = \tilde{P}_a^I \tilde{P}_a^U = \tilde{P}_a(A_i)[\alpha] = [\tilde{P}^L_{k,A_i}[\alpha], \tilde{P}^U_{k,A_i}[\alpha]] \quad i = I, II
\]
\[
\tilde{P}^L_{k,A_i}[\alpha] = \min \{P(A_i) | S\} \quad \text{and} \quad \tilde{P}^U_{k,A_i}[\alpha] = \max \{P(A_i) | S\}, i = I, II,
\]
Where S denotes the statement \( p \in \mathbb{P} \cup \mathbb{Q} \), \( p + q = 1 \), and the event \( A_i \) is the event of acceptance of a lot in terms of the sample \( i \)th and the FOC band which in terms of the first sample is plotted according to the calculation of the following fuzzy probability:
\[
\tilde{p}_a^I = \tilde{P}_a^I(A_i)[\alpha] = [\tilde{P}^L_{k,A_i}(\alpha), \tilde{P}^U_{k,A_i}(\alpha)]
\]
\[
P^L_{k,A_i}(\alpha) = \min \left\{ d_0, \frac{e^{-x_0} \lambda_0}{\lambda_0} \right\}
\]
\[
P^U_{k,A_i}(\alpha) = \max \left\{ d_0, \frac{e^{-x_0} \lambda_0}{\lambda_0} \right\}
\]

**Example 2:** Suppose that \( n_1 = 10, c_1 = 1, n_2 = 5, c_2 = 2, b_2 = 0.01, b_3 = 0.02 \) then we have
\[
\tilde{p}(\alpha) = (k + 0.01\alpha, 0.02 + k - 0.01\alpha), 0 < k < 0.98
\]
then fuzzy probability of acceptance on first sample is:
\[
\tilde{p}_a^I(\alpha) = \tilde{P}_a^I[0,1][\alpha] = \{10p + 1\}e^{-10p} | S\},
\]
\[
f_1(p) = (10p + 1)e^{-10p}
\]
according to that the \( f_1(p) \) decreasing, then:
\[
\tilde{p}_a^I[0] = \{10k + 1.2\}e^{-(10k + 0.2)} , (10k + 1)e^{-10k}
\]
and fuzzy probability of acceptance on the second sample is:
\[
\tilde{p}_a^U[\alpha] = \tilde{P}(D_1 = 2, D_2 = 0) = \{50p^2e^{-15p} | S\},
\]
\[
f_2(p) = 50p^2e^{-15p}
\]

Then, by studying \( f_2(p) \) function, we will have the \( \alpha \)-cut in the following fashion:
\[
\begin{array}{|c|c|}
\hline
\text{Cut} & \text{Range} \\
\hline
\frac{2}{15} & 0 < k < 0.98 \\
\frac{1}{17} & 0.12358 \\
\frac{17}{10} & 0.1203 \\
\frac{1}{15} & 0 < k < 0.12358 \\
\frac{15}{10} & 0.1203 \\
\frac{15}{10} & 0.02 \\
\hline
\end{array}
\]

Figure 2 represents a state when the process quality decreases from a perfect state to a moderate state and then the FOC band will be wider. Using fuzzy arithmetic and simplification, Table 1 will be calculated.

**FUZZY AVERAGE SAMPLE NUMBER**

The main advantage of a double sampling plan is the reduced average sample size, required to arrive to a good decision. In this plan the sample number (SN) can be only \( n_1 \) or \( n_1 + n_2 \). The \( \tilde{p}_l \) is the fuzzy probability of drawing a "first" sample which occurs when arriving at a decision at the first sample (with fuzzy probability \( \tilde{P}(D_1 < c_1) \)). The fuzzy probability \( \tilde{p}_l \) of having to draw a second sample, totaling a size of \( n_1 + n_2 \) occurs when we have an "inconclusive" outcome from the first sample (with fuzzy probability \( \tilde{P}(D_1 = c_1) \)).

The FASN is calculated with the formula of the fuzzy mean:
\[
\text{FASN} = \tilde{\mu}_{SN}[\alpha] = \{n_1p_l + (n_1 + n_2)p_{II} | S\}
\]

Where, as before, \( S \) denotes the statement \( "p_l \in \tilde{p}(\alpha)\}_{s = I, II, p_l + p_{II} = 1" \). Hence we get
\[
\text{FASN} = \{n_1 + n_2p_{II} | s\}
\]
Table 1: Fuzzy probability of acceptance with $c_1=1, n_1=10$, $c_2=2, n_2=5$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$p_a^I$</th>
<th>$p_a^H$</th>
<th>$p_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.02]</td>
<td>[0.9825,1]</td>
<td>[0.0048]</td>
<td>[0.9976,1]</td>
</tr>
<tr>
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<td>[0.9631,0.9953]</td>
<td>[0.0043,0.0287]</td>
<td>[0.9918,0.9996]</td>
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<td>[0.02,0.04]</td>
<td>[0.9384,0.9825]</td>
<td>[0.0148,0.0439]</td>
<td>[0.9823,0.9973]</td>
</tr>
<tr>
<td>[0.03,0.05]</td>
<td>[0.9098,0.9631]</td>
<td>[0.0287,0.059]</td>
<td>[0.9688,0.9918]</td>
</tr>
<tr>
<td>[0.04,0.06]</td>
<td>[0.8781,0.9384]</td>
<td>[0.0439,0.0732]</td>
<td>[0.9513,0.9823]</td>
</tr>
<tr>
<td>[0.05,0.07]</td>
<td>[0.8442,0.9098]</td>
<td>[0.0595,0.0857]</td>
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</tr>
<tr>
<td>[0.06,0.08]</td>
<td>[0.8088,0.8781]</td>
<td>[0.0732,0.0964]</td>
<td>[0.9052,0.9513]</td>
</tr>
</tbody>
</table>

**Example 1:** Let that $c_1 = 0$, $c_2 = 1$, $n_1 = n_2 = 10$ and $b_1 = 0.01$, $b_2 = 0.02$. Then FASN is obtained as follows:

$$\tilde{p}(\alpha) = [0.01 + 0.01\alpha, 0.03 - 0.01\alpha],$$

$$\text{FASN} = \{10 + 100pe^{-10p} | s\}$$

$$\text{FASN}[\alpha] = [10 + (1 + \alpha)e^{-(0.1 + 0.1\alpha)}, 10 + (3 - \alpha)e^{-(3 - 0.1\alpha)}].$$

Since we are using a double sampling plan, the sample size depends on whether or not a second sample is required, an important consideration for this kind of sampling is the average sample number band. According to the defined structure for $\tilde{p}$ in section 4, we can draw the FASN band in terms of $\tilde{p}$, the fuzzy fraction defective in an incoming lot.

**Example 3:** Let that $c_1 = 0$, $c_2 = 1$, $n_1 = n_2 = 10$ and $b_1 = 0.01$, $b_2 = 0.02$. Then FASN is obtained as follows:

$$\tilde{p}(\alpha) = [k + 0.01\alpha, k + 0.02 - 0.01\alpha]$$

And $\alpha$-cut of FASN is:

$$\text{FASN} = 10 + 100(k + 0.01)\alpha e^{-(10k + 0.1\alpha)}$$

And $\text{FASN}^*$ is:

$$\text{FASN}^* = 10 + 100(k + 0.02 - 0.01\alpha)e^{-(10k + 0.2 - 0.1\alpha)}.$$
FUZZY AVERAGE OUTGOING QUALITY

In programs of acceptance sampling, we can do rectifying inspection in order to improve the quality level of the lot. In one way of rectifying inspection, if the lot is accepted, the defective items in the sample will be substituted with safe items, if the lot is rejected, we do one hundred percent inspecting and if we are faced with defective items, we substitute them with safe items. The average outgoing quality is the quality level of the lot after the rectifying inspection process. It is the average outgoing rate of nonconforming items. Assume the size of lot to be \( N \) and the probability of defective items \( \hat{p} \), then, by selecting a sample with the size \( n_1 \), with the probability \( \hat{p}_d \) the lot will be accepted and with the probability \( 1 - \hat{p}_d \), we will do the secondary stage of sampling. If in this stage the lot is accepted, then \( n_1 \) items were investigated and became without defective items, finally, \( N - n_1 \) the remaining items which will be accepted without investigating, will have \( \hat{p}(N - n_1) \) of defective items averagely. If we have to select a secondary sample, we will select a random sample of size of \( n_2 \), which we will then investigate. If the lot is rejected at this stage, we will investigate one hundred percent and substitute the defective items with safe items. Then the number of defective items will be equal to zero. If the lot is accepted, according to the inspection the \( n_1 + n_2 \) items and were got without defective items, finally, \( N - n_1 - n_2 \) remaining items that are accepted without inspection, have \( \hat{p}(N - n_1 - n_2) \) of defective items, on the average. Thus in the outgoing process, the number of defective items with the probability of \( \hat{p}_d \) equal \( \hat{p}(N - n_1) \) and with the probability of \( \hat{p}_d \) equal \( \hat{p}(N - n_1 - n_2) \) and with the probability of \( \hat{p}_d^{III} \) equal zero. There are \( p_d^j \in \hat{p}_d : j = I, II, III \) such that \( p_d^I + p_d^{II} + p_d^{III} = 1 \). Then by using the definition of a fuzzy mean, the \( \alpha \)-cut of FAOQ is as follows:

\[
FAOQ \ [\alpha] = \left[ \left( p_d^I(N - n_1) + p_d^{II}(N - n_1 - n_2) + p_d^{III}(N - n_1 - n_2) \right) p \in \hat{p}[\alpha] \right]
\]

Where

\[
FAOQL [\alpha] = \min \left( \left( p_d^I(N - n_1) + p_d^{II}(N - n_1 - n_2) + p_d^{III}(N - n_1 - n_2) \right) p \in \hat{p}[\alpha] \right),
\]

and

\[
FAOQU [\alpha] = \max \left( \left( p_d^I(N - n_1) + p_d^{II}(N - n_1 - n_2) + p_d^{III}(N - n_1 - n_2) \right) p \in \hat{p}[\alpha] \right).
\]

For \( 0 \leq \alpha \leq 1 \), where S stands for the statement “\( p_d^i \in \hat{p}_d : i = I, II, III \) and \( p_d^I + p_d^{II} + p_d^{III} = 1 \)”,

\[
\hat{p}_d^{[\alpha]} = \{ e^{-10p} \mid p \in \hat{p}[\alpha] \}
\]

and

\[
\hat{p}_d^{[\alpha]} = \{ 10p e^{-20p} \mid p \in \hat{p}[\alpha] \}
\]

\[
FAOQL[\alpha] = \left\{ 0.95pe^{-10p} + 0.9p^2e^{-20p} \mid p \in \hat{p}[\alpha] \right\}
\]

and

\[
FAOQU[\alpha] = \left\{ 0.95(0.03 - 0.01)e^{-10(0.03 - 0.01\alpha)} + 0.9(0.03 - 0.01\alpha)^2 e^{-20(0.03 - 0.01\alpha)} \right\},
\]

under \( \alpha = 0 \) we obtain \( FAOQL[0] = [0.0087, 0.0216] \) and i.e., it is expected that for every 50 lots in such a process, 87 to 216 products will be defective items. And under \( \alpha = 1 \) we obtain \( FAOQL[1] = [0.0158, 0.0158] \). Figure 5 shows the FAOQ in comparison with the input quality process has improved.
FAOQ is the function of the quality of the lot and with it changing, the FAOQ will be change. If the FAOQ is drawn in terms of the proportion of defective items of the input lot, then the diagram will be a band which has downward and upward bounds and it is called the FAOQ band. According to the defined structure for $\hat{p}$ in section 4, we have

$$\hat{p}_{a,k}^f = \tilde{p}_k (\alpha_1 \leq \alpha \leq \alpha_2)$$

and

$$\hat{p}_{a,k}^u = \tilde{p}_k (\alpha_1 \leq \alpha \leq \alpha_2)$$

$$FAOQ[\alpha] = \left[ FAOQ_L[\alpha], FAOQ_U[\alpha] \right]$$

$$= \left[ \frac{p_{a,k}^f (N-n_i) + p_{a,k}^u (N-n_i-n_2)}{N} \right] p \in \tilde{p}[\alpha]$$

Where

$$FAOQ_L[\alpha] = \min \left\{ \frac{p_{a,k}^f (N-n_i) + p_{a,k}^u (N-n_i-n_2)}{N} \right\} p \in \tilde{p}[\alpha]$$

and

$$FAOQ_U[\alpha] = \max \left\{ \frac{p_{a,k}^f (N-n_i) + p_{a,k}^u (N-n_i-n_2)}{N} \right\} p \in \tilde{p}[\alpha]$$

Example 4: Suppose that $n_1 = 20$, $c_1 = 0$, $N = 200$, $b_1 = 0.01$, $b_1 = 0.02$ and $n_2 = 20$, $c_2 = 1$ then we have the following result:

$$\tilde{p}[\alpha] = [k + 0.01 \alpha, k + 0.02 - 0.01 \alpha]$$.

$$FAOQ[\alpha] = [0.9 pe^{20p} + 16 p^2 e^{-40p}]$$.

If we investigate the function cut of FAOQ $\alpha^*$, $f(p) = 0.9 pe^{-20p} + 16 p^2 e^{-40p}$ will be calculated as: $\alpha = 0$ with

$$[FAOQ^*, FAOQ^{**}]$$, $0 \leq k < 0.03$

$$FAOQ[0] = [FAOQ^*, 0.02196798]$$, $0.03 \leq k < 0.04065$

$$[FAOQ^{**}, 0.02196798]$$, $0.04065 \leq k < 0.05$

$$[FAOQ^{**}, FAOQ*]$$, $0.05 \leq k < 0.98$

and

$$FAOQ^* = 0.9 ke^{-20k} + 16 k^2 e^{-40k}$$

$$FAOQ^{**} = 0.9(k + 0.02)ke^{-20(k+0.02)} + 16(k + 0.02)^2 e^{-40(k+0.02)}$$

Figure 6 shows the FAOQ band for the sampling plan with a fuzzy parameter. We observe that if the proportion of defective items of the input lot is perfect or poor, the FAOQ will be perfect. One measure of how the sampling plans perform is the average outgoing quality limit (AOQL). The AOQL is the maximum percentage of defective items that can be expected in the lots examined by the plan. The maximum amount of FAOQ is the worse amount of FAOQ which will be earned in terms of an amount like $\hat{p}^*$ that will be called FAOQL. In Example 4 it is obtained as:

$$\hat{p}^* = [0.04065, 0.06065]$$

$$FAOQL = [0.0214, 0.0219]$$.

**FUZZY AVERAGE TOTAL INSPECTION**

The fuzzy average total inspection is an important criterion in the rectifying inspection for sampling plans with a fuzzy parameter. If the lot is accepted in the first stage (with fuzzy probability $\hat{p}^f_a$), the number of inspection items is equal to $n$, and if the lot is accepted in the secondary stage, the number of inspected items is equal to $n_1 + n_2$ (the fuzzy probability being $\hat{p}^u_a$), otherwise, it is equal to $N$ (with fuzzy probability $1 - \hat{p}^f_a - \hat{p}^u_a$). Finally, the random variable of inspected items has a fuzzy probability function as follows:
 Consequently, the FATI according to the definition of a fuzzy mean is as follows:

\[
FATI^{L} [\alpha] = \tilde{\mu}_{T '[\alpha]}
\]
\[
= \left[ n_{1}p_{a}^{I} + (n_{1} + n_{2})p_{a}^{H} + Np_{a}^{III} \right] S
\]
\[
= \left[ N - (N - n_{1})p_{a}^{I} - (N - n_{1} - n_{2})p_{a}^{II} \right] \left[ p \in \tilde{\mu}_{\alpha} \right]
\]
\[
= \left[ FATI^{L} [\alpha], FATI^{U} [\alpha] \right].
\]

Where

\[
FATI^{L} [\alpha] = \min \left\{ N - (N - n_{1})p_{a}^{I} - (N - n_{1} - n_{2})p_{a}^{II} \right\} S,
\]
\[
FATI^{U} [\alpha] = \max \left\{ N - (N - n_{1})p_{a}^{I} - (N - n_{1} - n_{2})p_{a}^{II} \right\} S,
\]

for \( 0 \leq \alpha \leq 1 \), where \( S \) stands for the statement 
\[
\mu_{a} = \frac{1}{1 + \alpha}, \mu_{a}^{II} = \frac{1}{1 + \alpha}.
\]

FATI in the Example 1 with \( N = 200 \) will be calculated as:

\[
FATI = \left[ 200 - 190e^{-10p} - 1800pe^{-20p} \right] p \in \tilde{\mu}_{[\alpha]}.
\]

Where the lower and upper limits of \( \alpha \)-cut is given as follows:

\[
FATI^{L} [\alpha] = 200 - 190e^{-(0\cdot1+0.1\alpha)} - (18 + 18\alpha)e^{-(0\cdot2+0.2\alpha)}
\]
\[
FATI^{U} [\alpha] = 200 - 190e^{-(0\cdot3-0.1\alpha)} - (54 - 18\alpha)e^{-(0\cdot6-0.2\alpha)}.
\]

Under \( \alpha = 0 \) we obtain FATI \([0] = [13, 30]\) and under \( \alpha = 1 \) we obtain FATI \([1] = 20\). It means we expect in accepting of every lot, 20 item will inspected. Figure 7 illustrates a fuzzy average total inspection in the Example 1.

According to the defined structure for \( \tilde{\mu}_{\alpha} \) in Section 4, we can draw the FATI band in terms of \( \tilde{\mu}_{\alpha} \). Then this diagram is a band which has upward and downward bounds. The degree of uncertainty of a proportion parameter is one of the factors that the bandwidth of FAOQ and FATI depends on. The lesser uncertainty value results in less bandwidth and if the proportion parameter gets a crisp value, the lower and upper bounds will become equal, which means that AOQ and ATI curves are in a classic state. The FATI band is the increasing function of the proportion of defective items of the input lot. The FATI band in the Example 3 is obtained as follows:

\[
\tilde{\mu}_{[\alpha]} = [k + 0.01\alpha, k + 0.02 - 0.01\alpha),
\]
\[
FATI = \left[ 200 - 190e^{-10p} - 1800pe^{-20p} \right] p \in \tilde{\mu}_{[\alpha]},
\]

we obtain \( \alpha = 0 \) Under

\[
FATI^{L} [0] = 200 - 190e^{-10k} - 1800ke^{-20k},
\]
\[
FATI^{U} [0] = 200 - 190e^{-10k + 0.2} - 1800(k + 0.02)e^{-20k + 0.4}.
\]

Figure 8 shows three FATI bands for \( N = 100 \), \( N = 200 \) and \( N = 300 \). It shows that the FATI is increasing in term of the proportion of defective items. These figures represent the state when the process quality decreases, then the FATI band will be narrower and we observed that when that the quality of the process is high, the FATI is near to the size of the sample and if the quality of the process is very low then most of the lots will be rejected and the FATI will be near the size of the lot.
CONCLUSION

In the present paper, we have proposed a method for designing acceptance double sampling plans with fuzzy quality characteristics by using fuzzy Poisson distribution. These plans are well defined since, if the fraction of defective items is crisp, the results agree with classical plans. We have shown that in our plan, amounts of the acceptance probability, the average sample number, the average outgoing quality and the average total inspection are fuzzy numbers. It was shown that the OC, ASN, AOQ and ATI curves of the plan are like band having high and low bounds. In this plan, if the process quality is perfect or poor, then the FAOQ will be perfect and the FASN will have lower value. The method of used in this paper applied for other fuzzy numbers, like fuzzy trapezoidal fuzzy numbers.

REFERENCES