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New Solutions for Positive and Negative Gardner-KP Equation

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Abstract: In this works we construct the travelling wave solutions for a nonlinear evolution equation. The (G'/G)-expansion method is used to construct the travelling wave solutions of the fifth order positive and negative Gardner-KP equation. The rational hyperbolic and other functions methods can be applied directly which the exact answers may have some physical interoperation. These characteristics make these methods so exceptional in exact solutions.

Key words: Gardner-KP equation · Direct algebraic method · (G' / G)-expansion method · Travelling wave solutions

INTRODUCTION

The (G' / G)-expansion method was developed by Mingliang Wang (2007). The method is now used by many researchers in a variety of scientific fields. In recent years, quite a few methods for obtaining explicit travelling and solitary wave solutions of nonlinear evolutions equations have been proposed. A variety of powerful methods, such as Bcklund and Darboux transformation [1-5], the tanh-sech method [6-8], extended tanh method [9], Exp-function method [10-13], the sine-cosine method [14-16], the Jacobi elliptic function method [17-18], the (G'/G) expansion method [19], He's homotopy perturbation method [20-22], homogeneous balance method [23-24], adomian decomposition method [25-27] and so on...

Description of Our Method: Considering the nonlinear partial differential equation in the form

$$P(u,u_x,u_y,u_t,u_{tt},u_{xt},u_{xx},....)=0 \qquad (1)$$

Where $u = u(x,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the $(\frac{G'}{G})$ -expansion method.

Step1: Combining the independent variables x and t into one variable $\xi = x\text{-}vt$, we suppose that

$$u = u(x,y,t) = u(\xi) \qquad \xi = k\,(x+y-vt) \qquad (2)$$

The travelling wave variable (2) permits us to reduce Eq(1) to an ODE for $G = G(\xi)$, namely

$$P(u,ku',-vku',ku',v^2k^2u'',-vk^2u'',....)=0 \qquad (2)$$

Step2: Suppose that the solution of ODE (3) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows

$$u(\xi)=\alpha_m(\frac{G'}{G})+...., \qquad (4)$$

Where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \qquad (50$$

a_m,....,$\lambda$and μ are constants to be determined later $a_m \neq 0$, the unwritten part in 4 is also a polynomial in, $(\frac{G'}{G})$ but the degree of which is generally equal to or less than m-1, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (3).

Step 3: By substituting (4) into Eq. (3) and using the second order linear ODE (5), collecting all terms with the same order $(\frac{G'}{G})$ together, the left-hand side of Eq. (3) is converted into another polynomial in $(\frac{G'}{G})$.

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Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $a_m,\ldots,\lambda$ and μ.

Step 4: Assuming that the constants $a_m,\ldots,\lambda$ and μ canbe obtained by solving the algebraic equations in Step 3, since the general solutions of the second order LODE (5) have been well known for us, then substituting $a_m,\ldots,v$ and the general solutions of Eq. (5) into (4) we have more travelling wave solutions of the nonlinear evolution equation (1).

Application of Method for Gardner-KP Equation: At first we consider the Gardner-KP Equation as follows

$$(u_t + 6uu_x \pm 6u^2u_x + u_{xxx})_x + u_{yy} = 0$$

Permits us converting Eq. (6) into an ODE in positive case for $u = u(\xi)$, $\xi = k(x+y-vt)$ and integrating we have

$$(-v+1)u + 3u^2 \pm 2u^3 + k^2u'' + c = 0 \qquad (7)$$

And for simplicity the first integral constant considered by zero.By considering the homogeneous balance between μ'' and μ^3 in Eq. (7), we required that $3m = m+2$, so we can write (4) as

$$u(\xi) = \alpha_1(\frac{G'}{G}) + \alpha_0$$

So we have

$$u^3 = \alpha_1^3(\frac{G'}{G})^3 + 3\alpha_1^2\alpha_0(\frac{G'}{G})^2 + 3\alpha_1\alpha_0^2(\frac{G'}{G}) + \alpha_0^3 \qquad (8)$$

$$u^2 = \alpha_1^2(\frac{G'}{G})^2 + 2\alpha_1\alpha_0(\frac{G'}{G}) + \alpha_0 \qquad (9)$$

By using (5) it is derived that

$$u'' = 2\alpha_1(\frac{G'}{G})^3 + 3\alpha_1\lambda(\frac{G'}{G})^2 + (\alpha_1\lambda^2 + 2\alpha_1\mu)(\frac{G'}{G}) + \alpha_1\lambda\mu \qquad (10)$$

In this case we substituting the relation above into equation (7) and collecting all terms with the same power of (G'/G) together, the left-hand side of Eq. (8) is converted into another polynomial in (G'/G). Equating each coefficient of this polynomial to zero yields a set of simultaneous algebraic equations for a_1, a_0 v, λ, μ and c as follows:

Positive Case:

$$(\frac{G'}{G})^3: \quad 2\alpha_1^3 + 2k^2\alpha_1 = 0$$

$$(\frac{G'}{G})^2: \quad 3\alpha_1^2 + 6\alpha_1^2\alpha_0 + 3k^2\alpha_1 = 0$$

$$(\frac{G'}{G})^1: \quad (-v+1)\alpha_1 + 6\alpha_0\alpha_1 + 6\alpha_1\alpha_0^2 + k^2(\alpha_1\lambda^2 + 2\alpha_1\mu) = 0$$

$$(\frac{G'}{G})^0: \quad (-v+1)\alpha_0 + 3\alpha_0^2 + 2\alpha_0^3 + k^2\alpha_1\lambda\mu + c = 0$$

By solving algebraic relations above by maple package we obtain,

$$a_1 = \pm \mathrm{ki}$$

For $a_1 = \mathrm{ki}$ we have

$$\alpha_0 = -\frac{1}{2i}(\mathrm{k}\lambda + \mathrm{i})$$

$$v = -\frac{1}{2}(k^2\lambda^2 - 4k^2\mu + 1)$$

$$c = \frac{1}{4}(-4k^2\mu + 1 + k^2\lambda^2)$$

λ is arbitrary constant. By substituting a_1, a_0 into equation (10) we obtain

$$u(\xi) = ki(\frac{G'}{G}) - \frac{1}{2i}(\mathrm{k}\lambda + \mathrm{i}), \qquad (11)$$

Substituting the general solutions of Eq. (5) as follows

$$\frac{G}{G} = \frac{1}{2}\sqrt{\lambda^2 - 4\mu} \times (\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}) - \frac{\lambda}{2}$$

Into (11) we have three types of travelling wave solutions of the (3+1)-dimensional Burgers system (6) as follows:

When $\lambda^2 - 4\mu > 0$

$$u(\xi) = \frac{ki}{2}\sqrt{\lambda^2 - 4\mu} \times (\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}) - \frac{1}{2}(\mathrm{k}\lambda i - 1 + \lambda)$$

Where $\xi = k(x+y+\frac{1}{2}(k^2\lambda^2-4k^2\mu+1)t)$ .C_1 and C_2 are arbitrary constants.

In particular, if $C_1 \neq 0$, $\lambda \succ 0$, $\mu = 0$, μ, become

$$u(\xi)=\frac{k\lambda i}{2}tgh\frac{1}{2}\lambda\xi-\frac{1}{2}(\mathrm{k}\lambda i-1+\lambda)$$

When λ^2 - $4\mu \prec 0$

$$u(\xi)=\frac{ki}{2}\sqrt{\lambda^2-4\mu}\times(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi})-\frac{1}{2}(\mathrm{k}\lambda i-1+\lambda)$$

When $\lambda^2 = 4\mu$

$$u(\xi)=\frac{kiC_2}{C_1+C_2\xi},$$

And for a_1 = - ki we have

$$\alpha_0 = \frac{1}{2i}(\mathrm{k}\lambda-\mathrm{i})$$

$$v=-\frac{1}{2}(k^2\lambda^2-4k^2\mu+1)$$

$$c=\frac{1}{4}(-4k^2\mu+1+k^2\lambda^2)$$

In this case we obtain three travelling wave solution as previous section in following form:

When λ^2 - $4\mu \succ 0$

$$u(\xi)=\frac{-ki}{2}\sqrt{\lambda^2-4\mu}\times(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi})+\frac{1}{2}(\mathrm{k}\lambda i+1-\lambda)$$

Where $\xi = k(x+y+\frac{1}{2}(k^2\lambda^2-4k^2\mu+1)t)$ . .C_1 and .C_2 are arbitrary constants.

In particular, if $C_1 \neq 0$, $C_2 = 0$, $\lambda \succ 0$, $\mu = 0$, μ, become

$$u(\xi)=\frac{-k\lambda i}{2}tgh\frac{1}{2}\lambda\xi+\frac{1}{2}(\mathrm{k}\lambda i+1-\lambda)$$

When λ^2 - $4\mu \prec 0$

$$u(\xi)=\frac{-ki}{2}\sqrt{\lambda^2-4\mu}\times(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi})+\frac{1}{2}(\mathrm{k}\lambda i+1-\lambda)$$

When λ^2 - $4\mu = 0$

$$u(\xi)=\frac{-kiC_2}{C_1+C_2\xi},$$

Negative Case: In this section by substituting the relations (8-10) in following negative type

$$(-v+1)u+3u^2-2u^3+k^2u''+c=0$$

And collecting all terms with the same power of (G'/G) together, the left-hand side of Eq. (8) is converted into another polynomial in (G'/G). Equating each coefficient of this polynomial to zero yields a set of simultaneous algebraic equations for a_1, a_0 v, λ, μ and c as positive case we have:

$$a_1 = \pm \text{ki}$$

For a_1 = ki

$$\alpha_0 = \frac{1}{2}(\text{k}\lambda{+}1)$$

$$v = \frac{1}{2}(4k^2\mu - k^2\lambda^2 + 5)$$

$$c = \frac{1}{4}(k^2\mu - k^2\lambda^2 + 1)$$

When $\lambda^2 - 4\mu < 0$

$$u(\xi) = \frac{\pm ki}{2}\sqrt{\lambda^2 - 4\mu} \times \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right) + \frac{1}{2}(1 \pm \text{k}\lambda\text{-}\lambda)$$

Where $\xi = k(x + y - \frac{1}{2}(4k^2\mu - k^2\lambda^2 + 5)t)$. C_1 and C_2 are arbitrary constants.

In particular, if $C_1 \neq 0$, $C_2 = 0$, $\lambda > 0$, $\mu = 0$, μ, become

$$u(\xi) = \frac{\pm k\lambda i}{2} tgh\frac{1}{2}\lambda\xi + \frac{1}{2}(1 \pm \text{k}\lambda\text{-}\lambda)$$

When $\lambda^2 - 4\mu < 0$

$$u(\xi) = \frac{\pm ki}{2}\sqrt{\lambda^2 - 4\mu} \times \left(\frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}\right) + \frac{1}{2}(1 \pm \text{k}\lambda\text{-}\lambda)$$

When $\lambda^2 - 4\mu = 0$

$$u(\xi) = \frac{\pm kiC_2}{C_1 + C_2\xi},$$

CONCLUSION

In this work we have seen that three types of travelling solutions of the Gardner-KP equation. the (G'/G)-expansion method has its own advantages: direct, concise, elementary that the general solutions of the second order LODE have been well known for the researchers and effective that it can be used for many other nonlinear evolution equations.

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