

Radiation Effects on Natural Convection in an Inclined Porous Surface with Internal Heat Generation

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Abstract: This paper focuses attention on natural convection heat transfer and fluid flow past an inclined semi-infinite porous surface considering internal heat generation and radiation. The governing equations are solved by Nachtsheim-Swigert shooting iterating technique with sixth order Runge-Kutta integration scheme for different parameters involved in this study. The velocity and temperature function are represented graphically for various values of parameters. The skin friction and heat transfer rate along the surface have been calculated.

Key words:

INTRODUCTION

Heat transfer flow of an incompressible viscous fluid in a porous medium with internal heat generation is present in several applications are in the field of nuclear energy and also to fire and combustion modeling. Several researches have been done on natural convection driven by internal heat generation [1, 2]. On the other hand, radiation effects can not be neglected due to the processes take place at a high temperature [3]. Lin *et al.* [4] studied the problem of free convection in an inclined plate with uniform surface flux. The problem of MHD heat transfer by mixed convection in an inclined permeable continuously stretching sheet in the presence of heat generation / absorption is investigated by Abo-Eldahab and Aziz [5]. They found that the local skin friction coefficient and the Nusselt number are increased with increasing buoyancy parameter. The local Nusselt number decreases while local skin friction coefficient increases with increasing heat generation parameter.

Natural convection heat transfer flow past an inclined surface is investigated by Lie group analysis and the resulting equations are solved numerically using the fourth order Runge Kutta scheme with shooting method by Sivasankaran *et al.* [6, 7]. They have found Grashof

number and Prandtl number effects on flow regimes. Pop and Na [8] studied the free convection from an arbitrarily inclined plate in a porous medium. They considered on both cases of positively inclined plate and negatively inclined plate at small angles to the horizontal. Postelnicu and Rees [9] analyzed the onset of convection in a porous layer inclined a small angle from horizontal.

Ferdows *et al.* [10] studied the forced and free convection of an electrically conduction viscous compressible fluid past a porous plate with suction. They found that temperature decreases with an increase of the suction and buoyancy parameters. A radiation effect on the free convection flow along a semi-infinite vertical plate was studied by Abo-eldahab [11]. They examined the effects of various parameters on the velocity and temperature fields as well as heat flux and shearing stress at the plate. The thermal radiation effects in a boundary layer flow past a vertical plate with variable suction has been studied by Ferdows *et al.* [12]. It is examined that the interaction of the thermal radiation intensifies (increase in R), the velocity increases at the wall. Hossain and Pop [13] investigated the effect of radiation on buoyancy induced flow along a heated inclined flat surface placed in a porous medium. The heat transfer rate increases with a decrease in the radiation parameter.

Raptis [14] investigated the free convection flow through porous medium bounded by a vertical porous plate in the presence of radiation. It is found that velocity decreases when the radiation parameter increases. Hossain *et al.* [15] determined the effect of radiation on natural convection of an optically dense incompressible fluid along a uniformly heated vertical plate with uniform suction. Badruddin *et al.* [16] analyzed the effect of radiation on convection through a water saturated porous porous medium bounded by a vertical plate. They found that the heat transfer rate increases due to radiation effect. The present analysis aims to study the radiation effects on free convection in an inclined porous surface with internal heat generation.

Mathematical Formulations: Consider heat transfer flow of an viscous incompressible fluid along a semi infinite porous plate with an acute angle δ from the vertical with thermal radiation and internal heat generation. The flow is also assumed to be in the x -direction is taken along the plate in the upward direction and y -direction is normal to it and is shown in below figure. The fluid is considered to be gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation [17] is used to describe radiation in the energy equation. The radiative heat flux in the x direction is considered negligible in comparison to the y direction. Under the boundary layer approximation the fluid flow and heat transfer in the presence of radiation and internal heat generation are governed by the following equations:

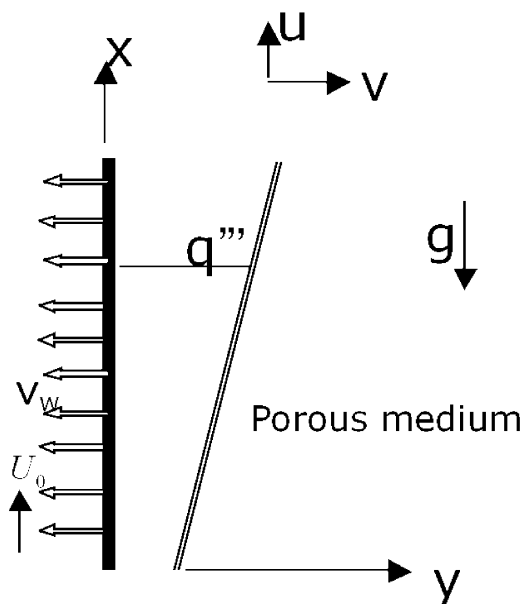


Fig. 1: Physical Configuration

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)\cos\delta - \frac{\nu}{K}u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial}{\partial y} \left(\frac{\partial T}{\partial y} - \frac{q_r}{\kappa} \right) + q'' \quad (3)$$

where u and v are the velocity components in the x and y directions respectively, ν is the kinematic viscosity, g is the acceleration due to gravity, β is the coefficient of volume expansion, T and T_∞ are the temperature of the fluid inside the thermal boundary layer and the fluid temperature in the free stream respectively. Also, K is the permeability of porous medium, α is the thermal diffusivity, q_r is the radiative heat flux, k is the thermal conductivity and q'' is the internal heat generation.

The boundary conditions for the model are:

$$\left. \begin{aligned} u &= U_0, v = v_0(x), T = T_w \text{ at } y = 0 \\ u &= 0, v = 0, T \rightarrow T_\infty, \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (4)$$

where U_0 is the uniform velocity, $v_0(x)$ is the velocity of suction at the plate and T_w is the plate temperature.

The radiative heat flux term is simplified by making use of Rosseland approximation and following Raptis [18] we have

$$q_r = -\frac{16\sigma^* T_\infty^3}{3kk^*} \frac{\partial T}{\partial y} \quad (5)$$

where σ^* and k are the Stefan-Boltzman constant and k^* mean absorption coefficient.

Here an exponential form is used for the internal heat generation and it is as follows

$$q'' = \frac{U_0(T_w - T_\infty)}{2x} e^{-\eta} \quad (6)$$

Using (5) and (6), equation (3) takes the form

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \left(1 + \frac{16\sigma^* T_\infty^3}{3\kappa k^*} \right) + \frac{U_0(T_w - T_\infty)}{2x} e^{-\eta} \quad (7)$$

Introduce the following dimensionless variables

$$\eta = y\sqrt{\frac{U_0}{2\nu x}}, u = U_0 f'(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (8)$$

the equations (1), (2) and (7) are reduced as follows

$$f''' + ff'' + Gr\theta \cos \delta - Kf' = 0 \quad (9)$$

$$(3R + 4)\theta'' + 3RPr f\theta' + 3RPr e^{-\eta} = 0 \quad (10)$$

where $Gr = \frac{g\beta(T_w - T_\infty)2x}{U_0^2}$ is the Grashof

number, $K = \frac{2\nu x}{K^* U_0}$ is the Permeability parameter,

$R = \frac{k^* \kappa}{4\delta^* T_\infty^3}$ is the radiation parameter and $Pr = \frac{\nu}{\alpha}$ is the

Prandtl Number: The corresponding boundary conditions are

$$\left. \begin{aligned} f &= f_w, f' = 1, \theta = 1 \text{ at } \eta = 0 \\ f' &= 0, \theta = 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (11)$$

where $f_w = -v_0 \sqrt{\frac{2x}{\nu U_0}}$ the dimensionless suction velocity

and prime denotes partial differentiation with respect to the variable η .

The physical quantities of the local skin-friction coefficient and the local Nusselt number are calculated respectively by the following equations,

$$\frac{1}{2} Re^{\frac{1}{2}} C_f = f''(0) \quad Nu(Re)^{-\frac{1}{2}} = -\theta'(0)$$

Where $Re = \frac{U_0 x}{\nu}$ is the Reynolds number.

Numerical Procedure: The set of nonlinear coupled differential equations (6)-(7) with boundary conditions (8) been solved numerically by applying Nachtsheim-Swigert [6] shooting iteration technique along with sixth order Runge-Kutta integration scheme. A step size of $\Delta\eta = 0.01$ was selected to be satisfactory for a convergence criterion of 10^{-6} in all cases. The value of η_∞ was found to each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞ to each group of parameters determined when the value of the unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-6} .

Table 1: Skin-friction coefficient and Nusselt number for different combinations of parameters

Gr	Pr	K	f _w	R	δ	f'(0)	-θ'(0)
0.2	0.71	1	1	1	30°	1.6155	0.1759
0.8						1.1435	0.2336
2.0						0.3259	0.3004
2.0	0.71	1	1	1	30°	0.3259	0.3004
		2				0.9149	0.2634
		4				1.7266	0.2137
2.0	0.71	1	1	1	30°	0.3259	0.3004
			2			1.0564	0.5032
			4			3.0413	1.0082
2.0	0.71	1	1	1	30°	0.3259	0.3004
	2.05					0.6651	0.5431
	4.24					0.8497	0.876
	13.6					1.0349	2.1396
2.0	0.71	1	1	1	0°	0.1397	0.3129
					30°	0.3259	0.3004
					45°	0.5646	0.2834
					60°	0.8926	0.257
					90°	1.7926	0.1462
2.0	0.71	1	1	1	30°	0.3259	0.3004
				2		0.4388	0.3583
				3		0.4851	0.3871

RESULTS AND DISCUSSION

Numerical solutions are carried out using Nachtsheim-Swigert shooting iterating technique together with sixth order Runge-Kutta integration scheme for various values of Grashof number, Prandtl number, inclination angle, porous, suction and radiation parameters. The Prandtl number is taken as 0.71 to 13.6, Grashof number is from 0.2 to 2.0, Porous parameter varies from 1 to 4, suction parameter from 1 to 4 and radiation parameter from 1 to 3 with the angle of inclination for the values 0, 30, 45, 60 and 90 degree. The values of the skin-friction coefficient and Nusselt number which are respectively proportional to $f'(0)$ and $-\theta'(0)$ are also sorted out and their numerical values are presented for different values of the parameters in Table 1.

From the table, it is observed that,

- With increasing Grashof number the Skin-friction coefficient decreases and Nusselt number increases.
- With increasing porous parameter and inclination angle the Skin-friction coefficient increases and Nusselt number decreases.
- With increasing Prandtl number and suction and radiation parameters the Skin-friction coefficient and Nusselt number both increase.

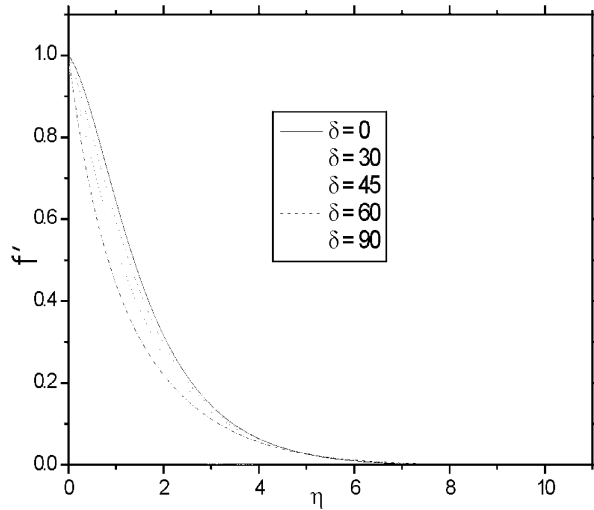


Fig. 2: Velocity profiles for different inclination angles

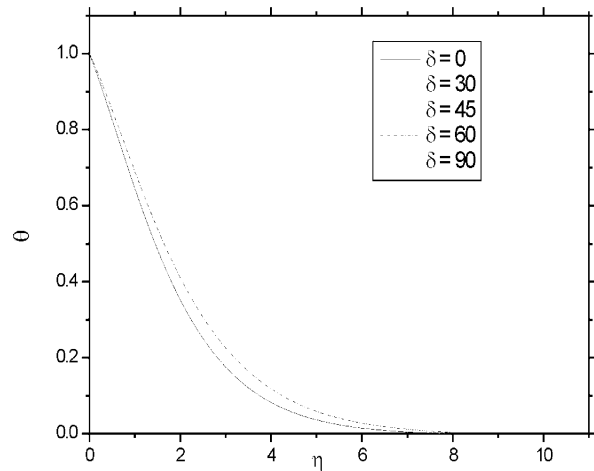


Fig. 3: Temperature profiles for different inclination angles

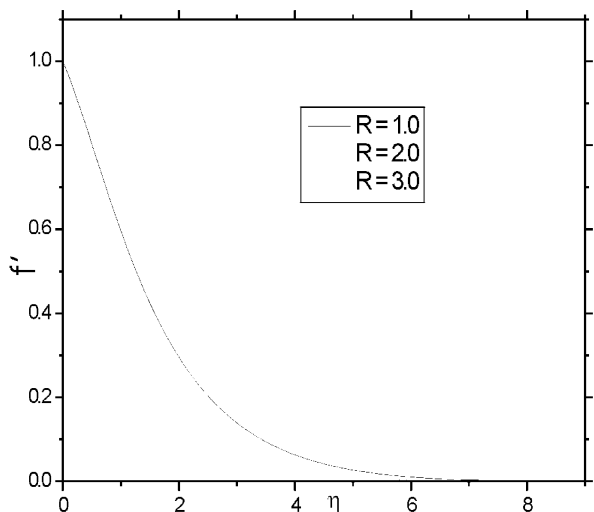


Fig. 4: Velocity profiles for different radiation parameter

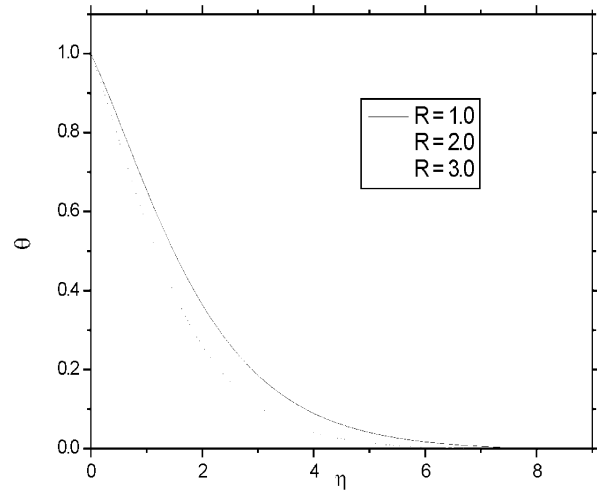


Fig. 5: Temperature profiles for different radiation parameter

The effect of inclination of the plate on the velocity and temperature field for $Gr = 2.0$, $Pr = 0.71$, $K = 1.0$, $fw = 1.0$ and $R = 1.0$ is depicted in Fig. 2 and Fig.3, respectively. It is seen that the velocity is decreased for all angles and there is a strong over shoot at an vertical angle. On the other hand, temperature is increased for increasing angle of the plate. The effect of radiation parameter of the fluid for $Gr = 2.0$, $K = 1.0$, $fw = 1.0$ and $\delta = 30^\circ$ is depicted in Fig. 4 and Fig. 5, respectively for velocity and temperature field. It is seen that the velocity and temperature fields are decreased for all R . From Figures when $\eta = 2.0$ and R decreases from 3 to 2 there are 9.44% and 11.50% increase in the velocity and temperature profiles respectively, whereas the corresponding increases are 25.62% and 24.20% when R decreases from 2 to 1.

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