

## Laplace Adomian Decomposition Method for Solving a Model Chronic Myelogenous Leukemia CML and T Cell Interaction

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**Abstract:** In this paper, Laplace Adomian Decomposition Method is implemented to give approximate solution of nonlinear ordinary differential equation system, we try to obtain analytic solution for model Chronic Myelogenous Leukemia, the model describe the interaction between naive T cells, effector E cells and CML cancer cells in the body. The technique is described and illustrated with numerical example.

**Key words:** Laplace Decomposition Method • Nonlinear differential equation system • A model for Chronic Myelogenous Leukemia (CML) and T cells interaction

### INTRODUCTION

Chronic Myelogenous Leukemia (CML) is cancer that affects cell circulating in the blood system [1] in our model consist of the system of three nonlinear ordinary differential equation. The system is given below, followed by explanation of the terms:

$$\begin{aligned} \frac{dT}{dt} &= s_n - d_n T - k_n T \left( \frac{C}{C + \eta} \right), \\ \frac{dE}{dt} &= \alpha_n k_n T \left( \frac{C}{C + \eta} \right) + \alpha_e E \left( \frac{C}{C + \eta} \right) - d_e E - \gamma_e CE, \quad (1) \\ \frac{dC}{dt} &= r_c C \ln \left( \frac{C_{\max}}{C} \right) - d_e C - \gamma_c CE, \end{aligned}$$

Each equation represents the rate of change with respect to time with the initial conditions.

$$T(0) = a_1, \quad E(0) = a_2, \quad C(0) = a_3, \quad (2)$$

In this paper,  $T(t)$ ,  $E(t)$  and  $C(t)$  denote the naive cells, effector E cells specific to CML and Chronic Myelogenous Leukemia CML cancer cells respectively, throughout, in this paper we set the solution with 12 parameter see [1]. A technique for calculating the approximate solution by Laplace Adomain Decomposition Method (ADM) introduced by Adomian [24] the convergence ADM was given by

[15] who using the fixed point theorem for abstract functional equation.

In this paper, LADM is applied to solve a model Chronic Myelogenous Leukemia this method construction an analytical solution in the form of a polynomial and definition of 12 parameter given in [1]. In section 2 we define the basic idea of LADM and polynomial. The applications of the proposed numerical scheme to model (1) are illustrated in section 3.

### Basic idea of Laplace Adomian Decomposition Method (LADM):

In this section, the basic idea of LADM are introduced [16]. In general nonlinear problem.

$$\begin{aligned} L^i \vec{u} + N^i \vec{u} &= \vec{g}(t) \\ \vec{u} &= (u^1, u^2, u^3) \end{aligned} \quad (3)$$

Given the vector describe the system  $\vec{g}(t) = (g^1(t), g^2(t), g^3(t))$  and  $L^i$  is linear operator and  $N^i$  nonlinear operator and  $\vec{g}(t)$  is analytic function. We apply formula for Laplace transforms and obtain:

$$\varphi L^i \vec{u} + \varphi N^i \vec{u} = \varphi \vec{g}(t) \quad (4)$$

In this paper,  $L$  is  $(\partial/\partial t)$  and the initial condition is  $\vec{u}(0) = (u^1(0), u^2(0), u^3(0))$ :

$$s\varphi \vec{u} - \vec{u}(0) + \varphi N^i \vec{u} = \varphi \vec{g}(t) \quad (5)$$

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And this result obtain by

$$\wp \vec{u} = \frac{\vec{u}(0)}{s} + \frac{1}{s} \wp \vec{g}(t) - \frac{1}{s} \wp N^i \vec{u} \quad (6)$$

With inverse Laplace given

$$\vec{u}_0 = \wp^{-1} \vec{u}(0) + \wp^{-1} \frac{1}{s} \wp \vec{g}(t) \quad (7)$$

$$u_{n+1} = -\wp^{-1} \frac{1}{s} \wp N^i \vec{u} \quad (8)$$

The exact solution may be obtained as:

$$u(t) = \sum_{n=0}^{\infty} u_n(t) \quad (9)$$

For nonlinear operator, a domain polynomial is given:

$$A_n(T_0, T_1, T_2, \dots, T_n; C_0, C_1, C_2, \dots, C_n) = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[ N \left( \sum_{i=0}^n \lambda^i T_i, \sum_{i=0}^n \lambda^i C_i \right) \right]_{\lambda=0} \quad (10)$$

**The Laplace Adomian Decomposition Method (LADM):** In this section, we will apply the Laplace Adomian Decomposition Method (LADM) to nonlinear differential system (1):

$$\begin{aligned} \wp \left\{ \frac{dT}{dt} \right\} &= \wp \{ s_n \} - \wp \{ d_n T \} - \wp \left\{ k_n T \left( \frac{C}{C+\eta} \right) \right\}, \\ \wp \left\{ \frac{dE}{dt} \right\} &= \wp \left\{ \alpha_n k_n T \left( \frac{C}{C+\eta} \right) \right\} + \wp \left\{ \alpha_e E \left( \frac{C}{C+\eta} \right) \right\} - \wp \{ d_e E \} - \wp \{ \gamma_e CE \}, \\ \wp \left\{ \frac{dC}{dt} \right\} &= \wp \{ r_c C \ln(C_{\max}) \} - \wp \{ r_c C \ln(C) \} - \wp \{ d_e C \} - \wp \{ \gamma_c CE \}, \end{aligned} \quad (11)$$

Applying the formulas for Laplace

$$\begin{aligned} s \wp \{ T \} - T(0) &= \wp \{ s_n \} - \wp \{ d_n T \} - \wp \left\{ k_n T \left( \frac{C}{C+\eta} \right) \right\}, \\ s \wp \{ E \} - E(0) &= \wp \left\{ \alpha_n k_n T \left( \frac{C}{C+\eta} \right) \right\} + \wp \left\{ \alpha_e E \left( \frac{C}{C+\eta} \right) \right\} - \wp \{ d_e E \} - \wp \{ \gamma_e CE \}, \\ s \wp \{ C \} - C(0) &= \wp \{ r_c C \ln(C_{\max}) \} - \wp \{ r_c C \ln(C) \} - \wp \{ d_e C \} - \wp \{ \gamma_c CE \}, \end{aligned} \quad (12)$$

Using the initial condition (2)

$$\begin{aligned} s \wp \{ T \} &= a_1 + \frac{s_n}{s} - d_n \wp \{ T \} - k_n \wp \left\{ T \left( \frac{C}{C+\eta} \right) \right\}, \\ s \wp \{ E \} &= a_2 + \alpha_n k_n \wp \left\{ T \left( \frac{C}{C+\eta} \right) \right\} + \alpha_e \wp \left\{ E \left( \frac{C}{C+\eta} \right) \right\} - d_e \wp \{ E \} - \gamma_e \wp \{ CE \}, \\ s \wp \{ C \} &= a_3 + r_c \ln(C_{\max}) \wp \{ C \} - r_c \wp \{ C \ln(C) \} - d_e \wp \{ C \} - \gamma_c \wp \{ CE \}, \end{aligned} \quad (13)$$

Where  $A = \frac{TC}{C+\eta}$ ,  $B = \frac{CE}{C+\eta}$ ,  $D = CE$ ,  $M = C \ln C$

$$T = \sum_{n=0}^{\infty} T_n, \quad E = \sum_{n=0}^{\infty} E_n, \quad C = \sum_{n=0}^{\infty} C_n \quad (14)$$

Also the nonlinear operators A, B, D and M are:

$$A = \sum_{n=0}^{\infty} A_n, \quad B = \sum_{n=0}^{\infty} B_n, \quad D = \sum_{n=0}^{\infty} D_n, \quad M = \sum_{n=0}^{\infty} M_n \quad (15)$$

Where  $A_n, B_n, D_n, M_n$  the Adomian polynomial, the first polynomial are given by

$$\begin{aligned} A_0 &= \frac{C_0 T_0}{C_0 + \eta}, \\ A_1 &= \frac{C_0^2 T_1 + \eta C_0 T_1 + \eta C_1 T_0}{(C_0 + \eta)^2}, \\ A_2 &= \frac{C_1^2 C_0 T_0}{(C_0 + \eta)^3} - \frac{C_1^2 T_0}{(C_0 + \eta)^2} - \frac{C_2 C_0 T_0}{(C_0 + \eta)^2} + \frac{C_2 T_0}{C_0 + \eta} - \frac{C_2 C_0 T_0}{(C_0 + \eta)^2} + \frac{C_1 T_1}{C_0 + \eta} + \frac{C_0 T_2}{C_0 + \eta}, \\ \dots\dots\dots \end{aligned} \quad (16)$$

$$\begin{aligned} B_0 &= \frac{C_0 E_0}{C_0 + \eta}, \\ B_1 &= \frac{C_0^2 E_1 + \eta C_0 E_1 + \eta C_1 E_0}{(C_0 + \eta)^2}, \\ B_2 &= \frac{C_1^2 C_0 E_0}{(C_0 + \eta)^3} - \frac{C_1^2 E_0}{(C_0 + \eta)^2} - \frac{C_2 C_0 E_0}{(C_0 + \eta)^2} + \frac{C_2 E_0}{C_0 + \eta} - \frac{C_2 C_0 E_0}{(C_0 + \eta)^2} + \frac{C_1 E_1}{C_0 + \eta} + \frac{C_0 E_2}{C_0 + \eta}, \\ \dots\dots\dots \end{aligned} \quad (17)$$

$$\begin{aligned} D_0 &= C_0 E_0, \\ D_1 &= C_1 E_0 + C_0 E_1, \\ D_2 &= C_2 E_0 + C_1 E_1 + C_0 E_2, \\ \dots\dots\dots \end{aligned} \quad (18)$$

$$\begin{aligned} M_0 &= C_0 \ln C_0, \\ M_1 &= C_1 + C_0 \ln C_0, \\ M_2 &= \frac{C_1^2}{2C_0} + C_2 + C_2 \ln C_0, \\ \dots\dots\dots \end{aligned} \quad (19)$$

Applying the formulas for Laplace

$$\begin{aligned} \mathcal{L}\left\{\sum_{n=0}^{\infty} T_n\right\} &= \frac{a_1}{s} + \frac{s_n}{s^2} - \frac{d_n}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} T_n\right\} - \frac{k_n}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} A_n\right\}, \\ \mathcal{L}\left\{\sum_{n=0}^{\infty} E_n\right\} &= \frac{a_2}{s} + \frac{\alpha_n k_n}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} A_n\right\} + \frac{\alpha_e}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} B_n\right\} - \frac{d_e}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} E_n\right\} - \frac{\gamma_e}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} D_n\right\}, \\ \mathcal{L}\left\{\sum_{n=0}^{\infty} C_n\right\} &= \frac{a_3}{s} + \frac{r_c \ln(C_{\max})}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} C_n\right\} - \frac{r_c}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} M_n\right\} - \frac{d_e}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} C_n\right\} - \frac{\gamma_c}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} D_n\right\}, \\ \dots\dots\dots \end{aligned} \quad (19)$$

It results

$$\begin{aligned}\wp\{T_0\} &= \frac{a_1}{s} + \frac{s_n}{s^2}, \\ \wp\{T_1\} &= -\frac{d_n}{s} \wp\{T_0\} - \frac{k_n}{s} \wp\{A_0\}, \\ \wp\{T_2\} &= -\frac{d_n}{s} \wp\{T_1\} - \frac{k_n}{s} \wp\{A_1\},\end{aligned}\quad (20)$$

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$$\begin{aligned}\wp\{E_0\} &= \frac{a_2}{s}, \\ \wp\{E_1\} &= \frac{\alpha_n k_n}{s} \wp\{A_0\} + \frac{\alpha_e}{s} \wp\{B_0\} - \frac{d_e}{s} \wp\{E_0\} - \frac{\gamma_e}{s} \wp\{D_0\}, \\ \wp\{E_2\} &= \frac{\alpha_n k_n}{s} \wp\{A_1\} + \frac{\alpha_e}{s} \wp\{B_1\} - \frac{d_e}{s} \wp\{E_1\} - \frac{\gamma_e}{s} \wp\{D_1\},\end{aligned}\quad (21)$$

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$$\begin{aligned}\wp\{C_0\} &= \frac{a_3}{s}, \\ \wp\{C_1\} &= \frac{r_c \ln(C_{\max})}{s} \wp\{C_0\} - \frac{r_c}{s} \wp\{M_0\} - \frac{d_e}{s} \wp\{C_0\} - \frac{\gamma_c}{s} \wp\{D_0\}, \\ \wp\{C_2\} &= \frac{r_c \ln(C_{\max})}{s} \wp\{C_1\} - \frac{r_c}{s} \wp\{M_1\} - \frac{d_e}{s} \wp\{C_1\} - \frac{\gamma_c}{s} \wp\{D_1\},\end{aligned}\quad (22)$$

**Application:** In this section, we present the numerical results with Figures (1-2).

The initial conditions are:

$$T(0) = 1510 \text{ cells}/\mu\text{l}, \quad E(0) = 20 \text{ cells}/\mu\text{l}, \quad C(0) = 10000 \text{ cells}/\mu\text{l} \quad (23)$$

We compare numerical results in Table (1).

We start to iteration with

$$\begin{aligned}T_0 &= 1510 + s_n t \\ E_0 &= 20 \\ C_0 &= 10000\end{aligned}\quad (24)$$

Adomain polynomials and inverse Laplace transformations give the solution

$$\begin{aligned}\wp\{T_{n+1}\} &= -\frac{d_n}{s} \wp\{T_n\} - \frac{k_n}{s} \wp\{A_n\}, \\ \wp\{E_{n+1}\} &= \frac{\alpha_n k_n}{s} \wp\{A_n\} + \frac{\alpha_e}{s} \wp\{B_n\} - \frac{d_e}{s} \wp\{E_n\} - \frac{\gamma_e}{s} \wp\{D_n\}, \\ \wp\{C_{n+1}\} &= \frac{r_c \ln(C_{\max})}{s} \wp\{C_n\} - \frac{r_c}{s} \wp\{M_n\} - \frac{d_e}{s} \wp\{C_n\} - \frac{\gamma_c}{s} \wp\{D_n\},\end{aligned}\quad (25)$$

By using Mathematica 5.2 software, we obtain following results:

Table 1: Parameter value used in graphs sees [1]:

Graph	$s_n$	$d_n$	$d_e$	$d_c$	$k_n$	$\eta$	$\alpha_n$	$\alpha_e$	$C_{\max}$	$r_c$	$\gamma_e$	$\gamma_c$
Fig. 1-2	0.37	0.23	0.30	0.024	.062	720	0.14	0.98	230000	0.0025	0.057	.0034
Fig. 3-4	.071	0.050	0.12	0.68	0.063	43	0.56	0.56	190000	0.23	0.0077	0.047

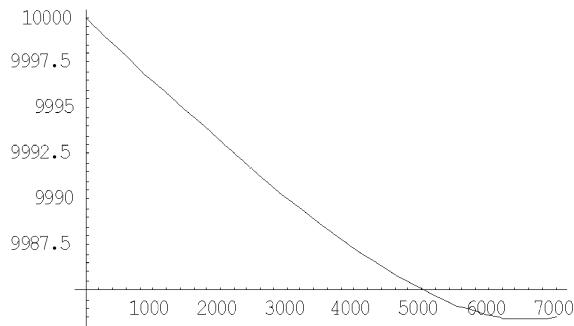


Fig. 1:  $C[t]$  and  $t$  decrease on the time

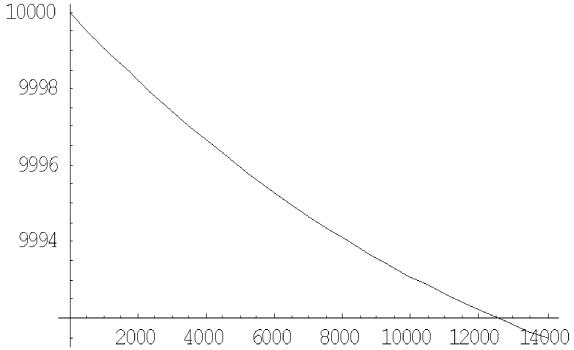


Fig. 2:  $C[t]$  and  $t$  decrease on the time but

## CONCLUSION

This work is to provide the series solution of the initial condition equation by using the Laplace Adomain decomposition method (LADM). The Laplace decomposition method (LDM) is a powerful tool to search for solutions of various nonlinear problems. We derived the same results by combining the series, obtained by the modified Laplace decomposition method. We obtain the convergent solution of model for chronic myelogenous leukemia (CML) and T cell interaction system and comparison the results by numerical solution.

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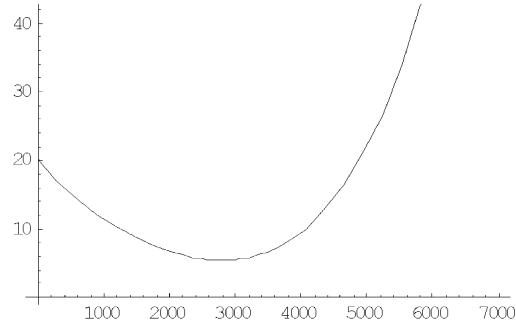


Fig. 3:  $E[t]$  and  $t$  increase on the time

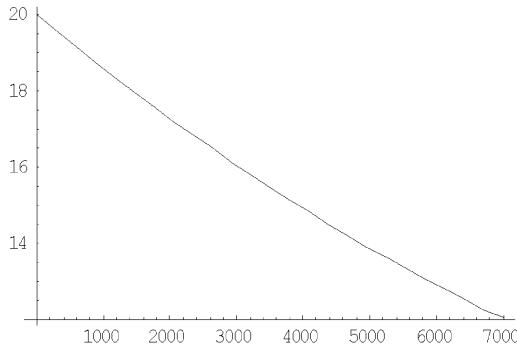


Fig. 4:  $E[t]$  and  $t$  increase on the time

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