Simulation of Static Characteristics of Self-Assembled Quantum Dot Lasers

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Abstract: Static-characteristics of self-assembled quantum-dot (SAQD) lasers are treated solving the rate equations numerically using fourth-order Runge-Kutta method. Energy level and size distributions of self-assembled quantum-dots are considered and their effects on Static-characteristics are investigated. Simulation results of static-characteristics show that nonlinearity appears in light-current characteristics whereas homogeneous broadening becomes equal to inhomogeneous broadening. Slope-efficiency increases as the homogeneous broadening heightens up to inhomogeneous broadening. Exceeding the homogeneous broadening from inhomogeneous broadening results in degradation of light-current characteristics. In fact, SAQD laser has the best performance when homogeneous broadening is equal to inhomogeneous broadening. Light-current characteristics degrade and threshold current increases as inhomogeneous broadening enhances. We also investigate the effects of quantum dot coverage on the laser performance and show that there is a threshold coverage for quantum-dots to begin lasing and an optimum quantum-dot coverage in which the SAQD lasers operate with lowest possible threshold current and maximum output power. In addition, we show that differential gain degrades as initial relaxation time increases and recombination times in quantum dots and wetting layer decrease. Consequently, relaxation oscillation frequency and modulation bandwidth decline.

Key words: Quantum-dot laser • Inhomogeneous and homogeneous broadenings • Static-characteristics

INTRODUCTION

Semiconductor lasers with quantum dots (nanostructures) in their active regions are expected to exhibit many superior properties, such as ultralow threshold currents, temperature insensitive and high-speed operation, high optical gain, broad modulation bandwidth and narrow spectrum linewidth due to three dimensional electronic confinement that results in completely discrete energy spectrum and allow the investigation of typical effects of atomic physics in a solid state environment [1-4]. However, we know that actual quantum dots (QDs) do not always satisfy our expectation because of the energy level broadening (homogeneous broadening) and size distributions (inhomogeneous broadening) and phonon bottleneck. Thus, for an accurate comprehension of quantum dot laser diode (QD-LD) performance, we must take into account all these actual aspects of QDs [5].

In this paper, simulating model of QD laser performance are described based on rate equations. We solve the rate equations considering homogeneous and inhomogeneous broadenings of the linear optical gain numerically using fourth-order Runge-Kutta method for a specific InGaAs/GaAs self-assembled quantum dot laser (SAQD-LD). We simulate static response and analyze the effects of inhomogeneous and homogeneous broadenings on static-characteristics. We also consider QD coverage as a variable parameter and inspect its effect on the laser performance. Finally, the effects of recombination times and initial relaxation time (phonon-bottleneck) are also treated on optical gain-current characteristics.

Simulation Model: Solving rate equations for carrier and photons is the most accurate way to treat static and dynamic characteristics in lasers. Fig. 1 illustrates the energy diagram of the conduction band of the quantum dot laser active region and diffusion, relaxation, recombination and escape processes of carriers [6, 7].

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Fig. 1: Energy diagram of the laser-active region and diffusion, recombination and relaxation processes

We consider an exciton model. It means that both an electron and a hole relax into the ground state simultaneously to form an exciton. We assume that only a single discrete electron and hole ground state is formed inside the QD and the charge neutrality always holds in each QD.

We divide the QD ensemble into \( j = 1, 2, \ldots, 2M+1 \) groups, depending on their resonant energy for the interband transition, over the longitudinal cavity photon modes. \( j = M \) corresponds to the group and the mode at \( E_{cv} \). We take the energy width of each group equal to the mode separation of the longitudinal cavity photon modes which equals to

\[
\Delta E = \alpha h \left( \frac{2n}{L_{cw}} \right)
\]

(1)

Where \( L_{cw} \) is the cavity length. The energy of the \( j \)th QDs group is represented by

\[
E_j = E_{cv} - (M-j)\Delta E
\]

(2)

Where \( j = 1, 2, \ldots, 2M+1 \). The QD density of \( j \)th group is given as

\[
N_j G_j = N_o G_j (E_j - E_{cv}) \Delta E
\]

(3)

Where \( N_o \) is the volume density of QDs and the energy fluctuation of QDs is given as

\[
G(E_j - E_{cv}) = \frac{1}{\sqrt{2\pi\xi_0}} \exp\left[-\left\{E_j - E_{cv}\right\}^2/2\xi_0^2\right]
\]

(4)

that takes a Gaussian distribution function and Whose FWHM is given by \( \Gamma_o = 2.35\xi_0 \). We consider \( N_j \) as the carrier number in \( j \)th QDs group, According to Pauli’s exclusion principle, the occupation probability in the ground state of the \( j \)th QDs group is defined as

\[
P_j = N_j / \left( 2N_o \varphi_{j} \right)
\]

(5)

Where \( \varphi_{j} \) is the total active region volume.

The rate equations are as follows [5-9]

\[
dN_j / dt = I / e - N_j / \tau_s - N_j / \tau_{wr} - N_j / \tau_{we} + N_o / \tau_{we}
\]

(6)

\[
dN_o / dt = N_o / \tau_s + \sum_j N_j / \tau_{er} D_j - N_o / \tau_{wr} - N_o / \tau_{we} - N_o / \tau_{d}
\]

(7)

\[
dN_j / dt = N_o G_j / \tau_{er} - N_j / \tau_s - N_j / \tau_{er} C_j - \frac{cG}{n_e} \sum_m G_m \text{\textsuperscript{S}_m}
\]

(8)

\[
dS_m / dt = \beta N_j / \tau_s + \frac{cG}{n_e} \sum_j G_j \text{\textsuperscript{S}_j} - S_m / \tau_p
\]

(9)

Where \( N_s, N_o, \) and \( N_j \) are the carrier number in SCH layer, wetting layer (WL) and \( j \)th QDs group, respectively, \( S_m \) is the photon number of \( m \)th mode, where \( m = 1, 2, \ldots, 2M+1 \), \( I \) is the injected current, \( G_j \) is the fraction of the \( j \)th QDs group type within an ensemble of different dot size populations, \( e \) is the electron charge, \( D_j \) is the degeneracy of the QD ground state without spin, \( \beta \) is the spontaneous-emission coupling efficiency to the lasing mode. The related time constants are, \( \tau_s \) diffusion in the SCH region, \( \tau_{wr} \) carrier recombination in the SCH region, \( \tau_{we} \) carrier reexcitation from the WL to the SCH region, \( \tau_{er} \) carrier recombination in the WL, \( \tau_d \) carrier relaxation into the \( j \)th QDs group, \( \tau \) carrier recombination in the QDs, \( \tau_p \) photon lifetime in the cavity. The average carrier relaxation time, \( \tau_d \) is given as

\[
\tau_d^{-1} = \sum_n \tau_n^{-1} G_n = \sum_n \Gamma_n^{-1} (1 - P_n) G_n
\]

(10)

Where \( \tau_n \) is the initial carrier relaxation time. The photon lifetime in the cavity is

\[
\tau_p^{-1} = \left( c n_e \right) \left[ \alpha_i + \ln \left( \frac{1}{R_1 R_2} \right) / \left( 2 L_{cw} \right) \right]
\]

(11)

Where \( R_1 \) and \( R_2 \) are the cavity mirror reflectivities and \( \alpha_i \) is the internal loss.

The linear optical gain which the \( j \)th QDs group gives to the \( m \)th mode photons is represented by
Table 1: Parameters Used for Simulation of InGaaS np-QD-LD [6]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>8mm</td>
</tr>
<tr>
<td>h</td>
<td>8mm</td>
</tr>
<tr>
<td>$E_{1\sigma}$</td>
<td>1eV</td>
</tr>
<tr>
<td>$V_\sigma$</td>
<td>$\pi\hbar$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>20$%$</td>
</tr>
<tr>
<td>$N_{p}$</td>
<td>3</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>4$%$</td>
</tr>
<tr>
<td>$R_{1,2}$</td>
<td>$10\mu m$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$6cm^{-1}$</td>
</tr>
<tr>
<td>d</td>
<td>10$\mu m$</td>
</tr>
<tr>
<td>$V_{\alpha}$</td>
<td>is the total active region volume $2.16 \times 10^{-16}$</td>
</tr>
<tr>
<td>$\tau_{\gamma}$</td>
<td>Photon cavity lifetime $8.8\mu s$</td>
</tr>
<tr>
<td>$L_{\infty}$</td>
<td>Cavity length $900_{\mu m}$</td>
</tr>
<tr>
<td>$\Gamma_{\theta}$</td>
<td>FWHM of inhomogeneous broadening $20meV$</td>
</tr>
<tr>
<td>$\tau_{\theta}$</td>
<td>Initial carrier relaxation lifetime $190\mu s$</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Spin-orbit interaction energy $0.35meV$</td>
</tr>
<tr>
<td>$m_e^*$</td>
<td>Electron effective mass $0.04m_0$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Spontaneous-emission coupling efficiency $10^{-4}$</td>
</tr>
<tr>
<td>$n_e$</td>
<td>Refractive index $3.5$</td>
</tr>
<tr>
<td>$E_{\alpha}$</td>
<td>Band gap of bulk semiconductor $0.8eV$</td>
</tr>
<tr>
<td>$\tau_{\alpha}$</td>
<td>Diffusion lifetime in the SCH region $150\mu s$</td>
</tr>
<tr>
<td>$\tau_{\alpha}$</td>
<td>Carrier recombination lifetime in the WL $5ns$</td>
</tr>
<tr>
<td>$\tau_{\alpha}$</td>
<td>Carrier recombination lifetime in the QDs $2.8ns$</td>
</tr>
</tbody>
</table>

\[
B_{nj} = \frac{2e^2\hbar N_{\sigma}}{cn_\alpha m_e^*} \left| \frac{P_{\sigma n}^2}{E_{\sigma n} - E_j} \right|^2 G_{\sigma n} \left( E_{mj} - E_j \right) \]

(12)

Where $n_e$ is the refractive index, $P_{\sigma n}^2$ is the transition matrix element, $E_{\sigma n}$ is the interband transition energy. The linear optical gain shows the homogeneous broadening of a Lorentz shape as

\[
B_{\sigma n} \left( E_{\alpha m} - E_j \right) = \frac{\hbar \gamma_{\alpha e}/\pi}{\left( E_{\alpha m} - E_j \right)^2 + \left( \hbar \gamma_{\alpha e} \right)^2}
\]

(13)

Where whose FWHM is given as $2\hbar \gamma_{\alpha e}$ with polarization dephasing or scattering rate $\gamma_{\alpha e}$. The transition matrix element is given as

\[
\left| P_{\sigma n}^2 \right|^2 = \left| T_{\sigma n} \right|^2 M^2
\]

(14)

Where $I_{\sigma n}$ represents the overlap integral between the envelope functions of an electron and a hole and

\[
M^2 = \frac{m_e^2}{12e^2} \frac{E_p(E_g + \Delta)}{E_g + 2\Delta/3}
\]

(15)

derived by the first-order k.p interaction between the conduction band and valence band. Here, $E_g$ is the band gap of bulk material, $m_e^*$ is the electron effective mass, $\Delta$ is the spin-orbit interaction energy of the QD material. Equation (14) holds as long as we consider QDs with a nearly symmetrical shape.

The coverage of QDs which is related to density and volume of QDs, $V_{\sigma}$ is given as

\[
\xi = N_{\sigma} V_{\sigma}
\]

(16)

The laser output power of the mth mode is also given as

\[
I_m = \hbar \omega_n c S_m \ln \left( 1/(1/R) / L_{\infty} \right)
\]

(17)

Where $\omega_n$ is the emitted photon frequency and $R$ is $R_1$ or $R_2$.

We solved the rate equations numerically using fourth-order Runge-Kutta method to simulate light-current characteristics and optical gain profile. The system reaches the steady-state after finishing the relaxation oscillation. We assume that all the carriers are injected into the WL, i.e., $\tau_{\omega c} = \tau_{\omega e} = \infty$ and do not consider the thermal carrier escape time, i.e., $\tau_{\omega c} = \infty$. It is also assumed that QDs have a cylindrical shape. The parameters we used at simulation are listed in Table 1 [6].

Simulation Results: Fig. 2 shows light-current (L-I) characteristics of the mentioned self-assembled QD laser diode (SAQD-LD) for FWHM of inhomogeneous broadening $\Gamma_{\alpha} = 20meV$ at (a) $h\gamma_{\alpha e} = 2, 4.5, 5, 5.5$ and $6.5meV$, (b) $h\gamma_{\alpha e} = 5, 7, 10$ and $20meV$ and also, FWHM of inhomogeneous broadenings (c) $\Gamma_{\alpha} = 30meV$ at $h\gamma_{\alpha e} = 7, 10, 15$ and $20meV$ and (d) $\Gamma_{\alpha} = 40meV$ at $h\gamma_{\alpha e} = 8$ and $10meV$.

As we can see from Fig. 2, with increase of homogeneous broadening, non-linearity appears at L-I characteristics and continues whereas the homogeneous broadening nearly becomes equal to inhomogeneous broadening. For large homogeneous broadenings, near to or larger than inhomogeneous broadening, output power increases infinitely as the injected current elevates. Besides, slope efficiency (external quantum differential efficiency) heightens as the homogeneous broadening increases up to the inhomogeneous broadening (Fig. 2(a) and Fig. 2(b)). In addition, when the homogeneous broadening exceeds the inhomogeneous broadening, static-characteristics degrade (Fig. 2(a) and Fig. 2(b)).
Fig. 2: light-current characteristics of SAOD-LD for FWHM of inhomogeneous broadening $\Gamma_0 = 20$ meV at (a) $\hbar \gamma_v = 2, 4.5, 5, 5.5, 6.5$ meV and (b) $\hbar \gamma_v = 2, 7, 10, 20$ meV and also FWHM of inhomogeneous broadenings (C) $\Gamma_0 = 30$ meV at $\hbar \gamma_v = 7, 10, 15, 20$ meV and (d) $\Gamma_0 = 40$ meV at $\hbar \gamma_v = 8$ and 10 meV.

Fig. 3: Output power as a function of injected current for homogeneous broadening 0.2 and 20 meV

Furthermore, Fig. 2 shows that slope efficiency decreases and threshold current also elevates with enhancement of inhomogeneous broadening as a result of decreasing the central group DOS. In some cases such as Fig. 2(a) for FWHM of homogeneous broadening 9 meV and at $I = 2.5$ mA or for FWHM of homogeneous broadening 10 meV and at $I = 4$ mA, all of the central group DOS occupies and the output power reaches its maximum amount, then, it decreases as the injected current increases, as a result of emitting of central group carriers within other modes.

Fig. 4: Shows L-I characteristics for QD coverage as a variable parameter $\xi = 0.04, 0.07, 0.1, 0.2, 0.4$ and 0.8.
Threshold current increases as the homogeneous broadening increases owing to elevating density of states (DOS) of the central group. This fact is revealed clearly in Fig. 3 where L-I characteristics is shown for homogeneous broadening 0.2 and 20 meV.

We can conclude that the SAQD laser has its best performance when homogeneous broadening is equal to inhomogeneous broadening.

Fig. 4. Shows L-I characteristics for QD coverage as a variable parameter $\xi = 0.04, 0.07, 0.1, 0.2, 0.4$ and 0.8.

As shown in Fig. 4, with increase of QD coverage from 0.1, threshold current increases owing to enhancing the number of QDs or the QD volume (the number of energy levels) and as a result, the need for more carriers to provide population inversion. Besides, output power and slope efficiency decline due to decreasing occupation probability and accordingly, increasing the relaxation time. We also see that a threshold QD coverage exists that enables output photons to overcome losses and to generate laser emission. It is clearly revealed that the QD coverage 0.1 has the best L-I characteristics and the lowest threshold current. It is concluded that there is an optimum QD coverage.

Fig. 5 shows optical gain-current characteristics of SAQD-LD for different initial relaxation times $\tau_e = 1, 10, 100, 300$ and 500 ps at different recombination times in QDs and WL (a) $\tau_e = 2.8$ ns, $\tau_{qr} = 3$ ns, (b) $\tau_e = 2.8$ ns, $\tau_{qr} = 0.5$ ns and (c) $\tau_e = 0.5$ ns, $\tau_{qr} = 0.5$ ns.

Optical gain increases to the threshold gain and then fixes with enhancement of injected current. Actually, what happens when the current is increased to a value above threshold is that the carrier density and optical gain initially (for on the order of a nanosecond) elevate to values above their threshold levels and the photon density grows. But then, the stimulated emission rate also heightens that leads to reducing of the carrier density and gain until a new steady-state balance is created [10]. Differential gain, before the threshold gain, declines as the initial relaxation time increases and recombination times in QDs and WL decrease. In fact, enhancing the initial relaxation time and decreasing the recombination times result in increasing the threshold
current. A small decrease occurs at threshold gain for lower recombination time in WL. After threshold, differential gain is the same for different initial relaxation times and different recombination times in WL (Fig. 5(a) and Fig. 5(b)). Since relaxation oscillation frequency has square direct relation with differential gain [11], relaxation oscillation frequency and modulation bandwidth decrease as the initial relaxation time enhances and recombination times decrease. Fig. 5(c) shows that light amplification is linear beneath the lasing threshold.

CONCLUSION

Considering energy level and size distributions of self-assembled quantum-dots, we solved the rate equations numerically using fourth-order Runge-Kutta method and simulated static-characteristics of InGaAs/GaAs SAQD-LD. Simulation results of the static-characteristics showed that slope efficiency elevates as the homogeneous broadening increases whereas it becomes equal to inhomogeneous broadening. Exceeding the homogeneous broadening from inhomogeneous broadening and elevating inhomogeneous broadening result in degradation of light-current characteristics. Threshold current increases as the homogeneous and inhomogeneous broadenings enhance. Nonlinearity appears in light-current characteristics whereas homogeneous broadening becomes equal to inhomogeneous broadening. This is the point that the SAQD laser has the best performance. In other words, the SAQD laser reveals its best performance when both homogeneous and inhomogeneous broadenings are the same. We also saw that increasing of homogeneous broadening from inhomogeneous broadening leads to declining of static-characteristics of InGaAs/GaAs SAQD-LD. We revealed that there is a threshold QD coverage and an optimum QD coverage in which the SAQD lasers operate with lowest possible threshold current and maximum output power. Optical gain increases to the threshold gain and then fixes as the injected current elevates. Differential gain also degrades as the initial relaxation time enhances and recombination times decrease, as a result, the relaxation oscillation frequency and modulation bandwidth degrade.

REFERENCES