Enhanced Sample Matrix Inversion Is a Better Beamformer for a Smart Antenna System

M. Yasin, Pervez Akhtar, M. Junaid Khan and S.H. Zaheer Naqvi

1Department of Electronics and Power Engineering
National University of Sciences and Technology, Islamabad, Pakistan
2Pakistan Navy Weapon Engineering School, KARSAZ, Karachi, Pakistan

Abstract: By using smart/adaptive antenna array system, limited available bandwidth can be used efficiently. Smart antenna radiates not only narrow beam towards desired users exploiting signal processing capability but also places null towards interferers, thus optimizing the signal quality and enhancing capacity. Sample matrix inversion (SMI), proposed enhanced sample matrix inversion (ESMI) and normalized sample matrix inversion (NSMI) are beamforming algorithms which are presented in this paper. Performance of SMI, ESMI and NSMI algorithms are investigated and simulation results verify the theoretical predictions. Simulation results confirmed that ESMI is found more efficient algorithm to implement in the mobile communication environment for CDMA/WCDMA to optimize the beam, enhancing capacity and service quality as compared to SMI and NSMI.

Keywords: Adaptive filtering · Adaptive signal processing algorithm · Sample matrix inversion (SMI) Algorithm · Enhanced sample matrix inversion (ESMI) Algorithm and Normalized sample matrix inversion (NSMI) Algorithm

INTRODUCTION

Since Radio Frequency (RF) spectrum is limited [1] and its efficient utilization is only possible by employing smart/adaptive antenna array system to exploit mobile systems capabilities for data and voice communication. The term smart refers to the use of signal processing in order to shape the antenna beam pattern in accordance to certain conditions. Basically smart means computer control of the antenna performance [2]. Adaptive signal processing forms vital part of the smart antenna system which controls the beam pattern by updating a set of antenna weights. Consider a smart antenna consists of antenna array elements equally spaced (d), terminated in an adaptive processor and user’s signal arrives from desired direction at an angle $\Phi_o$ as shown in Figure 1 [3]. SMI, ESMI and NSMI are the beamforming schemes, used for controlling weights adaptively to optimize signal to noise ratio (SNR) of the desired signal in look direction $\Phi_o$.

The array factor (AF) of the smart antenna [4-7] depends on the number of elements (Ne), the element spacing (d), amplitude and phase of the applied signal to each element which determines the surface area of the overall radiating structure. This surface area is called aperture which provides higher gain and the aperture efficiency quantifies how efficient the aperture is used. The influence of these parameters is explained by AF that is given by

Corresponding Author: M. Yasin, Department of Electronics and Power Engineering, National University of Sciences and Technology, Islamabad, Pakistan, E-mail: myasin@pucc.edu.pk.
$$\mathbf{AF}(\mathcal{D}) = \sum_{n=0}^{N-1} A_n \phi \left( \frac{2\pi d}{\lambda} \cos(\Phi + \alpha) \right)$$ (1)

Where $\alpha$ is the phase shift from element to element and is described as:

$$\alpha = \frac{-2\pi d}{\lambda} \cos \Phi_0$$ (2)

$\Phi_0$ is the desired direction of the beam and $d$ is the distance between elements.

This array factor spells out the effect of combining radiating elements in an array and results in certain directivity. This directivity ($\mathcal{D}$) linked through the efficiency ($\eta$) with the gain. The efficiency is given by

$$\eta = \frac{G}{\mathcal{D}}$$ (3)

Where directivity ($\mathcal{D}$) of an isotropic element is always unity or 0 dBi and thus $G \leq \mathcal{D}$ [8].

When smart antenna is deployed in mobile communications systems using code division multiple access (CDMA) environment, assigning different codes to different users, it radiates beam towards desired users only. Each beam becomes a channel, thus avoiding interference in a cell. Because of these, each coded channel reduces co-channel interference, due to the processing gain of the system. The processing gain (PG) of the CDMA system is described as:

$$PG = 10 \log (B/R_0)$$ (4)

Where $B$ is the CDMA channel bandwidth and $R_0$ is the information rate in bits per second.

If a single antenna is used for CDMA system, then this system supports a maximum of 31 users. When an array of five elements is employed instead of single antenna, then capacity of CDMA system can be increased more than four times. It can further be enhanced if array of more elements are used [9-17].

The next section introduces the theoretical predictions of SMI, ESMI and NNSM algorithms. Section 3 contains simulation results. Simulation analysis is presented in section 4. Finally, section 5 contains the conclusions.

**Algorithms Description**

**SMI Algorithm:** This algorithm was developed by Reed, Mallett and Brennen in 1974 [2]. It computes the array weights by replacing $R$ with its estimates [3, 18] and is based on an estimate of the correlation matrix as is given by

$$R(n) = (1/N) \sum_{n=0}^{N-1} \mathbf{x}(n)\mathbf{x}^H(n)$$ (5)

Where $\mathbf{x}$ is the signals arrived on array elements and is given by

$$\mathbf{x} = [x_1(n), x_2(n), ..., x_L(n)]^T$$ (6)

This signal array vector can also be written as

$$\mathbf{x}(n) = \mathbf{s}_c(n) a(\theta_c) + \sum_{l=1}^{L} \mathbf{s}_l(n) a(\theta_l) + \mathbf{N}(n)$$ (7)

Where $S_c$ & $S_l$ are the desired and interfering signals arriving at the array at an angle $\theta_c$ & $\theta_l$, respectively. $L$ is the number of interfering signals and $N$ is the white Gaussian noise at the array elements. $a(\theta_c)$ and $a(\theta_l)$ are the steering vectors for the desired and interfering signals respectively. The steering vector is described as

$$a(\theta) = [1, e^{\frac{-2\pi f d}{\lambda} \sin(\theta)}, ..., e^{\frac{-2\pi f d}{\lambda} (M-1) \sin(\theta)}]$$ (8)

Where $\lambda = \frac{c}{f}$ and $f$ is in Hertz.

The output of beamformer is given by

$$y = w^H x$$ (9)

Where $H$ denotes the Hermitian transpose (complex conjugate) operation. The array weight vector can be expressed as

$$w = [w_1, w_2, ..., w_M]^T$$ (10)

The estimate of $R$ is updated when new samples arrives, resulting in a new estimate of the weights using

$$R(n+1) = \frac{nR(n) + x(n+1)x^H(n+1)}{n+1}$$ (11)

As the number of samples grows, the matrix update approaches its true value and thus the estimated weights approaches the optimal weights [16] as given by

$$w(n) = R^{-1}(n)a(\theta)$$ (12)
That is \( n \to \infty, R(n) \to R \) and \( w(n) \to w \) or \( w_{\text{est}} \).

To calculate \( R^{-1} \), use matrix inversion lemma as given by

\[
R^{-1}(n) = R^{-1}(n-1) - \frac{R^{-1}(n-1)x(n)x^H(n)R^{-1}(n-1)}{1 + x^H(n)R^{-1}(n-1)x(n)}
\]  

(13)

Where \( R^{-1}(0) = \frac{I}{\epsilon_0} \) and \( \epsilon_0 > 0 \).

**ESMI Algorithm:** The proposed algorithm is designed in order to optimize the performance of existing SMI algorithm as described in subsection 2.1. The goal of this adaptive beamforming is to extract desired information \( S_d \) from signal array vector \( (x(n)) \) and to place null towards interferers of same frequency. This is achieved by adjusting weights of each antennas used in the array adaptively. Using (5) and (8) for computing optimum weight vectors, which is given by

\[
w(n) = R^{-1}(n)a(\theta) + \gamma R(n)
\]  

(14)

Where \( \gamma \) is known as the convergence stability factor, given by

\[0 < \gamma \leq 1
\]  

(15)

The convergence stability factor becomes a problem when it is too small or too large. Therefore it is better to select \( \gamma \) within bounded conditions as defined in (15).

Using matrix inversion lemma for \( R^{-1} \) computation as highlighted in (13) and \( R \) is estimated when new samples arrives, resulting in a new estimate of the weights using (11) where \( R \) is the autocorrelation matrix of filter input and is given by

\[
R(n) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)x^H(n)
\]  

(16)

Where \( x \) is the input samples arrived on the array system and is given by

\[
x = [x_1(n), x_2(n), ..., x_N(n)]^T
\]  

(17)

When the number of samples grows, the matrix update approaches its true value and thus the estimated weights approaches the optimal weights as given by (14), therefore it is expressed as \( n \to \infty, R(n) \to R \) and \( w(n) \to w \) or \( w_{\text{est}} \).

**NSMI Algorithm:** This proposed algorithm is also designed in order to enhance the performance of existing SMI algorithm as described in subsections 2.1. Again using (5) and (8) for computing optimum weight vectors, which is defined by

\[
w(n) = R^{-1}(n)a(\theta) + \mu \frac{R(n)}{R(n)}
\]  

(18)

Where \( R^{-1} \) is calculated as highlighted in (13) and \( R \) is updated when new samples arrives, resulting in a new estimate of the weights using (11) where \( R \) is the autocorrelation matrix of filter input and is given in (16). \( | \cdot | \) denotes absolute value and complex magnitude. \( a(\theta) \) is the steering vector which, for identical linearly spaced elements, as described in (8), \( \mu \) is the step size which relies on \( \lambda_{\text{max}} \). The \( \lambda_{\text{max}} \) is the largest Eigen value of autocorrelation matrix \( R \). If \( \mu \) is chosen to be very small, then convergence becomes fast. If \( \mu \) is kept large, then convergence becomes slow. Therefore it is better to select \( \mu \) within bounded conditions as defined below

\[
0 < \mu < \frac{1}{\lambda_{\text{max}}}
\]  

(19)

This is very similar to that of least mean square (LMS) algorithm.

The array output signal obtained with the sample weights is given by

\[y = w^T x
\]  

(20)

Where \( x \) is the input samples arrived on the array system and \( w \) is the weight vector, described as

\[w = [w_1, w_2, ..., w_N]^T
\]  

(21)

Where \( T \) denotes transpose of the weight matrix.

The weight matrix update approaches its true value \( (R(n) - R) \), when the number of samples grows i.e. \( n \to \infty \) and thus the estimated weights approaches the optimal weights \( (w(n) - w) \) or \( w_{\text{est}} \) as given by (18).

**Simulation Results:** Computer simulation is carried out, to illustrate that how various parameters such as number of elements \( (N_e) \) and element spacing \( (d) \), linked through the weights \( w \) to quantify the beam formation and null steering. The OOK signal as shown in Figure 2 and is given by
\[ S(t) = \begin{cases} A \cos(\omega t) & u(t) = 1 \\ 0 & u(t) = 0 \end{cases} \] (22)

Where \( \omega = 2\pi f \) and \( f \) is the frequency in Hertz.

The simulations are designed to analyze the properties of SMI, ESMI and NMSI algorithms. The applied signal modulation is amplitude shift keying (ASK), also called on-off keying (OOK) \([19-20]\) with \( \text{SNR} = 10 \ \text{dB} \), used for simulation purpose.

**Simulation for SMI**

**Effect of Number of Elements on Array Factor:** Uniform linear array is taken for simulation purpose with spacing between array elements as \( \lambda/2 \).

AOA for desired user is set at 20 degree and two interferers are at 40 and -70 degrees as shown in Figure 3 which provides deep null at -70 degrees and shallow null at 40 degrees but at the same time forms narrow beam in accordance to number of elements with constant space of \( \lambda/2 \) between elements but this happens at the cost of large number of sidelobes.

The data provided in Table 1 is taken on the basis of null depth performance, beam width and maximum side lobe level based on Figure 3 keeping effect of number of elements \( N \) on array factor.

The scatter plot for complex weights is also shown in Figure 4 for convergence analysis.

**Effect of Element Spacing on Array Factor:** The element spacing has a large influence on the array factor. Larger element spacing results in a higher directivity. The effect of array spacing is depicted in Figure 5 for \( Ne = 6 \) with two interferers \( (IAOA) \) at 20 and -20 degrees.

| Table 1: Effect of number of elements on null depth performance, beam width and maximum side lobe level |
|---|---|---|---|
| S. No. | Number of Elements *N* | Element Spacing *d* | Desired Angle of Arrival 'DAOA' (degree) | IAOA1 | IAOA2 |
| | | | | -70 | 40 | Max. Side lobe level (dB) | Beam width (degree) |
| 1. | 6 | \( \lambda/2 \) | 20 | -32 | -14 | -7 | 34 |
| 2. | 8 | \( \lambda/2 \) | 20 | -33 | -12 | -3 | 20 |
| 3. | 10 | \( \lambda/2 \) | 20 | -27 | -33 | -6 | 28 |

| Table 2: Effect of element spacing on null depth performance, beam width and maximum side lobe level |
|---|---|---|---|
| S. No. | Number of Elements *N* | Element Spacing *d* | Desired Angle of Arrival 'DAOA' (degree) | IAOA1 | IAOA2 |
| | | | | -20 | 20 | Max. Side lobe level (dB) | Beam width (degree) |
| 1. | 6 | \( \lambda/2 \) | 0 | -27 | -12 | -10 | 34 |
| 2. | 6 | \( \lambda/4 \) | 0 | -23 | -27 | -6 | 46 |
| 3. | 6 | \( \lambda/8 \) | 0 | -38 | -11 | -6 | 102 |
Table 3: Performance analysis of null depth performance, beam width and maximum side lobe level taking different number of elements

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Number of Elements 'N'</th>
<th>Element Spacing 'd'</th>
<th>Desired Angle of Arrival 'DOA' (degree)</th>
<th>Null depth performance (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>4</td>
<td>λ/2</td>
<td>20</td>
<td>L/AOA1: -29, L/AOA2: -23, Max. Side lobe level (dB): -8, Beam width (degree): 42</td>
</tr>
<tr>
<td>2.</td>
<td>6</td>
<td>λ/2</td>
<td>20</td>
<td>L/AOA1: -22, L/AOA2: -23, Max. Side lobe level (dB): -7, Beam width (degree): 34</td>
</tr>
<tr>
<td>3.</td>
<td>8</td>
<td>λ/2</td>
<td>20</td>
<td>L/AOA1: -33, L/AOA2: -35, Max. Side lobe level (dB): -11, Beam width (degree): 32</td>
</tr>
</tbody>
</table>

Fig. 4: Scatter plot for complex weights for SMI algorithm

Fig. 5: Normalized array factor plot for SMI algorithm for Ne = 6 with two interferers at 20 and -20 degrees

In Table 2, the performance analysis is provided based on the null depth performance, beam width and maximum side lobe level taken from Figure 5 keeping effect of element spacing on array factor.

Simulation for ESMI

Effect of Number of Elements on Array Factor:
The desired user is at 20 degree and two interferers are at 40 and -20 degrees as shown in Figure 6 which provides deep null at -10 degrees and at 40 degrees but at the same time forms good narrow beam in accordance to number of elements with constant space of λ/2 between elements but this happens at the cost of number of side lobes.

The data given in Table 3 is taken on the basis of null depth performance, beam width and maximum side lobe level extracted from Figure 6 keeping effect of number of elements (N) on array factor.

The scatter plot for complex weights is also shown in Figure 7 for convergence analysis with SMI and NSMI.

Effect of Element Spacing on Array Factor: The influence of array spacing is depicted in Figure 8 for Ne = 6 with two interferers at 30 and -30 degrees. AOA for desired user is set at 20 degrees.
Table 4: Performance analysis of null depth performance, beam width and maximum side lobe level taking different element spacing

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Number of Elements 'N'</th>
<th>Element Spacing 'd'</th>
<th>Desired Angle of Arrival 'DAOA' (degree)</th>
<th>IAOA1 30</th>
<th>IAOA2 30</th>
<th>Max. Side lobe level (dB)</th>
<th>Beam width (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>6</td>
<td>λ/2</td>
<td>20</td>
<td>-32</td>
<td>-20</td>
<td>-10</td>
<td>38</td>
</tr>
<tr>
<td>2.</td>
<td>6</td>
<td>λ/4</td>
<td>20</td>
<td>-27</td>
<td>-28</td>
<td>-8</td>
<td>50</td>
</tr>
<tr>
<td>3.</td>
<td>6</td>
<td>λ/8</td>
<td>20</td>
<td>-18</td>
<td>Nil</td>
<td>-13</td>
<td>132</td>
</tr>
</tbody>
</table>

Table 5: Comparison of null depth performance, beam width and maximum side lobe level taking different number of elements

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Number of Elements 'N'</th>
<th>Element Spacing 'd'</th>
<th>Desired Angle of Arrival 'DAOA' (degree)</th>
<th>IAOA1 30</th>
<th>IAOA2 40</th>
<th>Max. Side lobe level (dB)</th>
<th>Beam width (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>4</td>
<td>λ/2</td>
<td>20</td>
<td>-30</td>
<td>-35</td>
<td>-10</td>
<td>45</td>
</tr>
<tr>
<td>2.</td>
<td>6</td>
<td>λ/2</td>
<td>20</td>
<td>-43</td>
<td>-42</td>
<td>-15</td>
<td>40</td>
</tr>
<tr>
<td>3.</td>
<td>8</td>
<td>λ/2</td>
<td>20</td>
<td>-35</td>
<td>-43</td>
<td>-16</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 6: Comparison on Basis of Weights Convergence

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Figure No.</th>
<th>N</th>
<th>DAOA (degree)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>6,8,10</td>
<td>20</td>
<td>Good</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>4,6,8</td>
<td>20</td>
<td>Better</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>4,6,8</td>
<td>30</td>
<td>Good</td>
</tr>
</tbody>
</table>

Fig. 7: Scatter plot for complex weights for ESMI algorithm

Fig. 8: Normalized array factor plot for ESMI algorithm for Ne = 6 with two interferers at -30 and 30 degrees

**Simulation for NSMI**

**Effect of Number of Elements on Array Factor:** AOA for desired user is set at 20 degrees and two nulls are shown at 40 and -30 degrees for constant space of λ/2 between elements as depicted in Figure 9 for Ne = 4, 6, and 8. Deep null is achieved at 40 degrees.

In Table 5, the data given is taken on the basis of null depth performance, beam width and maximum side lobe level extracted from Figure 9 keeping effect of number of elements (N) on array factor.

The scatter plot for complex weights is also shown in Figure 10 for convergence analysis with SMI and ESMI and summary of analysis on the basis of weights convergence is given in Table 6.
Table 7: Comparison of null depth performance, beam width and maximum side lobe level taking different element spacing

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Number of Elements 'N'</th>
<th>Element Spacing 'd'</th>
<th>Desired Angle of Arrival 'DAOA' (degree)</th>
<th>Null depth performance (dB)</th>
<th>IAOA1</th>
<th>IAOA2</th>
<th>Max. Side lobe level (dB)</th>
<th>Beam width (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>6</td>
<td>λ/2</td>
<td>20</td>
<td>-37</td>
<td>-30</td>
<td>-13</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>6</td>
<td>λ/4</td>
<td>20</td>
<td>-22</td>
<td>-34</td>
<td>-15</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>6</td>
<td>λ/8</td>
<td>20</td>
<td>-25</td>
<td>Nil</td>
<td>-15</td>
<td>138</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 9:** Normalized array factor plot for NSMI algorithm with AOA for desired user is 20 degrees and 40 and -30 degrees for interferer with constant space of λ/2 between elements

**Fig. 10:** Scatter plot for complex weights for NSMI algorithm

**Fig. 11:** Normalized array factor plot for NSMI algorithm for Ne = 6 with two interferers at -40 and 40 degrees

**Effect of Element Spacing on Array Factor:** The effect of array spacing is shown in Figure 11 for Ne = 6. AOA is 20 degrees for desired user with two interferers at 40 and -40 degrees. Deep null is obtained at 40 degrees and shallow null is at -20 degrees.

The performance analysis is also made available in Table 7 based on the null depth performance, beam width and maximum side lobe level taken from Figure 11 keeping effect of element spacing on array factor.

**RESULTS AND DISCUSSION**

In this paper, three adaptive beamforming algorithms are compared on the basis of their beam formation, null steering and fast convergence capability.

It is confirmed from the simulation results that optimize narrow beam of smart antenna can be obtained towards the desired direction by steering beam angle $\Phi_0$, keeping elements spacing $d$ and number of elements $Ne$ with the help of beamformer merely consists of SMI, ESMI and NMSI beamforming algorithms. SMI algorithm has
good response for placing null in the desired direction as compared to beam formation. The proposed algorithms i.e. ESMI and NSMI have better capability to form beam in the desired direction as well as to place null towards interferers. However, ESMI has good potential as compared to NSMI and SMI algorithms for beam formation and null steering. Array directivity increases with the number of elements but at the same time, side lobes and the side lobe level also increases with the number of elements. The element spacing has a large influence on the array factor. Larger element spacing results in a higher directivity. An element spacing beyond \( \lambda \) becomes impractical and results in multiple unwanted grating lobes.

It is clear from the Figures 4, 7 and 10 that the convergence property of ESMI is good as compared to SMI and NSMI as the speed of convergence for ESMI does not depend on Eigen value (\( \lambda_{\text{max}} \)) of input correlation matrix \( R \) that play significant role in optimum solution \( w_i \) in case of NSMI. Therefore, the convergence capability of ESMI is better than NSMI and SMI.

It is also ascertained from Figures 3, 5, 6, 8, 9 and 10 that the performance of ESMI algorithm is better to enhance beam and place null keeping same number of elements and spacing maintained between elements. From the parameters summarized in Tables 1 to 7, it is confirmed that each algorithms is better but ESMI is the best among them.

It is to be noted that value of \( \mu \) and \( \gamma \) can be adjusted for NSMI and ESMI algorithm for getting a better simulation result than the one shown in Figures 6 to 11.

Therefore, ESMI is found the well-organized algorithm in terms of its beam formation, null steering and fast convergence capability.

**CONCLUSION**

ESMI is found the most efficient algorithm and also simple in computation. ESMI provides good beam forming capabilities due to which it has better directivity. Directivity (\( D \)) linked through the efficiency (\( \eta \)) with the gain; therefore, maximum bandwidth is utilizing to exchange information between transmitters and receivers, thus enhancing capacity and range. ESMI is, therefore, a better option to implement at base station of mobile communications systems using CDMA/WCDMA environment to avoid interference, system overloading and for better efficiency.

**REFERENCES**


Muhammad Yasin is enrolled for PhD in the field of electrical engineering majoring in telecommunication in Pakistan Navy Engineering College, National University of Science and Technology, Karachi, Pakistan. He is working in Pakistan navy as naval officer in the capacity of communication engineer since 1996. His research interests include signal processing, adaptive filtering, implementation of communication networking and its performance evaluation. He has received a B.Sc. degree in electrical engineering from NWFP University of Engineering and Technology, Peshawar (1994) and M.Sc. degree in electrical engineering from NED, University of Engineering and Technology, Karachi (2006). He has also done a Master degree in Economics (2002) from University of Karachi. In the past, he is involved in implementation of ISO 9000 on indigenous project of AGOSTA 90B Class Submarines along with French engineers. Currently, he is working on indigenous project of Acoustic System Trainer, being used for imparting Sonar related training.