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Factored Lorentz Matrix Via Infeld-Van Der Waerden Symbols

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Abstract: We use the Infeld-van der Waerden symbols to factorize an arbitrary Lorentz matrix.

Key words: Lorentz matrix - Infeld-van der Waerden symbols - Unimodular matrix - Special relativity

INTRODUCTION

We know that any Lorentz matrix can be generated with the Infeld-van der Waerden symbols [1-4] via the following expression [5-8]:

$$L^{\mu}{}_{\nu} = \sigma^{\mu}{}_{A\dot{R}}\, B^{A}{}_{C}\, \sigma_{\nu}{}^{C\dot{E}}\, B^{\dagger\dot{R}}_{\dot{E}}\,, \qquad B = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, \qquad B^{\dagger} = \begin{pmatrix} \alpha^{*} & \gamma^{*} \\ \beta^{*} & \delta^{*} \end{pmatrix}, \qquad \alpha\,\delta - \beta\,\gamma = 1, \tag{1}$$

Then:

$$L^{\mu}{}_{\nu} = \sigma^{\mu}{}_{A\dot{1}}\, B^{A}{}_{C}\, \sigma_{\nu}{}^{C\dot{E}} B^{\dagger\dot{1}}_{\dot{E}} + \sigma^{\mu}{}_{A\dot{2}}\, B^{A}{}_{C}\, \sigma_{\nu}{}^{C\dot{E}}\, B^{\dagger\dot{2}}_{\dot{E}} = {}_{1}P^{\mu}{}_{C}\;\, {}_{1}Q^{C}{}_{\nu} + {}_{2}P^{\mu}{}_{C}\;\, {}_{2}Q^{C}{}_{\nu}\,, \tag{2}$$

with the rectangular matrices:

$$({}_{r}P^{\mu}{}_{C})_{4x2} = (\sigma^{\mu}{}_{A\dot{r}}\, B^{A}{}_{C}), \qquad ({}_{r}Q^{C}{}_{\nu})_{2x4} = \left(\sigma_{\nu}{}^{C\dot{E}} B^{\dagger\dot{r}}_{\dot{E}}\right), \qquad r = 1, 2. \tag{3}$$

It is simple to construct the matrices (3), in fact:

$$\left({}_{1}P^{\mu}{}_{C}\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \\ i\gamma & i\delta \\ \alpha & \beta \end{pmatrix}, \qquad \left({}_{1}Q^{C}{}_{\nu}\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha^{*} & \beta^{*} & i\beta^{*} & \alpha^{*} \\ \beta^{*} & \alpha^{*} & -i\alpha^{*} & -\beta^{*} \end{pmatrix},$$

$$\left({}_{2}P^{\mu}{}_{C}\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} \gamma & \delta \\ \alpha & \beta \\ -i\alpha & -i\beta \\ -\gamma & -\delta \end{pmatrix}, \qquad \left({}_{2}Q^{C}{}_{\nu}\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} \gamma^{*} & \delta^{*} & i\delta^{*} & \gamma^{*} \\ \delta^{*} & \gamma^{*} & -i\gamma^{*} & -\delta^{*} \end{pmatrix}, \tag{4}$$

Hence from (2) and (4) the Lorentz matrix is factored in the form:

$$(L^{\mu}{}_{\nu}) = \begin{pmatrix} {}_{1}P & {}_{2}P \end{pmatrix} \begin{pmatrix} {}_{1}Q \\ {}_{2}Q \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ \gamma & \delta & \alpha & \beta \\ i\gamma & i\delta & -i\alpha & -i\beta \\ \alpha & \beta & -\gamma & -\delta \end{pmatrix} \begin{pmatrix} \bar{\alpha} & \bar{\beta} & i\bar{\beta} & \bar{\alpha} \\ \bar{\beta} & \bar{\alpha} & -i\bar{\alpha} & -\bar{\beta} \\ \bar{\gamma} & \bar{\delta} & i\bar{\delta} & \bar{\gamma} \\ \bar{\delta} & \bar{\gamma} & -i\bar{\gamma} & -\bar{\delta} \end{pmatrix}, \tag{5}$$

In accordance with [9-13]; therefore [7, 14-19]:

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$$L^0{}_0 = \frac{1}{2}\left(\alpha\bar{\alpha} + \beta\bar{\beta} + \gamma\bar{\gamma} + \delta\bar{\delta}\right), \quad L^1{}_0 = \frac{1}{2}\left(\bar{\alpha}\,\gamma + \bar{\beta}\delta\right) + cc, \quad L^2{}_0 = -\frac{i}{2}\left(\alpha\bar{\gamma} - \bar{\beta}\delta\right) + cc\,,$$

$$L^0{}_1 = \frac{1}{2}(\bar{\alpha}\beta + \bar{\gamma}\delta) + cc, \quad L^1{}_1 = \frac{1}{2}(\bar{\alpha}\delta + \beta\bar{\gamma}) + cc, \quad L^2{}_1 = -\frac{i}{2}\left(\alpha\bar{\delta} + \beta\bar{\gamma}\right) + cc,$$

$$L^0{}_2 = -\frac{i}{2}(\bar{\alpha}\beta + \bar{\gamma}\delta) + cc, \quad L^1{}_2 = -\frac{i}{2}(\bar{\alpha}\delta + \beta\bar{\gamma}) + cc, \quad L^2{}_2 = \frac{1}{2}\left(\bar{\alpha}\delta - \bar{\beta}\gamma\right) + cc\,, \tag{6}$$

$$L^0{}_3 = \frac{1}{2}\left(\alpha\bar{\alpha} - \beta\bar{\beta} + \gamma\bar{\gamma} - \delta\bar{\delta}\right), \quad L^1{}_3 = \frac{1}{2}\left(\bar{\alpha}\gamma - \bar{\beta}\delta\right) + cc, \quad L^2{}_3 = -\frac{i}{2}\left(\alpha\bar{\gamma} + \bar{\beta}\delta\right) + cc\,,$$

$$L^3{}_0 = \frac{1}{2}\left(\alpha\bar{\alpha} + \beta\bar{\beta} - \gamma\bar{\gamma} - \delta\bar{\delta}\right), \quad L^3{}_1 = \frac{1}{2}(\bar{\alpha}\beta - \bar{\gamma}\delta) + cc, \quad L^3{}_2 = -\frac{i}{2}(\bar{\alpha}\beta - \bar{\gamma}\delta) + cc\,,$$

$$L^3{}_3 = \frac{1}{2}\left(\alpha\bar{\alpha} - \beta\bar{\beta} - \gamma\bar{\gamma} + \delta\bar{\delta}\right), \qquad \alpha\delta - \beta\gamma = 1,$$

where *cc* means the complex conjugate of all the previous terms.

REFERENCES

1. Van Der Waerden, B.L., 1929. Spinoranalyse, Nachr. Ges. Wiss. Göttingen Math.-Phys., pp: 100-109.
2. Infeld, L., 1932. Die verallgemeinerte spinorenrechnung und die Diracschen gleichungen, Phys. Z., 33: 475-483.
3. Infeld, L. and B.L. Van Der Waerden, 1933. Die wellengleichung des elektrons in der allgemeinen relativitätstheorie, Sitz. Ber. Preuss. Akad. Wiss. Physik-Math. Kl., 9: 380-401.
4. O'Donnell, P., 2003. Introduction to 2-spinors in general relativity, World Scientific, Singapore.
5. Veblen, O., 1933. Geometry of two-component spinors, Proc. Nat. Acad. Sci. USA, 19: 462-474.
6. Carvajal-Gámez, B.E., M. Galaz and J. López-Bonilla, 2007. On the Lorentz matrix in terms of Infeld-van der Waerden symbols, Scientia Magna, 3(3): 56-57.
7. Ahsan, Z., J. López-Bonilla and B.M. Tuladhar, 2014. Lorentz transformations via Pauli matrices, J. of Advances in Natural Sci., 2(1): 49-51.
8. Hernández-Galeana, A., J. López-Bonilla, R. López-Vázquez and G.R. Pérez-Teurel, 2015. Faraday tensor and Maxwell spinor, Prespacetime J., 6(2): 88-97.
9. López-Bonilla, J. and D. Morales-Cruz, 2020. Rodrigues-Cartan's expression for Lorentz transformations, Studies in Nonlinear Sci., 5(3): 41-42.
10. López-Bonilla, J. and M. Morales-García, 2020. Factorization of the Lorentz matrix, Comput. Appl. Math. Sci., 5(2): 32-33.
11. López-Bonilla, J., D. Morales-Cruz and S. Vidal-Beltrán, 2021. On the Lorentz matrix, Studies in Nonlinear Sci., 6(1): 1-3.
12. Cruz-Santiago, R., J. López-Bonilla and S. Vidal-Beltrán, 2021. Lorentz transformation and its associated unimodular matrix, Comput. Appl. Math. Sci., 6(1): 5-8.
13. Cruz-Santiago, R., J. López-Bonilla and N. Mondragón-Medina, 2021. Unimodular matrix for a given Lorentz transformation, Studies in Nonlinear Sci., 6(1): 4-6.
14. Ju. Rumer, 1936. Spinorial analysis, Moscow.
15. Aharoni, J., 1959. The special theory of relativity, Clarendon Press, Oxford.
16. Synge, J.L., 1965. Relativity: the special theory, North-Holland, Amsterdam.
17. Penrose, R. and W. Rindler, 1984. Spinors and space-time. I, Cambridge University Press.
18. López-Bonilla, J., J. Morales and G. Ovando, 2002. On the homogeneous Lorentz transformation, Bull. Allahabad Math. Soc., 17: 53-58.
19. Acevedo, M., J. López-Bonilla and M. Sánchez, 2005. Quaternions, Maxwell equations and Lorentz transformations, Apeiron, 12(4): 371-384.