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Dirac Matrices and the Shifted Eigenvalue Problem

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Abstract: We obtain two shifted eigenvalue problems for the Dirac supermatrix.

Key words: Dirac matrices - Singular value decomposition

INTRODUCTION

The sixteen Dirac matrices, in the standard representation [1-4]:

$$I, \qquad \gamma^{\mu}, \qquad \gamma^{5}, \qquad \gamma^{\mu}\gamma^{5}, \qquad \sigma^{\mu\nu}, \tag{1}$$

Allow construct the supermatrix [5, 6]:

$$D_{16\,x\,16} = \begin{pmatrix} I & -i\gamma^3\gamma^5 & -\gamma^0\gamma^5 & \sigma^{20} \\ i\sigma^{13} & -\gamma^0 & i\gamma^2 & -\gamma^5 \\ -\gamma^3 & i\sigma^{01} & \sigma^{21} & \gamma^1\gamma^5 \\ -\gamma^1 & i\sigma^{30} & \sigma^{32} & -\gamma^3\gamma^5 \end{pmatrix} = (\vec{u}_1 \; \cdots \; \vec{u}_4), \tag{2}$$

and its transpose:

$$D^T = \begin{pmatrix} I & i\sigma^{31} & \gamma^3 & \gamma^1 \\ i\gamma^2\gamma^5 & -\gamma^0 & i\sigma^{01} & i\sigma^{30} \\ \gamma^0\gamma^5 & i\gamma^2 & \sigma^{21} & \sigma^{32} \\ \sigma^{02} & -\gamma^5 & \gamma^1\gamma^5 & -\gamma^3\gamma^5 \end{pmatrix} = (\vec{r}_1 \; \cdots \; \vec{r}_4). \tag{3}$$

It is possible to show the property:

$$D^T D = \Lambda\, D, \qquad \Lambda = \Lambda^T = 4\begin{pmatrix} I & 0 & 0 & 0 \\ 0 & -\gamma^0 & 0 & 0 \\ 0 & 0 & \sigma^{21} & 0 \\ 0 & 0 & 0 & -\gamma^3\gamma^5 \end{pmatrix}, \qquad \Lambda\,\Lambda^T = 16\, I_{16\,x\,16}, \tag{4}$$

Which is equivalent to the following expression:

$$D^T\,\vec{u}_j = \Lambda\,\vec{u}_j\,, \qquad j = 1, \dots, 4, \tag{5}$$

That is:

$$D^T\begin{pmatrix} I \\ i\sigma^{13} \\ -\gamma^3 \\ -\gamma^1 \end{pmatrix} = 4\begin{pmatrix} I \\ i\gamma^2\gamma^5 \\ \gamma^0\gamma^5 \\ \sigma^{02} \end{pmatrix} = 4\,\vec{v}_1, \qquad D^T\begin{pmatrix} -i\gamma^2\gamma^5 \\ -\gamma^0 \\ i\sigma^{01} \\ i\sigma^{30} \end{pmatrix} = 4\begin{pmatrix} -i\gamma^2\gamma^5 \\ I \\ \sigma^{02} \\ -\gamma^0\gamma^5 \end{pmatrix} = 4\,\vec{v}_2, \tag{6}$$

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$$D^T \begin{pmatrix} -\gamma^0\gamma^5 \\ i\gamma^2 \\ \sigma^{21} \\ \sigma^{32} \end{pmatrix} = 4 \begin{pmatrix} -\gamma^0\gamma^5 \\ \sigma^{20} \\ I \\ i\gamma^2\gamma^5 \end{pmatrix} = 4\,\vec{v}_3, \qquad D^T \begin{pmatrix} \sigma^{20} \\ -\gamma^5 \\ \gamma^1\gamma^5 \\ -\gamma^3\gamma^5 \end{pmatrix} = 4 \begin{pmatrix} \sigma^{20} \\ \gamma^0\gamma^5 \\ -i\gamma^2\gamma^5 \\ I \end{pmatrix} = 4\,\vec{v}_4\,.$$

Besides, it is simple to prove the inverse of (6):

$$D\vec{v}_j = 4\,\vec{u}_j\,, \qquad 1 \le j \le 4, \tag{7}$$

Hence (6) and (7) represent the ‘shifted eigenvalue problem’ [7-10] for the Dirac supermatrix (2).

Similarly:

$$DD^T = \Lambda\, D^T \qquad \therefore \qquad D\,\vec{r}_k = \Lambda\,\vec{r}_k\,, \quad 1 \le k \le 4, \tag{8}$$

That is:

$$D \begin{pmatrix} I \\ i\gamma^2\gamma^5 \\ \gamma^0\gamma^5 \\ \sigma^{02} \end{pmatrix} = 4 \begin{pmatrix} I \\ i\sigma^{13} \\ -\gamma^3 \\ -\gamma^1 \end{pmatrix} = 4\,\vec{w}_1, \qquad D \begin{pmatrix} i\sigma^{31} \\ -\gamma^0 \\ i\gamma^2 \\ -\gamma^5 \end{pmatrix} = 4 \begin{pmatrix} i\sigma^{31} \\ I \\ -\gamma^1 \\ \gamma^3 \end{pmatrix} = 4\,\vec{w}_2,$$

$$D \begin{pmatrix} \gamma^3 \\ i\sigma^{01} \\ \sigma^{21} \\ \gamma^1\gamma^5 \end{pmatrix} = 4 \begin{pmatrix} \gamma^3 \\ \gamma^1 \\ I \\ i\sigma^{13} \end{pmatrix} = 4\,\vec{w}_3, \qquad D \begin{pmatrix} \gamma^1 \\ i\sigma^{30} \\ \sigma^{32} \\ -\gamma^3\gamma^5 \end{pmatrix} = 4 \begin{pmatrix} \gamma^1 \\ -\gamma^3 \\ i\sigma^{31} \\ I \end{pmatrix} = 4\,\vec{w}_4\,, \tag{9}$$

and it is immediate to obtain the inverse of (9):

$$D^T\,\vec{w}_k = 4\,\vec{r}_k\,, \qquad k = 1, \dots, 4, \tag{10}$$

Thus (9) and (10) generate an alternative shifted eigenvalue problem for (2).

Finally, it is interesting to exhibit the following factorizations of the Dirac supermatrices (2) and (3):

$$D = \tfrac{1}{4}\,\Lambda\, V, \quad V = (\vec{v}_1 \ \cdots \ \vec{v}_4) \qquad \& \qquad D^T = \tfrac{1}{4}\,\Lambda\, W, \quad W = (\vec{w}_1 \ \cdots \ \vec{w}_4). \tag{11}$$

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