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On Some Propagators in Quantum Physics

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Abstract: We use certain function as candidate to generate propagators in quantum mechanics.

Key words: Green function - Schrödinger equation - Gaussian function

INTRODUCTION

Here we consider the following kernel:

$$K(x,t;x_0,t_0) = \frac{1}{\sqrt{i\,\pi\,\hbar\,F}} \exp[\frac{1}{i\,\hbar}(-\frac{G}{F}(x-x_0)^2 + P\,x + Q], \tag{1}$$

where F, G, P, Q are functions of t, t_0, x_0 such that:

$$lim_{\,t\to t_0}\ F = lim_{\,t\to t_0} P = lim_{\,t\to t_0}\ Q = 0, lim_{\,t\to t_0}\ G = 1, \tag{2}$$

Then (1) and (2) imply the property:

$$lim_{\,t\to t_0}\ K(x,t;x_0,t_0) = \delta(x-x_0). \tag{3}$$

Now we ask the verification of the Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 K}{\partial x^2} + V(x)\,K = i\,\hbar\,\frac{\partial K}{\partial t}, \tag{4}$$

Which gives the following structure for the corresponding potential:

$$V(x) = a\,x^2 + b\,x, \tag{5}$$

where *a* and *b* are constants satisfying the expressions:

$$a = -\frac{d}{dt}\left(\frac{G}{F}\right) - \frac{2}{m}\left(\frac{G}{F}\right)^2, b = P' + \frac{2\,P}{m}\left(\frac{G}{F}\right) - 2a\,x_0\,, \tag{6}$$

$$P'\,x_0 - \frac{P^2}{2m} + Q' - \frac{i\hbar}{2G}\,(G' + a\,F) - a\,x_0^2 - b\,x_0 = 0.$$

Now we consider three special cases:

- $F = \frac{2}{m}(t-t_0), G = 1, P = Q = 0$, in agreement with (2).

Then (5) and (6) imply $a = b = 0$ and $V = 0$, thus (1) gives the propagator for free particle [1-8]:

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$$K(x,t;x_0,t_0) = \sqrt{\frac{m}{2\pi i\hbar\,(t-t_0)}}\ \exp\left[\frac{im}{2\hbar\,(t-t_0)}(x-x_0)^2\right]. \tag{7}$$

- $F = \frac{2}{m}(t-t_0), G = 1, P = \frac{b}{2}(t-t_0), Q = \frac{b}{2}\,x_0\,(t-t_0) + \frac{b^2}{24\,m}(t-t_0)^3$, in according with (2).

Thus (5) and (6) give $a = 0$ and $V(x) = bx$, hence (1) is the propagator for a linear potential [2, 4, 7-10]:

$$K(x,t;x_0,t_0) = \sqrt{\frac{m}{2\pi i\hbar\,(t-t_0)}}\ \exp\left\{\frac{i}{2\hbar}\left[\frac{m}{t-t_0}(x-x_0)^2 - b\,(x+x_0)(t-t_0) - \frac{b^2}{12\,m}(t-t_0)^3\right]\right\}. \tag{8}$$

- $F = \frac{2}{m\omega}\,Sin[\omega(t-t_0)], G = Cos[\omega(t-t_0)], P = m\omega\,x_0\tan\left[\frac{\omega}{2}(t-t_0)\right], Q = 0,$

In harmony with (2). Therefore, $a = \frac{m\,\omega^2}{2}$, $b = 0$ and $V(x) = \frac{m\omega^2}{2}\,x^2$, then (1) gives the propagator for the harmonic oscillator [2, 4, 6-9, 11-15]:

$$K(x,t;x_0,t_0) = \sqrt{\frac{m\omega}{2\pi i\hbar\,Sin(\omega(t-t_0))}}\ \exp\{\frac{im\omega}{2\hbar\,Sin(\omega(t-t_0))}[(x^2+x_0^2)\,Cos(\omega(t-t_0)) - 2x_0\,x]\}. \tag{9}$$

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