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New Treatment of Adomian Decomposition Method with Compaction Equations

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Abstract: The Adomian Decomposition Method (ADM) shows a very good ability in dealing with nonlinear differential equations but it is still have some drawbacks. In this paper, it is shown how we can improve the efficiency of the method by using Improved Adomian Decomposition Method (IADM) and linking the method with other techniques like Padé approximants and Laplace transform. The results shows efficiency of the new treatment with a special type of Nonlinear Partial Differential Equations (NPDE) like compacton equations *K(n,n), B(n,n) and* Zakharov-Kuznetsov *ZK(n,n)*. The technique is powerful with these equations. It gives the exact solution directly.

Key words: Nonlinear partial differential equation . adomian decomposition method . compacton . laplace transform . padé technique

INTRODUCTION

In recent years, nonlinear phenomena play a crucial role in applied mathematics and physics. Directly searching for exact solutions of nonlinear partial differential equations has become more and more attractive partly due to the availability of computer symbolic systems Like mathematica or maple that allow us to perform some complicated and tedious algebraic calculation on a computer as well as helping us to find exact solutions of partial differential equations [1-9].

Since the beginning of the 1980s, Adomian has presented and developed the so-called decomposition method for solving linear or nonlinear problems such as ordinary differential equations and partial differential equations. Adomian's decomposition method consists of splitting the given equation into linear and nonlinear parts, inverting the highest-order derivative operator contained in the linear operator on both sides, identifying the initial and the terms involving the independent variable alone as initial approximation, decomposing the unknown function into a series whose components are to be determined, decomposing the nonlinear function in terms of special polynomials called Adomian's polynomials and finding the successive terms of the series solution by recurrent relation using Adomian's polynomials. Some applications of the method show its advantages in dealing with nonlinear differential equation.[10-16].

ADM can't deal with nonlinear differential equation in general. Abassy shows what type of equation it can deal with effectively and introduced Improved ADM for dealing with equations which ADM can't deal with. In this paper, IADM is used in solving one of the case studies.

The series solution that obtained by ADM or IADM like any series it has a restricted region of convergence. The obtained series contains some information about the exact solution. Abassy *et al*. [11, 12] extract more information from the obtained series solution using Padé technique, which enables us to view data can't be seen by using ADM alone.

In this paper, we will apply the new treatment to compacton equations, where Rosenau and Hyman [17] investigated the role of nonlinear dispersion in the formation of patterns in liquid and introduced a class of solitary waves with compact support, which they called compactons, by introducing and studying a family of nonlinear KdV like equations of the form

$$u_t + a\left(u^n\right)_x + \left(u^n\right)_{3x} = 0 \quad n > 1 \qquad (1.1)$$

calling it K(n,n). The focusing branch (a = 1) of equation (1.1) exhibits compact solitary traveling structures, whereas the defocusing branch (a =-1) admits solitary patterns having cusps or infinite slopes. The K(n,n) equation (1.1) cannot be derived from a first-order Lagrangian except for n = 1 and did not possess the usual conservation laws of energy that KdV equation possessed [17]. Numerous works followed in

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[17-26] to investigate the deep qualitative change in the genuinely nonlinear phenomena caused by the purely nonlinear dispersion. The stability analysis has revealed that compactons are stable structures.

The purpose of the paper is improving the efficiency of ADM and solving its drawbacks. In this paper, the exact solution of special type of NPDE is obtained by using more treatments on the truncated series solution. The treatment uses the Laplace transform and Padé approximants.

BASICS OF IMPROVED ADOMIAN DECOMPOSITION METHOD

Consider the following general non-linear initial value problem

$$\begin{aligned} &Lu(x,t)+Ru(x,t)+N\big(u(x,t)\big)=0\\ &u(x,0)=f_0(x)\\ &\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=f_1(x)\\ &\quad\vdots\\ &\left.\frac{\partial^{s-1}u(x,t)}{\partial t^{s-1}}\right|_{t=0}=\frac{f_{s-1}(x)}{(s-1)!}\end{aligned} \tag{1}$$

where

$$L=\frac{\partial^s}{\partial t^s},\ s=1,2,3,\ldots$$

is the highest partial derivative with respect to t, R is a linear operator and Nu (x,t) is the nonlinear term. Ru (x, t) and N(u(x,t)) are free of partial derivative with respect to t.

Following the usual analysis of standard Adomian [10-12, 27],

The inverse operator L^{-1} is an integral operator which is given by

$$L^{-1}(.)=\int_0^t\cdots\int_0^t(.)dt\ldots(sfold)\ldots dt \tag{2}$$

Applying L^{-1} on equation (1) and using the constrains lead to

$$\begin{aligned}u(x,t)&=f_0(x)+f_1(x)t+\cdots\\&+f_{s-1}(x)t^{s-1}-L^{-1}(N\big(u(x,t)\big)+Ru(x,t))\end{aligned} \tag{3}$$

The Adomian decomposition method assumes that the unknown function $u(x,t)$ can be expressed by an infinite series of the form

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t) \tag{4}$$

and the nonlinear term N(u(x,t)) can be decomposed by an infinite series of polynomials given by

$$N\big(u(x,t)\big)=\sum_{n=0}^{\infty}A_n \tag{5}$$

where the components $u_n(x,t)$ will be determined recurrently and A_n are the so-called Adomian polynomials of $u_0, u_1, u_2,\ldots, u_n$ and are defined by Adomian [10] in the form

$$A_n(x,t)=\frac{1}{n!}[\frac{d^n}{d\lambda^n}N(\sum_{i=0}^{\infty}\lambda^i u_i(x,t))]_{\lambda=0},\ \ n=0,1,2,3... \tag{6}$$

which are redefined by Abassy [27] in the form

$$A_n=\frac{1}{ns!}[\frac{d^{ns}}{d\lambda^{ns}}N(\sum_{i=0}^{\infty}\lambda^i f_i t^i)]_{\lambda=0}+\frac{1}{(ns+1)!}[\frac{d^{(ns+1)}}{d\lambda^{(ns+1)}}N(\sum_{i=0}^{\infty}\lambda^i f_i t^i)]_{\lambda=0}+\ldots+\frac{1}{(ns+s-1)!}[\frac{d^{(ns+s-1)}}{d\lambda^{(ns+s-1)}}N(\sum_{i=0}^{\infty}\lambda^i f_i t^i)]_{\lambda=0},\ \ n=0,1,2,3... \tag{7}$$

where f_i is the coefficient of t^i in $u_n(x,t)$ components. For example when s = 1 leads to

$$A_n=\frac{1}{n!}[\frac{d^n}{d\lambda^n}N(\sum_{i=0}^{\infty}\lambda^i f_i t^i)]_{\lambda=0},\ \ n=0,1,2,3... \tag{8}$$

when s = 2 leads to

$$A_n=\frac{1}{2n!}[\frac{d^{2n}}{d\lambda^{2n}}N(\sum_{i=0}^{\infty}\lambda^i f_i t^i)]_{\lambda=0}+\frac{1}{(2n+1)!}[\frac{d^{(2n+1)}}{d\lambda^{(2n+1)}}N(\sum_{i=0}^{\infty}\lambda^i f_i t^i)]_{\lambda=0},\ \ n=0,1,2,3... \tag{9}$$

when s = 3 leads to

$$A_n=\frac{1}{3n!}[\frac{d^{3n}}{d\lambda^{3n}}N(\sum_{i=0}^{\infty}\lambda^i f_i t^i)]_{\lambda=0}+\frac{1}{(3n+1)!}[\frac{d^{(3n+1)}}{d\lambda^{(3n+1)}}N(\sum_{i=0}^{\infty}\lambda^i f_i t^i)]_{\lambda=0}+\frac{1}{(3n+2)!}[\frac{d^{(3n+2)}}{d\lambda^{(3n+2)}}N(\sum_{i=0}^{\infty}\lambda^i f_i t^i)]_{\lambda=0},\ \ n=0,1,2,3... \tag{10}$$

and so on ….

Substituting by equations (4) and (5) into equation (3) gives

$$\sum_{n=0}^{\infty} u_n(x,t) = f_0(x) + f_1(x)t + \cdots + f_{s-1}(x)t^{s-1} - L^{-1}\Big(\sum_{n=0}^{\infty} A_n + R\big(\sum_{n=0}^{\infty} u_n\big)\Big) \quad (11)$$

The component of $u_n(x,t)$ follows immediately upon setting

$$u_0(x,t) = f_0(x) + f_1(x)t + \cdots + f_{s-1}(x)t^{s-1} \quad (12)$$

$$u_{n+1}(x,t) = -L^{-1}(A_n + Ru_n), \qquad n \ge 0 \quad (13)$$

The new Adomian polynomial (7) gives the same result of old Adomian polynomials when s = 1 but different polynomials when s = 2,3,4,… TheADM does not give the exact power series for equation (1) when s = 2,3,4,… IADM gives the exact power series solution for equation (1) when s = 2,3,4,… and cancels the calculations of all the non accurate terms which consume time, effort and deteriorate the convergences.

CASE STUDIES

Many nonlinear partial differential equations are used in the following case studies.

Case-Study 1: " k(2,2)": Consider the k (2,2) equation [17], which takes the form

$$u_t + (u^2)_x + (u^2)_{3x} = 0$$
$$u(x,0) = \begin{cases} \left(\sqrt{\frac{4c}{3}}\mathrm{Cos}\left[\frac{x}{4}\right]\right)^2, & \left|\frac{x}{4}\right| \le \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

Following the analysis of Adomian [10, 27], equation (14) can be re-written in an operator form as

$$Lu(x,t) + N1(u) + N2(u) = 0 \quad (15)$$

where the differential operator L = ∂/∂t and the nonlinear terms $N1(u) = (u^2)_x$ and $N2(u) = (u^2)_{xxx}$. The nonlinear terms N1(u) and N2(u) can be decomposed by an infinite series of polynomials A_n and B_n using (8) respectively because s = 1. Adomian polynomials that represent the nonlinear term $N1(u) = (u^2)_x$ are

$$\begin{aligned} A_0 &= 2f_{0_x}f_0 \\ A_1 &= \left(2f_{0_x}f_1 + 2f_{1_x}f_0\right)t \\ A_2 &= \left(2f_{0_x}f_2 + 2f_{1_x}f_1 + 2f_{2_x}f_0\right)t^2 \\ A_3 &= \left(2f_{0_x}f_3 + 2f_{1_x}f_2 + 2f_1f_{2x} + 2f_{3_x}f_0\right)t^3 \\ A_4 &= \left(2f_{0_x}f_4 + 2f_{1_x}f_3 + 2f_{2_x}f_2 + 2f_{3_x}f_1 + 2f_{4_x}f_0\right)t^4 \\ &\vdots \end{aligned} \quad (16)$$

polynomials that represent the nonlinear term $N2(u) = (u^2)_{xxx}$ are

$$\begin{aligned} B_0 &= (6f_{0_x}f_{0_{xx}} + 2f_0f_{0_{xxx}}) \\ B_1 &= (6f_{1_x}f_{0_{xx}} + 2f_1f_{0_{xxx}} + 6f_{0_x}f_{1_{xx}} + 2f_0f_{1_{xxx}})t \\ B_2 &= (6f_{2_x}f_{0_{xx}} + 6f_{1_x}f_{1_{xx}} + 6f_{0_x}f_{2_{xx}} \\ &\quad + 2f_2f_{0_{xxx}} + 2f_1f_{1_{xxx}} + 2f_0f_{2_{xxx}})t^2 \\ B_3 &= (6f_{3_x}f_{0_{xx}} + 6f_{2_x}f_{1_{xx}} + 6f_{1_x}f_{2_{xx}} + 6f_{0_x}f_{3_{xx}} + 2f_3f_{0_{xxx}} \\ &\quad + 2f_2f_{1_{xxx}} + 2f_1f_{2_{xxx}} + 2f_0f_{3_{xxx}})t^3 \\ &\vdots \end{aligned} \quad (17)$$

Other polynomials can be generated in a like manner. The components $u_n(x,t)$ obtained immediately by using

$$u_0 = \left(\sqrt{\frac{4c}{3}}\mathrm{Cos}\left[\frac{x}{4}\right]\right)^2 \quad (18)$$

and substituting in the equation

$$u_{n+1}(x,t) = -L^{-1}(A_n + B_n) \quad (19)$$

Following the above procedure we obtain the following components

$$\begin{aligned} u_0(x,t) &= \frac{4c}{3}\mathrm{Cos}^2\left[\frac{x}{4}\right] \\ u_1(x,t) &= \frac{1}{3}c^2\mathrm{Sin}\left[\frac{x}{2}\right]t \\ u_2(x,t) &= -\frac{1}{12}c^3\mathrm{Cos}\left[\frac{x}{2}\right]t^2 \\ u_3(x,t) &= -\frac{1}{72}c^4\mathrm{Sin}\left[\frac{x}{2}\right]t^3 \\ u_4(x,t) &= \frac{1}{576}c^5\mathrm{Cos}\left[\frac{x}{2}\right]t^4 \\ &\vdots \end{aligned} \quad (20)$$

and so on. Considering these components, the solution can be approximated as:

$$u(x,t) \simeq U_n = \sum_{m=0}^{n} u_m(x,t) \quad (21)$$

Now, taking U_5, which has the form

$$U_5(x,t)=\frac{4c}{3}\mathrm{Cos}^2\left[\frac{x}{4}\right]+\frac{1}{3}c^2\mathrm{Sin}\left[\frac{x}{2}\right]t-\frac{1}{12}c^3\mathrm{Cos}\left[\frac{x}{2}\right]t^2-\frac{1}{72}c^4\mathrm{Sin}\left[\frac{x}{2}\right]t^3+\frac{1}{576}c^5\mathrm{Cos}\left[\frac{x}{2}\right]t^4+\frac{1}{5760}c^6\mathrm{Sin}\left[\frac{x}{2}\right]t^5 \quad (22)$$

It is a partial sum of the Taylor series of the exact solution u(x,t) at t = 0. Appling Laplace transformation to $U_5(x,t)$, which yields

$$\pounds[U_5(x,t)]=\frac{4c}{3s}\mathrm{Cos}^2\left[\frac{x}{4}\right]+\frac{c^2}{3s^2}\mathrm{Sin}\left[\frac{x}{2}\right]-\frac{c^3}{6s^3}\mathrm{Cos}\left[\frac{x}{2}\right]-\frac{c^4}{12s^4}\mathrm{Sin}\left[\frac{x}{2}\right]+\frac{c^5}{24s^5}\mathrm{Cos}\left[\frac{x}{2}\right]+\frac{c^6}{48s^6}\mathrm{Sin}\left[\frac{x}{2}\right] \quad (23)$$

For the sake of simplicity, let s = 1/t then

$$\pounds[U_5(x,t)]=\frac{4c}{3}\mathrm{Cos}^2\left[\frac{x}{4}\right]t+\frac{c^2}{3}\mathrm{Sin}\left[\frac{x}{2}\right]t^2-\frac{c^3}{6}\mathrm{Cos}\left[\frac{x}{2}\right]t^3-\frac{c^4}{12}\mathrm{Sin}\left[\frac{x}{2}\right]t^4+\frac{c^5}{24}\mathrm{Cos}\left[\frac{x}{2}\right]t^5+\frac{c^6}{48}\mathrm{Sin}\left[\frac{x}{2}\right]t^6 \quad (24)$$

Its [L/M Padé approximant with L≥3 and M≥2 yields

$$\left[\frac{L}{M}\right]=\frac{8c\left(1+\mathrm{Cos}\left[\frac{x}{2}\right]\right)t+4c^2\mathrm{Sin}\left[\frac{x}{2}\right]t^2+2c^3t^3}{12+3c^2t^2} \quad (25)$$

Recalling t = 1/s, we obtain [L/M] in terms of s
By using the inverse Laplace transformation to [L/M], the true solution is obtained.

$$u(x,t)=\left(\sqrt{\frac{4c}{3}}\mathrm{Cos}\left[\frac{(x-ct)}{4}\right]\right)^2 \quad (26)$$

Because the equation is a compacton equation so the final solution takes the form of compacton

$$u(x,t)=\begin{cases}\left(\sqrt{\frac{4c}{3}}\mathrm{Cos}\left[\frac{(x-ct)}{4}\right]\right)^2, & \left|\frac{(x-ct)}{4}\right|\le\frac{\pi}{2}\\ 0, & \text{otherwise}\end{cases} \quad (27)$$

Figure 1 illustrates the surface solution of (27)

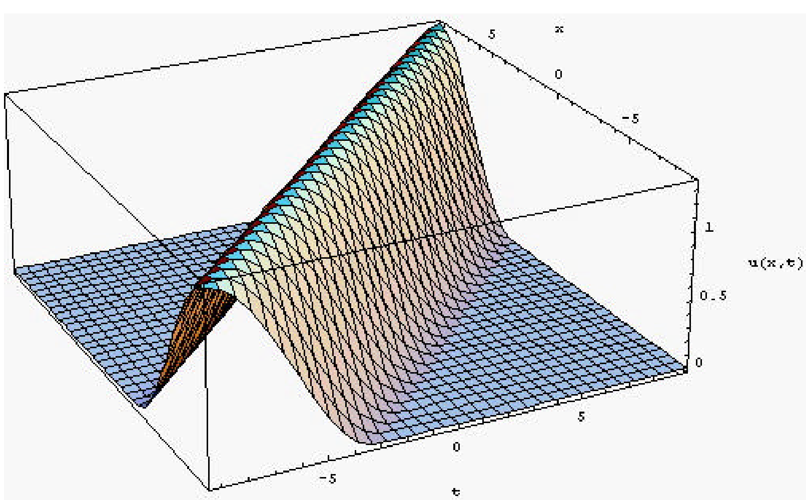

Fig. 1: Plot of K(2,2) solution where c = 1

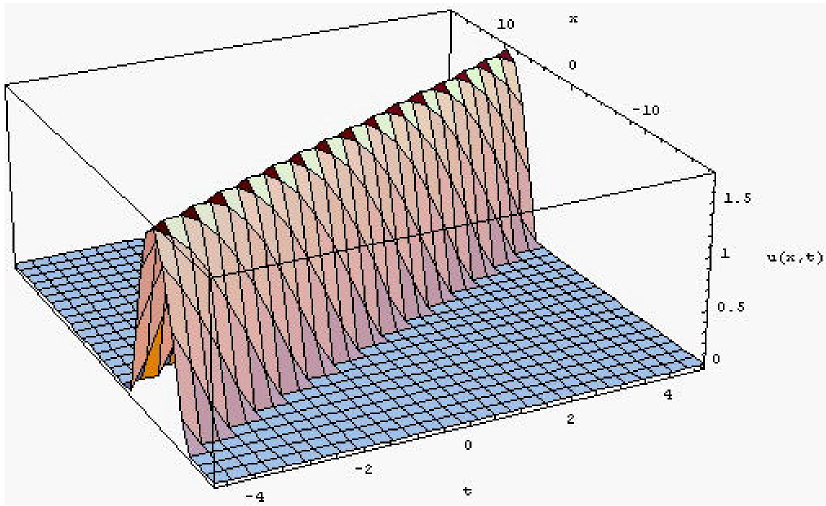

Fig. 2: Plot of K(3,3) solution where c = 2

Case-Study 2: " k(3,3)": Consider the k(3,3) equation [17], which takes the form

$$u_t + \left(u^3\right)_x + \left(u^3\right)_{3x} = 0$$

$$u(x,0) = \begin{cases} \sqrt{\dfrac{3c}{2}}\,\mathrm{Cos}\left[\dfrac{x}{3}\right], & \left|\dfrac{x}{3}\right| \le \dfrac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

Following the same procedures in case-study 1, we obtain the exact compacton solution.

$$u(x,t) = \begin{cases} \sqrt{\dfrac{3c}{2}}\,\mathrm{Cos}\left[\dfrac{(x-ct)}{3}\right], & \left|\dfrac{(x-ct)}{3}\right| \le \dfrac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \tag{29}$$

Figure 2 illustrates the surface solution of (29)

Case-study 3: "Compacton: the (2+1) dimensions Zk(2,2)": Consider the compacton Zk(2,2) equation [28, 29], which takes the form

$$u_t + a(u^2)_x + b(u^2)_{xxx} + k(u^2)_{yyx} = 0,$$

$$u(x,y,0) = \begin{cases} \left(\sqrt{\dfrac{4c}{3a}}\,\mathrm{Cos}\left[\sqrt{\dfrac{a}{b+k}}\,\dfrac{x+y}{4}\right]\right)^2, & \left|\sqrt{\dfrac{a}{b+k}}\,\dfrac{x+y}{4}\right| \le \dfrac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \tag{30}$$

Following the analysis of Adomian [13], equation (30) can be re-written in an operator form as

$$Lu(x,t) + a\,N1(u) + bN2(u) + kN3(u) = 0 \tag{31}$$

where the differential operator L = ∂/∂t and the nonlinear terms $N1(u) = (u^2)_x$, $N2(u) = (u^2)_{xxx}$ and $N3(u) = (u^2)_{yyx}$ The nonlinear terms N1(u), N2(u) and N3(u) can be decomposed by an infinite series of polynomials A_n, B_n and C_n using (8) respectively. Adomian polynomials that represent the nonlinear term $N1(u) = (u^2)_x$ given by:

$$\begin{aligned} A_0 &= 2f_{0_x} f_0 \\ A_1 &= (2f_{0_x} f_1 + 2f_{1_x} f_0)t \\ A_2 &= (2f_{0_x} f_2 + 2f_{1_x} f_1 + 2f_{2_x} f_0)t^2 \\ A_3 &= (2f_{0_x} f_3 + 2f_{1_x} f_2 + 2f_1 f_{2_x} + 2f_{3_x} f_0)t^3 \\ A_4 &= (2f_{0_x} f_4 + 2f_{1_x} f_3 + 2f_{2_x} f_2 + 2f_{3_x} f_1 + 2f_{4_x} f_0)t^4 \\ &\vdots \end{aligned} \tag{32}$$

polynomials that represent the nonlinear term $N2(u) = (u^2)_{xxx}$ are

$$\begin{aligned} B_0 &= 6f_{0_x} f_{0_{xx}} + 2f_0 f_{0_{xxx}} \\ B_1 &= (6f_{1_x} f_{0_{xx}} + 2f_1 f_{0_{xxx}} + 6f_{0_x} f_{1_{xx}} + 2f_0 f_{1_{xxx}})t \\ B_2 &= (6f_{2_x} f_{0_{xx}} + 6f_{1_x} f_{1_{xx}} + 6f_{0_x} f_{2_{xx}} + 2f_2 f_{0_{xxx}} + 2f_1 f_{1_{xxx}} + 2f_1 f_{3_{xxx}})t^2 \\ B_3 &= (6f_{3_x} f_{0_{xx}} + 6f_{2_x} f_{1_{xx}} + 6f_{1_x} f_{2_{xx}} + 6f_{0_x} f_{3_{xx}} \\ &\quad + 2f_3 f_{0_{xxx}} + 2f_2 f_{1_{xxx}} + 2f_1 f_{2_{xxx}} + 2f_0 f_{3_{xxx}})t^3 \\ &\vdots \end{aligned} \tag{33}$$

polynomials that represent the nonlinear term $N3(u) = (u^2)_{yyx}$ are

$$\begin{aligned} C_0 &= 2f_{0_X} f_{0_{YY}} + 4f_{0_{XY}} f_{0_Y} + 2f_0 f_{0_{YYX}} \\ C_1 &= 2(f_{1_{YY}} f_{0_X} + f_{1_X} f_{0_{YY}} + 2f_{1_Y} f_{0_{YX}} + 2f_{0_Y} f_{1_{YX}} + f_1 f_{0_{YYX}} + f_0 f_{1_{YYX}})t \\ C_2 &= 2(f_{2_{YY}} f_{0_X} + 2f_{1_{YY}} f_{1_X} + 2f_{0_{YY}} f_{2_X} + 2f_{2_Y} f_{0_{YX}} + 2f_{1_Y} f_{1_{YX}} \\ &\quad + 2f_{0_Y} f_{2_{YX}} + f_2 f_{0_{YYX}} + f_1 f_{1_{YYX}} + f_0 f_{2_{YYX}})t^2 \\ C_3 &= 2(f_{3_{YY}} f_{0_X} + f_{2_{YY}} f_{1_X} + f_{1_{YY}} f_{2_X} + f_{0_{YY}} f_{3_X} + 2f_{3_Y} f_{0_{YX}} + 2f_{2_Y} f_{1_{YX}} \\ &\quad + 2f_{1_Y} f_{2_{YX}} + 2f_{0_Y} f_{3_{YX}} + f_3 f_{0_{YYX}} + f_2 f_{1_{YYX}} + f_1 f_{2_{YYX}} + f_0 f_{3_{YYX}})t^3 \\ &\vdots \end{aligned} \tag{34}$$

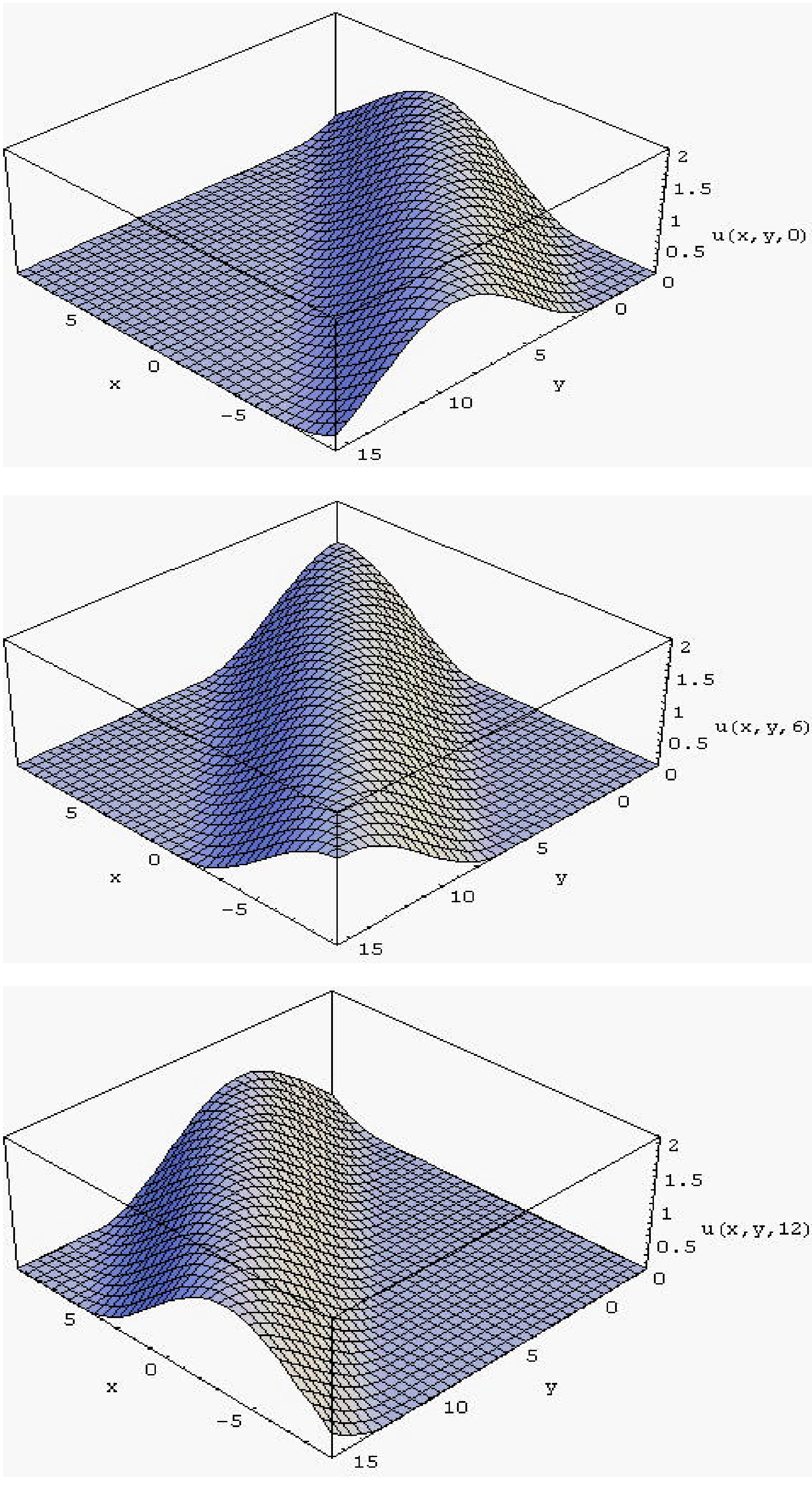

Fig. 3: Plot of Zk(2,2) solution at successive time interval where

Other polynomials can be generated in a like manner. The components $u_n(x, t)$ obtained immediately by using

$$u_0 = \left(\sqrt{\frac{4c}{3a}}\mathrm{Cos}\left[\sqrt{\frac{a}{b+k}}\frac{x+y}{4}\right]\right)^2$$

and substituting in the equation

$$u_{n+1}(x,y,t) = -L^{-1}\left(a\,A_n + b\,B_n + k\,C_n\right) \tag{35}$$

Following the same procedure and taking U_5 and using [L/M] Padé approximant with L≥3 and M≥2, the true solution is obtained

$$u(x,y,t) = \left(\sqrt{\frac{4c}{3a}}\mathrm{Cos}\left[\sqrt{\frac{a}{b+k}}\frac{(x+y-ct)}{4}\right]\right)^2 \tag{36}$$

Because the equation is a compacton equation so the final solution takes the form of compacton

$$u(x,y,t) = \begin{cases} \left(\sqrt{\frac{4c}{3a}}\mathrm{Cos}\left[\sqrt{\frac{a}{b+k}}\frac{(x+y-ct)}{4}\right]\right)^2, & \left|\sqrt{\frac{a}{b+k}}\frac{(x+y-ct)}{4}\right| \le \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \tag{37}$$

Figure 3 illustrate the surface solution of Zk(2,2) at successive time interval.

Case-study 4: "Compacton: the (2+1) dimensions Zk(3,3)": Consider the compacton Zk(3,3) equation [28, 29], which takes the form

$$u_t + a(u^3)_x + b(u^3)_{xxx} + k(u^3)_{yyx} = 0,$$
$$u(x,y,0) = \begin{cases} \sqrt{\frac{3c}{2a}}\mathrm{Cos}\left[\sqrt{\frac{a}{b+k}}\frac{x+y}{3}\right], & \left|\sqrt{\frac{a}{b+k}}\frac{x+y}{3}\right| \le \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \tag{38}$$

Following the same procedure in case-study 3, we obtain the exact compacton solution

$$u(x,y,t) = \begin{cases} \sqrt{\frac{3c}{2a}}\mathrm{Cos}\left[\sqrt{\frac{a}{b+k}}\frac{(x+y-ct)}{3}\right], & \left|\sqrt{\frac{a}{b+k}}\frac{(x+y-ct)}{3}\right| \le \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \tag{39}$$

Case-Study 5: "Coupled Burgers' system": Consider the Coupled Burgers' system of equations [30], which takes the form

$$\begin{aligned} &u_t - \left(u^2\right)_x - u_{xx} + (uv)_x = 0 \\ &v_t - \left(v^2\right)_x - v_{xx} + (uv)_x = 0 \\ &u(x,0) = \mathrm{Sin}(x) \quad \text{and} \quad v(x,0) = \mathrm{Sin}(x) \end{aligned} \tag{40}$$

Following the same procedure but with two simultaneous equation, we obtain the exact solution

$$v(x,t) = e^{-t}\mathrm{Sin}(x),\ \ u(x,t) = e^{-t}\mathrm{Sin}(x) \tag{41}$$

Case-study-6: B(2,2) Equation: Consider the compacton B(2,2) equation [31], which takes the form

$$u_{tt} - \left(u^2\right)_{xx} - \left(u^2\right)_{4x} = 0,$$
$$u(x,0) = \begin{cases} \left(\sqrt{\frac{4c^2}{3}}\cos\left[\frac{x}{4}\right]\right)^2, & \left|\frac{x}{4}\right| \le \frac{\pi}{2}, \\ 0, & \text{otherwise} \end{cases} \qquad u_t(x,0) = \begin{cases} \frac{-1}{3}c^3\sin\left[\frac{x}{2}\right], & \left|\frac{x}{4}\right| \le \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \tag{42}$$

where c is an arbitrary constant.

Solving (42) using IADM where the Adomian polynomials are calculated using (9). Taking the IADM series solution and following the same procedure in previous case-studies, we obtain the compacton solution

$$u(x,t)=\begin{cases}\left(\sqrt{\frac{4c^2}{3}}\cos\left[\frac{(x-ct)}{4}\right]\right)^2, & \left|\frac{(x-ct)}{4}\right|\le\frac{\pi}{2}\\ 0, & \text{otherwise}\end{cases} \quad (43)$$

CONCLUSION

ADM can deal with highly nonlinear differential equations with no need to small parameter or linearization. The solution procedure is very simple by means of ADM and few iteration leads to high accurate solutions in a restricted region of convergence [11, 12]. We can't trust in ADM solution, when the solution is needed outside the convergence region. So, ADM needs some treatment. In [11, 12], a treatment is done by using Padé technique, which increase the region of convergence. It enable us to view data can't seen by using ADM alone. In this paper more treatment using Laplace transform and Padé technique is added. The Laplace transforms and Padé technique treatment leads to obtain the exact solution.

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