Middle-East Journal of Scientific Research 13 (8): 983-992, 2013
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2013.13.8.3454

Natural Convection Heat Transfer Flow Past a Magnetized Vertical Permeable Plate for Liquid Metals

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Abstract: This paper considers the natural convection flow of, viscous, incompressible electrically conducting fluid in which the Prandtl number (Pr) and magnetic Prandtl number (*Pm*) have been chosen in the range of liquid metals in the presence of magnetic field parameter (*S*) along with a magnetized porous plate. The effect of above mentioned parameters have also been calculated in terms of coefficients of skin friction, rate of heat transfer and current density. Moreover, the effects of Prandtl number (Pr), magnetic Prandtl number (*Pm*) and magnetic force parameter (*S*) on velocity, temperature distribution and transverse component of magnetic field for different values of transpiration parameter ξ have also been investigated. The well known numerical techniques finite difference method for primitive variable formulation and asymptotic series solution for stream function formulation have been used in this investigation. Later, the results obtained by both methods have been compared and outcome found to be in quite excellent agreement.

Key words: Natural convection · Coefficient of skin friction · Rate of heat transfer · Current density · Transverse component of magnetic field · Transpiration · Liquid metals

INTRODUCTION

In the presence of strong magnetic field the natural convection boundary layer flow of an electrically conducting fluid has been investigated [1-4] because of its wide applications in nuclear engineering in connection with the cooling reactors. Further contribution to the problem was given by Cramer and Pai [5] with varying the surface temperature. The problem of viscous, electrically conducting liquid past a fixed semi-infinite unmagnetized plate has been considered by Davies [6] and [8]. Gribben [9] studied the magnetohydrodynamic boundary layer flow when an external magnetohydrodynamic pressure gradient effect the flow pattern. The boundary layer flow of electrically conducting gas with an aligned magnetic field at large distances from the plate was determined by Ingham [10]. He examined that when the magnetic field increases or decreases for a given Mach number which results to thickens the momentum and thermal boundary layer thicknesses. The magnetohydrodynamic boundary layer in uniform flow past a magnetized plate, when uniform magnetic field in the stream direction is applied examined by Glauert [11]. He observed that the exceed in the strength of magnetic field implies that the separation in boundary layer is occurred. Chawla [12] has been investigated the laminar boundary layer flow on a magnetized plate for low frequency fluctuating by using series expansion has been investigated and observed that the phase angle of surface current decreases and its amplitude increases with frequency. Later, Wilks [13] studied the uniform heat flux past a plate by using perturbation series expansion and obtained results near the leading edge and downstream regime. The combined effect of forced and free convection fluid flow in the presence of magnetic filed with uniform heat flux has been carried out by Hossain and Ahmad cite{Hossain and Ahmad}. The viscous and joule heating effect on free convection flow has been examined by Takhar and Soundalgekar [14] by using regular perturbation series expansion technique. The viscous and joule heating on

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the flow of an electrically conducting and viscous incompressible fluid past a semi-infinite plate has been examined by Hossain [15]. The effect of thermal radiation on two dimensional natural convection flow by using implicit finite difference method has been investigated by Molla *et al*. [16]. Gorla and Hossain [17] studied the natural convection flow of a non newtonian fluid past a uniformly vertical heated slotted surface and obtained solution by using finite difference scheme. The numerical solution of the flow and heat transfer in a triangular enclosure filled with a fluid in saturated porous medium with conducting thin fin on the hot vertical wall has been analyzed by Varol *et al*. [18]. In the presence of magnetic field the natural convection flow in an inclined rectangular enclosure with isothermal vertical wall has been considered by Ece *et al*. [19]. Krisna *et al* [20] studied the natural convective heat transfer in a rectangular porous duct with differentially heated sidewalls. Similarly, the case of natural convective heat transfer along a horizontal cylinder under isothermal conditions when one part of its surface is adiabatic has been studied by Gusev *et al*. [21]. Recently, [22-26] discussed the boundary layer heat transfer in the presence of magnetic field for different fluids.

In light of above literature survey the hydromagnetic natural convection flow along a magnetized vertical porous plate has not been treated previously. In present study the effects of varying the Prandtl number, Pr, magnetic Prandtl number *Pm* and magnetic force parameter *S*, on coefficient of skin friction $Gr_L^{-3/4}x^{-1/4}C_f$, rate of heat transfer $Gr_L^{1/4}x^{1/4}Nu_x$ and current density $Gr_L^{-3/4}x^{-1/4}J_w$ are shown. The effects of above mentioned parameters on velocity profile, temperature distribution and transverse component magnetic field are also examined. The numerical solutions for intermediate range of transpiration parameter ξ have been obtained by using finite difference method. The asymptotic series solution for small and large value of transpiration parameter ξ has been compared with the numerical solutions that have been obtained by finite difference method and found to be in concordance.

Basic Equations and Flow Model: In present study, We consider a steady two-dimensional magnetohydrodynamic natural convection flow of an electrically conducting, viscous and incompressible fluid past a uniformly heated vertical porous plate.

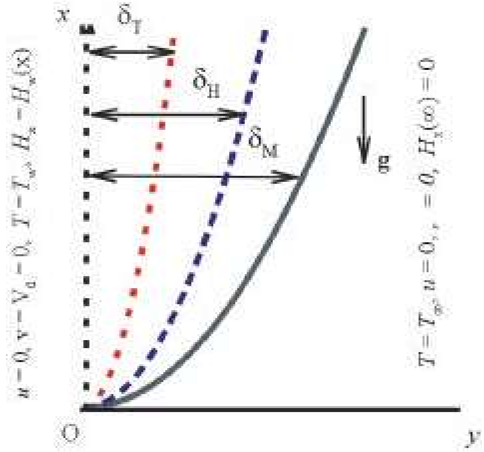

Fig. 1: The coordinate system and flow configuration

The coordinate system of the flow model is shown in Fig. 1. Here, the *x*-axis is taken along the surface and *y*-axis is normal to it.

In Fig.1 δ_M, δ_T and δ_H are momentum, thermal and magnetic field boundary layer thicknesses. The dimensioned boundary layer equations purposed by Davies [6-7] are given as under

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\bar{\mu}}{\rho}\left(\bar{G}_x\frac{\partial \bar{G}_x}{\partial \bar{x}} + \bar{G}_y\frac{\partial \bar{G}_x}{\partial \bar{y}}\right) + g\beta(\bar{T} - \bar{T}_\infty) \tag{2}$$

$$\frac{\partial \bar{G}_x}{\partial \bar{x}} + \frac{\partial \bar{G}_y}{\partial \bar{y}} = 0 \tag{3}$$

$$\bar{u}\frac{\partial \bar{G}_x}{\partial \bar{x}} + v\frac{\partial \bar{G}_x}{\partial \bar{y}} - \bar{G}_x\frac{\partial \bar{u}}{\partial \bar{x}} - \bar{G}_y\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{1}{\gamma}\frac{\partial^2 \bar{G}_x}{\partial \bar{y}^2} \tag{4}$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \tag{5}$$

The boundary conditions

$$\begin{aligned} &\bar{u}(x,0) = 0, \bar{v}(x,0) = V_0, \bar{G}_x(x,0) = \bar{G}_w(x), \bar{G}_y(x,0) = 0, \\ &\bar{T}(x,0) = T_w \ \bar{u}(x,\infty) = 0, \bar{H}_x(x,\infty) = 0, \bar{T}(x,\infty) = 0 \end{aligned} \tag{6}$$

Now, we introduce the following dependent and independent variables to make dimensionless above equations

$$\bar{u} = \frac{\nu}{L} Gr_L^{\frac{1}{2}} u, \quad \bar{v} = \frac{\nu}{L} Gr_L^{\frac{1}{4}} v, \quad \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}$$

$$\bar{G}_x = \frac{G_0}{L} Gr_L^{\frac{1}{2}} G_x, \quad \bar{G}_y = \frac{G_0}{L} Gr_L^{\frac{1}{4}} G_y, \quad \bar{y} = \frac{y}{L} Gr_L^{\frac{1}{4}}, \quad x = \frac{\bar{x}}{L} \tag{7}$$

By using (7) into (1)-(6) we get the dimensionless boundary layer equations and boundary conditions in the following form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \theta + \frac{\partial^2 u}{\partial y^2} + S\left(G_x \frac{\partial G_x}{\partial x} + G_y \frac{\partial G_x}{\partial y} \right) \tag{9}$$

$$\frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} = 0 \tag{10}$$

$$u\frac{\partial G_x}{\partial x} + v\frac{\partial G_x}{\partial y} - G_x\frac{\partial u}{\partial x} - G_y\frac{\partial u}{\partial y} = \frac{1}{Pm}\frac{\partial^2 G_x}{\partial y^2} \tag{11}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}$$

$$\begin{aligned} &u(x,0) = 0, v(x,0) = V_0, G_x(x,0) = 1, G_y(x,0) = 0, \theta(x,0) = 1 \\ &u(x,\infty) = 0, G_x(x,\infty) = 0, \theta(x,\infty) = 0 \end{aligned} \tag{12}$$

Here, u and v are dimensionless fluid velocity components in x and y-direction respectively, G_x and G_y are the dimensionless x and y-components of magnetic field, θ is being the dimensionless temperature of the fluid in boundary layer. Here, Pr, S, Pm and G_{rL} are Prandtl number, magnetic force parameter and Grashof number respectively which are defined as $Pr = \frac{\nu}{\alpha}, \quad S = \frac{\mu G_0^2}{\rho L^2}, \quad Pm = \frac{\nu}{\gamma}, \quad Gr_L = \frac{g\beta \Delta T L^3}{\nu^2}$ where, μ, ν, γ and L are dynamic fluid viscosity, kinematic viscosity, magnetic diffusion and characteristic length respectively. In equation (6), V_0 is surface mass flux, which is assumed to be uniform, when fluid is being withdrawn through the surface it is negative and when fluid is being blown through it is positive. In our present investigation we shall consider that V_0 is for the case of withdrawal of fluid.

Solution Methodologies: To get the numerical solutions of the problem, we will use two methods namely (i) Primitive variable transformation for finite difference method and (ii) Stream function formulation for asymptotic series solutions near and far from the leading edge of the plate.

Primitive Variable Transformation (PVF): The primitive variable formulation is used to convert the above system of equation in convenient form and then these equations are discretized by using finite difference method to get the numerical solutions for entire value of transpiration parameter ξ. For this purpose, we define the following transformations for the dependent and independent variables:

$$\begin{aligned} &u = x^{\frac{1}{2}}\psi(\xi,Y), \quad v = x^{-\frac{1}{4}}(V(\xi,Y)+\xi), \quad Y = x^{-\frac{1}{4}}V(\xi,Y) \\ &G_x = x^{\frac{1}{2}}\phi_1(\xi,Y), \quad G_y = x^{-\frac{1}{4}}\phi_2(\xi,Y), \quad \theta = \bar{\theta}(\xi,Y), \\ &\xi = V_0 x^{\frac{1}{4}} \end{aligned} \tag{14}$$

By substituting (14) into equations (8)-(12) with boundary conditions (13) we have

$$\frac{1}{2}\psi + \xi\frac{\partial \psi}{\partial \xi} - \frac{1}{4}Y\frac{\partial \psi}{\partial Y} + \frac{\partial V}{\partial Y} \tag{15}$$

$$\begin{aligned} &\frac{1}{2}\psi^2 + \frac{1}{4}\xi\psi\frac{\partial \psi}{\partial \xi} + (V - \frac{1}{4}Y\psi)\frac{\partial \psi}{\partial Y} - \xi\frac{\partial \psi}{\partial Y} = \bar{\theta} + \frac{\partial^2 U}{\partial Y^2} + \\ &S\left[\frac{1}{2}\phi_1^2 + \frac{1}{4}\xi\phi_1\frac{\partial \phi_1}{\partial \xi} + \left(\phi_2 - \frac{1}{4}Y\phi_1\right)\frac{\partial \phi_1}{\partial Y}\right] \end{aligned} \tag{16}$$

$$\frac{1}{2}\phi_1 + \frac{1}{4}\xi\frac{\partial \phi_1}{\partial \xi} - \frac{1}{4}Y\frac{\partial \phi_1}{\partial Y} + \frac{\partial \phi_2}{\partial Y} = 0 \tag{17}$$

$$\begin{aligned} &\frac{1}{4}\xi\psi\frac{\partial \phi_1}{\partial \xi} + (V - \frac{1}{4}Y\psi)\frac{\partial \phi_1}{\partial Y} - \xi\frac{\partial \phi_1}{\partial Y} - \frac{1}{4}\xi\phi_1\frac{\partial \psi}{\partial \xi} \\ &-(\phi_2 - \frac{1}{4}Y\phi_1)\frac{\partial \psi}{\partial Y} = \frac{1}{Pm}\frac{\partial^2 \phi_1}{\partial Y^2} \end{aligned} \tag{18}$$

$$\frac{1}{4}\xi\psi\frac{\partial \bar{\theta}}{\partial \xi} + (V - \frac{1}{4}Y\psi)\frac{\partial \bar{\theta}}{\partial Y} - \xi\frac{\partial \bar{\theta}}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 \bar{\theta}}{\partial Y^2} \tag{19}$$

The transformed set of boundary conditions are given below

$$\psi(\xi,0)=V(\xi,0)=0,\phi_1(\xi,0)=1,\phi_2(\xi,0)=0,\bar{\theta}(\xi,0)=1$$
$$\psi(\xi,\infty)=0,\phi_1(\xi,\infty)=0,\bar{\theta}(\xi,\infty)=0 \tag{20}$$

We will discretize the equations (15)-(19) and the boundary conditions (20) by using the finite difference method using central difference for convective terms, out of which we get a system of tri-diagonal algebraic equations. The Gaussian elimination technique is used to solve these tri diagonal equations. Here we adjust $\Delta\xi$=0.025 and ΔY=0.01 for ξ and Y grids respectively. The solutions are obtained for small values of Prandtl number Pr and magnetic Prandtl number *Pm* in the range of liquid metals that are often used in nuclear cooling system and for strong magnetic force parameter *S*. The solutions are then obtained for different values of pertinent physical parameters, namely, the magnetic field parameter, *S*, the magnetic Prandtl number, *Pm*, the Prandtl, Pr. Finally, the results are obtained in coefficients of skin friction, $Gr_L^{-3/4}x^{-1/4}C_f$, rate of heat transfer, $Gr_L^{1/4}x^{1/4}Nu_x$ and current density $Gr_L^{-3/4}x^{-1/4}J_w$ defined in equation (21). Effect of different physical parameters are also obtained in form of velocity, temperature and transverse component of magnetic field and shown graphically in Figures 4-6. Once we know the solutions of the equations (15)-(19), we are at the position to measure the physical quantities such as of coefficients skin friction, rate of heat transfer and current density from the relation given below, which are important from the application point of view, from the following dimensionless expressions

$$Gr_L^{-3/4}x^{-1/4}C_f=\left(\frac{\partial u}{\partial Y}\right)_{Y=0},\quad Gr_L^{1/4}x^{1/4}Nu_x=-\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0}$$
$$Gr_L^{-3/4}x^{-1/4}J_w=\left(\frac{\partial \phi_1}{\partial Y}\right)_{Y=0} \tag{21}$$

Now, in the following section the solution will be obtained for small and large value of local transpiration parameter ξ.

Stream Function Formulation (SFF): To get the numerical solutions for small and large values of transpiration parameter ξ for the steady state equations, we define the flow variables as given below:

$$Y=x^{-\frac{1}{4}}y,\quad u=x^{\frac{1}{2}}f'(Y),$$
$$v=-x^{-\frac{1}{4}}\left(\frac{3}{4}f(Y)-\frac{1}{4}Yf'(Y)+\frac{1}{4}\xi\frac{\partial f}{\partial \xi}+\xi\right)$$
$$\theta_x=x^{-1}\left(\frac{1}{4}\xi\frac{\partial\bar{\theta}}{\partial\xi}-\frac{1}{4}Y\bar{\theta}'\right),\quad \theta_y=x^{-\frac{1}{4}}\bar{\theta}'$$
$$H_x=x^{\frac{1}{2}}\phi'(Y),\quad H_y=-x^{-\frac{1}{4}}\left(\frac{3}{4}\phi(Y)-\frac{1}{4}Y\phi'(Y)+\frac{1}{4}\xi\frac{\partial\phi}{\partial\xi}\right),$$
$$\xi=V_0x^{\frac{1}{4}} \tag{22}$$

Which reduces the set of equations to

$$f'''+\frac{3}{4}ff''-\frac{1}{2}f'^2+\bar{\theta}-S\left(\frac{3}{4}\phi\phi''-\frac{1}{2}\phi'^2\right)+\xi f''$$
$$=\frac{1}{4}\xi\left[f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}-S\left(\phi'\frac{\partial\phi'}{\partial\xi}-\phi''\frac{\partial\phi}{\partial\xi}\right)\right] \tag{23}$$

$$\frac{1}{Pm}\phi'''+\frac{3}{4}f\phi''-\frac{3}{4}f''\phi+\xi\phi''$$
$$=\frac{1}{4}\xi\left(f'\frac{\partial\phi'}{\partial\xi}-\phi'\frac{\partial f'}{\partial\xi}+f''\frac{\partial\phi}{\partial\xi}-\phi''\frac{\partial f}{\partial\xi}\right) \tag{24}$$

$$\frac{1}{Pr}\bar{\theta}''+\frac{3}{4}f\bar{\theta}'+\xi\bar{\theta}'=\frac{1}{4}\xi\left(f'\frac{\partial\bar{\theta}}{\partial\xi}-\bar{\theta}'\frac{\partial f}{\partial\xi}\right) \tag{25}$$

Boundary conditions to be satisfied by the above equations are

$$f(\xi,0)=f'(\xi,0)=0,\quad \phi(\xi,0)=0,\quad \phi'(\xi,0)=1,\quad \bar{\theta}(\xi,0)=1$$
$$f'(\xi,\infty)=0,\quad \phi'(\xi,\infty)=0,\quad \bar{\theta}(\xi,\infty) \tag{26}$$

When ξ is small: We can expand all the depending functions in power of \$\xi\$ by considering (ξ<<1) near the leading edge. Accordingly, we consider that

$$f(\xi,Y)=\sum_{i=0}^{\infty}\xi^i f_i(Y),$$
$$\phi(\xi,Y)=\sum_{i=0}^{\infty}\xi^i\phi_i(Y),\quad \bar{\theta}(\xi,Y)=\sum_{i=0}^{\infty}\xi^i\theta_i(Y) \tag{27}$$

sing (27) into (23)-(26) and taking the term up to $O(\xi)$ following sets of equations are obtained: $O(\xi)$

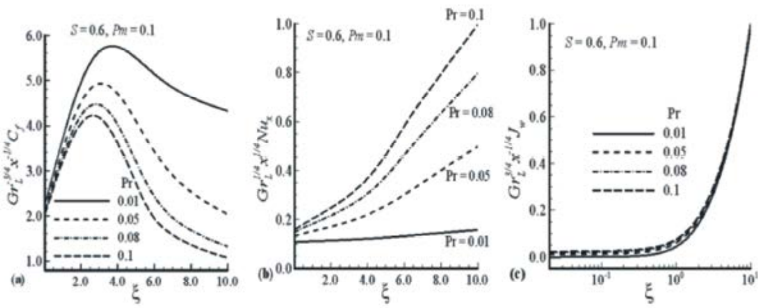

Fig. 2: The behavior of coefficients of (a) skin friction $Gr_L^{-3/4}x^{-1/4}C_f$ (b) rate of heat transfer $Gr_L^{1/4}x^{1/4}Nu_x$ and (c) current density $Gr_L^{-3/4}x^{-1/4}J_w$ at the surface against ξ for different values of Prandtl number Pr=0.01, 0.05, 0.08, 0.1 when *Pm*=0.1 and *S*=0.6.

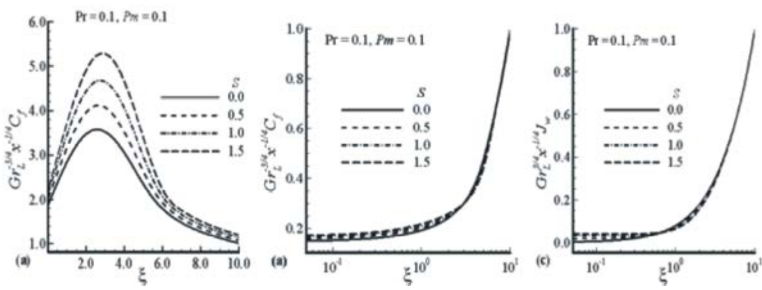

Fig. 3: The behavior of coefficients of (a) skin friction $Gr_L^{-3/4}x^{-1/4}C_f$ (b) rate of heat transfer $Gr_L^{1/4}x^{1/4}Nu_x$ and (c) current density $Gr_L^{-3/4}x^{-1/4}J_w$ at the surface against ξ for different values of magnetic force parameter *S* = 0.0, 0.5, 1.0, 1.5 when *Pm*=0.1 and Pr=0.1

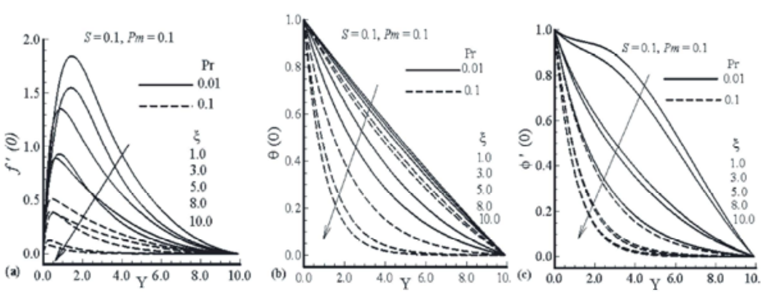

Fig. 4: a) Velocity profile (b) temperature distribution and (c) transverse component of magnetic field against for various values of ξ = 1.0, 3.0, 5.0, 8.0, 10.0 against *Y* for different values of Prandtl number Pr=0.01, 0.1 when *Pm*=0.1, *S*=0.1.

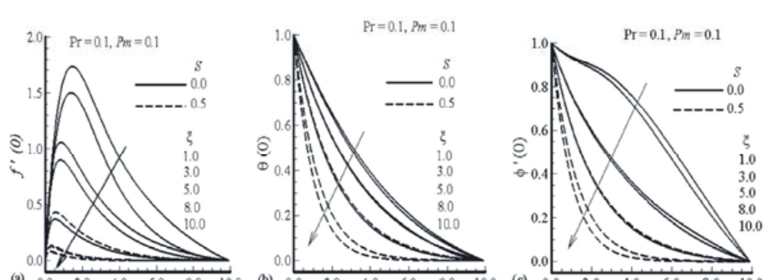

Fig. 5: a) Velocity profile (b) temperature distribution and (c) transverse component of magnetic field against for various values of ξ = 1.0, 3.0, 5.0, 8.0, 10.0 against *Y* for different values of magnetic force parameter *S*=0.0, 0.5 when *Pm*=0.1, Pr=0.1.

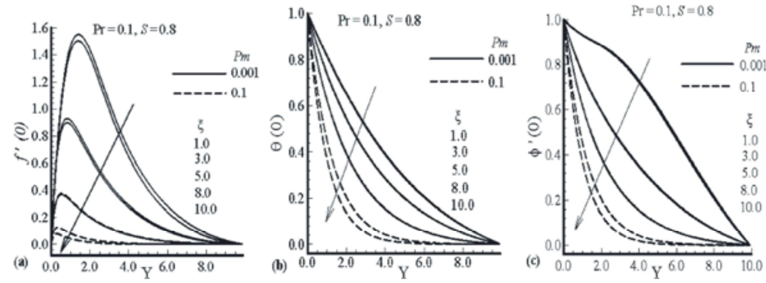

Fig. 6: a) Velocity profile (b) temperature distribution and (c) transverse component of magnetic field against Y for various values of ξ = 1.0, 3.0, 5.0, 8.0, 10.0 against Y for different values of magnetic Prandtl number Pm=0.001

$$f_{0'''} + \frac{3}{4} f_0 f_{0''} - \frac{1}{2} f_{0'}^2 + \theta_0 - S\left(\frac{3}{4}\phi_0\phi_{0''} - \frac{1}{2}\phi_{0'}^2\right) = 0 \tag{28}$$

$$\frac{1}{Pm}\phi_{0'''} + \frac{3}{4} f_0 \phi_{0''} - \frac{3}{4} f_{0''}\phi_0 = 0 \tag{29}$$

$$\theta_{0''} + \frac{3}{4} Pr f_0 \theta_{0'} = 0 \tag{30}$$

the boundary conditions for $O(\xi^0)$ are

$$f_0(0) = f_{0'}(0) = 0, \quad \phi_0(0) = 0, \quad \phi_{0'}(0) = 1, \quad \theta_0(0) = 1$$
$$f_{0'}(\infty) = 0, \quad \phi_{0'}(\infty) = 0, \quad \theta_0(\infty) = 0 \tag{31}$$

$O(\xi^1)$

$$f_{1'''} + \frac{3}{4}(f_0 f_{1''} - S\phi_0\phi_{1''}) + (f_{0''} f_1 - S\phi_{0''}\phi_1)$$
$$-\frac{5}{4}(f_{0'} f_{1'} - S\phi_{0'}\phi_{1'}) + \theta_1 + f_{0''} = 0 \tag{32}$$

$$\frac{1}{Pm}\phi_{1'''} + \frac{3}{4} f_0\phi_{1''} + \frac{5}{4} f_1\phi_{0''} - \frac{3}{4} f_{0''}\phi_1 - \frac{5}{4} f_{0''}\phi_1$$
$$-\frac{1}{4} f_{0'}\phi_{1'} + \frac{1}{4} f_{1'}\phi_{0'} + \phi_{0''} = 0 \tag{33}$$

$$\theta_{1'} + Pr\left(\frac{3}{4} f_0\theta_{1'} + f_1\theta_{0'} - \frac{1}{4} f_{0'}\theta_1 + \theta_{0'}\right) = 0 \tag{34}$$

The boundary conditions for $O(\xi^1)$ are

$$f_1(0) = f_{1'}(0) = 0, \quad \phi_1(0) = 0, \quad \phi_{1'}(0) = 0, \quad \theta_1(0) = 0$$
$$f_{1'}(\infty) = 0, \quad \phi_{1'}(\infty) = 0, \quad \theta_1(\infty) = 0 \tag{35}$$

The equations (31)-(35) are nonlinear coupled equations, the solutions of these equations are obtained by Nactsheim-Swigert iteration technique together with six order implicit Runge-Kutta-Butcher initial value solver. We can calculate the values of the coefficients skin friction, rate of heat transfer and current density at the surface in the region near the leading edge against ξ with the help of following relation.

$$Gr_L^{-3/4} x^{-1/4} C_f = f''(0)$$
$$Gr_L^{-3/4} x^{-1/4} J_w = \phi''(0)$$
$$Gr_L^{1/4} x^{1/4} N_u = -\theta'(0) \tag{36}$$

The results obtained with the help of the equations (31)-(35) for different values of magnetic Prandtl number Pm=0.01, 0.05, 0.1 and keeping other parameter constant i.e. Prandtl number Pr=0.1 and magnetic force parameter S=0.2 are entered in Tables 1-3 in terms of coefficients of skin friction, rate of heat transfer and current density for for small values of ξ.

When ξ is large: In this subsection, we derive the simplified form of equations to finding the solution of equation (23)-(25) along with boundary conditions (26) for large value of transpiration parameter ξ. The order of magnitude analysis of various terms in these equations shows that largest terms are f''' and $\xi f'''$ in equation (23), ϕ''' and $\xi\phi''$ in equation (24) and θ'', $\theta\xi'$ in equation (25) and it needs to be balanced in magnitude. For this purpose we may assume that η is small and hence its derivative is large. It is essential to find the appropriate scaling for, f, ϕ, θ and η. On balancing f''' and $\xi f''$ in equation (23), ϕ'' and $\xi\phi''$ in equation (24) and θ'' and $\xi\theta'$ in (25), it is found that $\eta = O(\xi^{-1})$, $f = O(\xi^{-3})$ and $\phi = O(\xi^{-3})$. Therefore following transformations may be introduced to overcome this difficult situation.

$$Y = \xi^{-1}\eta, \quad f(\xi, Y) = \xi^{-3} F(\xi, \eta)$$
$$\phi(\xi, Y) = \xi^{-3}\Phi(\xi, \eta), \quad \theta(\xi, Y) = \Theta(\xi, \eta) \tag{37}$$

Table 1: Numerical values of coefficient of skin friction $Gr_L^{-3/4}x^{-1/4}Cf_x$ obtained for *Pm*= 0.01, 0.05, 0.1 when *S*=0.2, Pr=0.1against ξ by two methods.

	Pm=0.01		Pm=0.05		Pm=0.1	
ξ	FDM	ASS	FDM	ASS	FDM	ASS
0.0	0.93534	0.93511ss	0.93662	0.93152ss	0.93651	0.93126ss
0.1	0.96380	0.96259ss	0.96364	0.95316ss	0.96345	0.95312ss
0.5	1.06860	1.01312ss	1.06804	1.07321ss	1.06734	1.07384ss
1.0	1.16956	1.14562ss	1.16815	1.13296ss	1.16640	1.15318ss
2.0	3.64394	-	3.64782	-	3.65670	-
4.0	3.39722	-	3.29929	-	3.21341	-
8.0	1.51303	1.01250Ls	1.33319	1.21713Ls	1.29372	1.13471Ls
10.0	1.21663	1.01000Ls	1.04678	1.01000Ls	1.01901	1.01000Ls

Table 2: Numerical values of coefficient of heat transfer $Gr_L^{-3/4}x^{-1/4}Nu_x$ obtained for *Pm*= 0.01, 0.05, 0.1 when *S*=0.2, Pr=0.1against ξ by two methods.

	Pm=0.01		Pm=0.05		Pm=0.1	
ξ	FDM	ASS	FDM	ASS	FDM	ASS
0.0	0.36757	0.36538ss	0.36757	0.36571ss	0.36757	0.36557ss
0.1	0.37233	0.37231ss	0.37233	0.37231ss	0.37233	0.37203ss
0.5	0.39262	0.39251ss	0.39262	0.39241ss	0.39261	0.39141ss
1.0	0.41865	0.41437ss	0.41863	0.41319ss	0.41861	0.41208ss
2.0	0.24558	-	0.24473	-	0.24095	-
4.0	0.37852	-	0.37592	-	0.37474	-
8.0	0.79408	0.80000Ls	0.79483	0.80000Ls	0.79519	0.80000Ls
10.0	0.99680	1.00000Ls	0.99729	1.00000Ls	0.99744	1.00000Ls

Table 3: Numerical values of coefficient of current density $Gr_L^{-3/4}x^{-1/4}J_w$ obtained for *Pm*= 0.01, 0.05, 0.1 when *S*=0.2, Pr=0.1against ξ by two methods.

	Pm=0.01		Pm=0.05		Pm=0.1	
ξ	FDM	ASS	FDM	ASS	FDM	ASS
0.0	0.36401	0.36217ss	0.36550	0.36419ss	0.36733	0.36109ss
0.1	0.36445	0.36391ss	0.36772	0.36592ss	0.37177	0.37115ss
0.5	0.36635	0.36271ss	0.37728	0.37732ss	0.39111	0.39471ss
1.0	0.36877	0.36201ss	0.38972	0.38831ss	0.41678	0.41541ss
2.0	0.10729	-	0.13645	-	0.17849	-
4.0	0.12171	-	0.22577	-	0.39003	-
8.0	0.14577	0.14357Ls	0.40626	0.40000Ls	0.79790	0.80000Ls
10.0	0.15864	0.15000Ls	0.50268	0.50000Ls	0.99873	1.00000Ls

Here ss and Ls are presented the small solution and large solution for the values of ξ

B substituting (37) into (23)-(36), we will obtain the following system of equations

$$F''' + F'' + \Theta = \frac{1}{4}\xi^{-3}\left[F'\frac{\partial F'}{\partial \xi} - F''\frac{\partial F}{\partial \xi} - S\left(\Phi\frac{\partial \Phi'}{\partial \xi} - \Phi''\frac{\partial \Phi}{\partial \xi}\right)\right] \tag{38}$$

$$\frac{1}{Pm}\Phi''' + \Phi'' = \frac{1}{4}\xi^{-3}\left[F'\frac{\partial \Phi'}{\partial \xi} - \Phi'\frac{\partial F'}{\partial \xi} + F''\frac{\partial \Phi}{\partial \xi} - \Phi''\frac{\partial F}{\partial \xi}\right] \tag{39}$$

$$\frac{1}{Pr}\Theta'' + \Theta' = \frac{1}{4}\xi^{-3}\left[F'\frac{\partial \Theta}{\partial \xi} - \Theta'\frac{\partial F}{\partial \xi}\right] \tag{40}$$

The transformed form of boundary equations that can satisfied by the above equations are

$$\begin{gathered} F(\xi,0) = F'(\xi,0) = 0, \quad \Phi(\xi,0) = 0, \quad \Phi'(\xi,0) = 1, \quad \Theta(\xi,0) = 1 \\ F'(\xi,\infty) = 0, \quad \Phi'(\xi,\infty) = 0, \quad \Theta(\xi,\infty) = 0 \end{gathered} \tag{41}$$

Now the expansion of functions F, Φ, Θ in powers of ξ^{-1} by using (42) into (38)-(41) can be written in the form

$$\begin{gathered} F(\xi,\eta) = \sum_{m=0}^{1}\xi^{-3m}F_m(\eta), \Phi(\xi,\eta) = \sum_{m=0}^{1}\xi^{-3m}\Phi_m(\eta), \\ \Theta(\xi,\eta) = \sum_{m=0}^{1}\xi^{-3m}\Theta_m(\eta) \end{gathered} \tag{42}$$

By equating like powers of ξ from both sides we have

$$F_{0'''} + F_{0''} + \Theta_0 = 0 \quad (43)$$

$$\Phi_{0'''} + Pm\Phi_{0''} = 0 \quad (44)$$

$$\Theta_{0''} + Pr\Theta_{0'} = 0 \quad (45)$$

and the boundary conditions are

$$F_0(0) = F_{0'}(0) = 0, \quad \Phi_0 = 0, \quad \Phi_{0'}(0) = 1, \quad \Theta_0(0) = 1$$
$$F_{0'}(\infty) = 0, \quad \Phi_0(\infty) = 0, \quad \Theta_0(\infty) = 0 \quad (46)$$

from which we see that

$$F_{1'''} + F_{1''} + \Theta_1 = 0 \quad (47)$$

$$\Phi_{1'''} + Pm\Phi_{1''} = 0 \quad (48)$$

$$\Theta_{1''} + Pr\Theta_{1'} = 0 \quad (49)$$

$$F_1(0) = F_{1'}(0) = 0, \quad \Phi_1 = 0, \quad \Phi_{1'}(0) = 0, \quad \Theta_1(0) = 0$$
$$F_{1'}(\infty) = 0, \quad \Phi_1(\infty) = 0, \quad \Theta_1(\infty) = 0 \quad (50)$$

The solution obtained by these equations enables us to calculate the solution of physical quantities that are important in many physical phenomena in terms of different parameters for large values of transpiration parameter ξ with the help of following expressions

$$Gr_L^{-3/4} x^{-1/4} C_f = F''(0)$$
$$Gr_L^{-3/4} x^{-1/4} J_w = \Phi''(0)$$
$$Gr_L^{1/4} x^{1/4} N_u = -\Theta'(0) \quad (51)$$

The numerical solutions obtained by expressions (43)-(50) are entered in Tables 1-3 for large values of transpiration parameter ξ and compared with the solution that obtained by finite difference method and found to be in excellent agreement.

RESULTS AND DISCUSSIONS

Equations (15)-(19) along with boundary conditions (20) have been solved numerically for all values of the transpiration parameter ξ by using finite difference method. Similarly equations (23)-(25) and the boundary conditions (26) sufficiently near to the plate and away from the plate have been solved by using asymptotic series solutions. Later, to test the accuracy of the results obtained by finite difference method is compared with the results obtained by asymptotic series solution and found to be in excellent agreement. We shall now give a brief discussion on the effects of Prandtl number Pr, magnetic force parameter *S* and magnetic parameter Pm on coefficients of skin friction $Gr_L^{-3/4} x^{-1/4} C_f$, rate of heat transfer $Gr_L^{1/4} x^{1/4} N_u$ and current density $Gr_L^{-3/4} x^{-1/4} J_w$ in section 3.1. The detail of velocity profile, temperature distribution and transverse component of the magnetic field for varying the parameters Pr, Pm and magnetic force parameter *S* for different values of transpiration parameter ξ is given in section 3.2.

The Effect of Physical Parameters on Skin Friction, Current Density and Rate of Heat Transfer: Figures 2(a)-2(c) exhibiting the influence of different values of Prandtl number Pr = 0.01, 0.05, 0.08, 0.1 for magnetic Prandtl number *Pm*=0.1 for the range of liquid metals and magnetic force parameter *S*=0.6 on coefficients of skin friction, rate of heat transfer and current density at the surface. From these figures, it is shown that the coefficient of skin friction decreases and coefficient rate of heat transfer and current density at the surface increases. From this phenomena we got the information that the increase in the value of Pr implies the rise in kinematic viscosity of the fluid and reduce the thermal diffusion for this reason the thermal and magnetic field boundary layer decreases and momentum boundary layer thickness increases. The increase in magnetic force parameter *S* increase the coefficient of skin friction actively in the middle range of the surface of the plate but the coefficients rate of heat transfer and current density increases at very low margin near the surface of the plate in Figures 3(a)-3(c). The reason is that with the increase of magnetic force parameter *S* the magnetic energy increases which extract the kinetic energy of the fluid, thus the coefficients of skin friction, rate of heat transfer and current density increases. current with in the boundary layer that tends to spread away from the surface and this results in thickening the momentum and magnetic field boundary layer thickness, but these effects have no significant role in the case of coefficient of rate of heat transfer. Finally, we can see that the results obtained by finite difference method and asymptotic series solution are with in good agreement near the leading edge and in down stream regime.

The Effects of Physical Parameters on Velocity, Temperature and Transverse Component of Magnetic Field: The velocity, temperature and transverse component of magnetic field distributions obtained by the finite difference method for various values of transpiration parameter ξ are displayed in Figs. 4-6. The aim of these figures is to display how the profiles vary in ξ to control the boundary layer thickness. The transpiration parameter ξ in present investigation is taken as positive for suction. It is shown that the values of velocity, temperature and transverse component of magnetic field decreases in magnitude as ξ increases in Figs 4-6. This phenomena occurs due to the very strong reason that the suction slow down the motion of the fluid in the down stream regime and the values of the aforementioned physical quantities decreases. Thus the numerical results in Figs. 4-6 indicates that the momentum, thermal and magnetic field boundary layers thicknesses decreases as ξ=1.0,3.0,5.0,8.0,10.0 increases for two different values Prandtl number Pr, magnetic Prandtl number *Pm* and magnetic force parameter *S*.

CONCLUSION

In the present work, several aspects of hydromagnetic natural convection flow past a magnetized vertical porous plate for the range of liquid metals have been presented. Here, we remarking the effects of different physical parameters such as magnetic force parameter S, Prandtl number Pr, magnetic Prandtl number *Pm* on coefficients of skin friction $Gr_L^{-3/4}x^{-1/4}C_f$, rate of heat transfer $Gr_L^{1/4}x^{1/4}N_u$ and current density $Gr_L^{-3/4}x^{-1/4}J_w$ on the surface. Moreover, the velocity profile, temperature distribution and transverse component of magnetic field for different values of transpiration parameter ξ has been studied very briefly which are given as below.

- The coefficient of skin friction decreases and the coefficient of rate of heat transfer and current density increases with the increase of Prandtl number Pr.
- The increase in magnetic force parameter *S* increase the coefficient of skin friction in the middle range of the flow domain and the coefficient of the rate of heat transfer and current density increases at very low difference near the surface of the plate.
- With varying the magnetic Pradtl number *Pm* the coefficient of skin friction and current density increases and the coefficient of rate of heat transfer remains unchanged.
- The values of velocity, temperature distribution and transverse component of magnetic field decreases with the increase of transpiration parameter ξ for different values of Prandtl number, Pr, magnetic Prandtl number *Pm* and magnetic force parameter *S*.
- The results obtained by finite difference method for primitive variable formulation for entire values of transpiration parameter ξ have been compared by asymptotic series solution for stream function formulation near the leading edge and in down stream regime and outcome has been found in quite agreement.

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