Middle-East Journal of Scientific Research 13 (8): 1065-1069, 2013
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2013.13.8.524

Some Problems Become Math Games and Some Math Games Become Problems

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Abstract: Interrelation between problems in mathematics and math games has ever had a great importance in teaching mathematics particularly at early stages of learning. Combinatorial games include double games where no random move and no bluffing are concerned. As well, these games have a finite and, in some cases, finite but so long and probably exhausting end and one of the two players will have to win the game following the rules. Theory of combinatorial games is still at the outset. In this paper, the contributors try to describe math games, explaining some cases of math-game meanwhile and to put in plain words how to create new games.

Key words: Game · Fair · Bead · Row

INTRODUCTION

Thus far, a host of analyses have been published of different games such as Nim analyzed by button in 1902 as a starting point. Integrated theory of fair games which will be described in part 3, had an independent formulation by Sprague and Grand in 1930's and was developed by Guy and Smith some while later. Subsequently, Conway developed the theory of guerrilla games which will be explained in part 3. Except from natural appeal of these games, this field has some links with other fields of mathematics such as Coding Theory, Graph Theory, the Numbers, Complexity Theory, Mathematical Logic, Metroid Theories, Network Theories and so on. In addition, game theories enjoy the advantage of having multiple would-be solutions which makes teaching mathematics more enhanced. [1-4].

The following may be referred to as some math games: chess, backgammon, Tower of Hanoi, Nymbl, Nim, Packer Nim, Lasker Nim, a Wad of Counterfeit, North Cut, Less Nonexistent Rule, game Grand, game Loves. No friend and many more games. Direct use of some of these games will lead to formation of some new games and there might be new games when two are combined. *See* 8 and 9. [1,2,4].

Fair Game and Guerrilla Game: Suppose that you meet two people in a deep reflection playing a curious game and you cannot understand how they play. When you see them pleased by the game, however, you would like to decode the game by observing their moves and participate in the game. But it takes a long time for one of them to make a move and so you go embarrassed. Then, one makes a move and you lose it; in this case, if you can identify the moving player, e.g. player A or player B; or white bead or black bead, the game will be called a guerrilla game, including Gu, checkers, chess, Tic Tac Toe or backgammon. But if you cannot identify the moving player, the game will be called a fair game in which opportunities are equal irrespective to whom is turn for. For example, Nim and Nymbl are classified as fair games and North Cut as a guerrilla one.

Game Loves. No Friend: This game is started by a few strings. A move involves tying open ends of the strings together in which case there should be either a longer string or a ring. The game will end when open ends of all strings are already tied together so that one or more rings are formed. One who ties the last knot wins the game. In Figure 1, the game has been started by 13 strings. [1,2,4].

Game Brussels Sprouts: Invented by John Conway and Michael Peterson, this game is played using paper and pencil. You start with some (×). A move involves connecting two arms (of a single cross or of two separate crosses) and creating a new cross by putting a transverse line over the conneting line between the two arms (Figure 2). [1,3].

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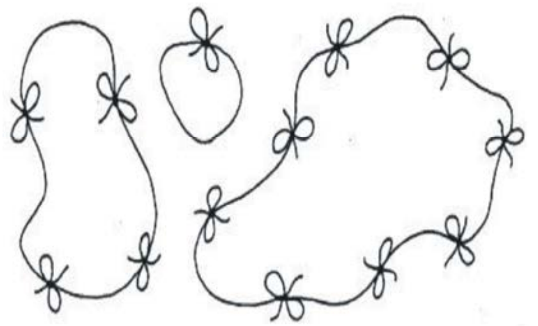


Fig. 1: End of Game Loves. No Friend

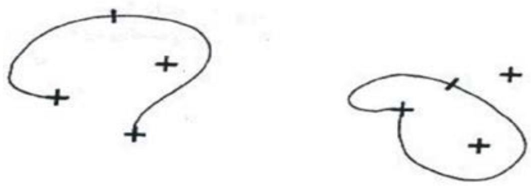


Fig. 2: Two possible first moves in game Brussels Sprouts with 3 crosses

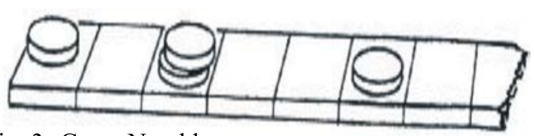


Fig. 3: Game Nymbl

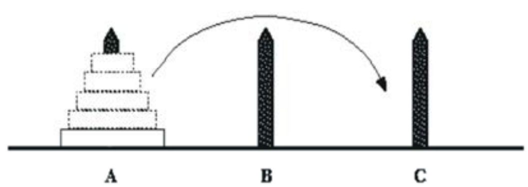


Fig. 4: Tower of Hanoi

Game Nymbl: Here, we intbaruce a simple game where you can win against your friends at least before they learn the mysteries of the game, too.

Put some coins or beads on a partitioned strip. Move, in turn, only one coin to the left in each move. No more limitation is concerned; you can put your coin on or jump \over another coin, even if jumping leaves the coin thrown out of the strip. You can put as many coins on each partition as you wish. One who makes the last move wins the game. [1,2].

Last player is the winner. If you fail to move, you will lose.

Knots: Suppose Loves. No Friend as reversed. We start the game with some strings tied together as in Figure 1; the move involves untying a knot. It takes a little while before you find out that this game is actually similar to the original one. As you might feel bored with untying knots, in his game known as Knots, Dean Thomas Allemang [5] allows for an extra move so that more interest is attracted. Instead of untying knots, take a scissors and cut the string between the two knots, if you will.

If you start with a knotted ring, like the middle item in Figure 1, you might untie the knot (in which case the game is over) or cut the ring (in which case your opponents will win by untying the knot). An untied string with one knot has a Nim value equal to 1; while, Nim value for a ring with one knot is 2. [1,2].

Game Tower of Hanoi: We have a board with three vertical bars and n disks of different diameters on it. Disks, arranged relative to their diameters, are fixed into a bar (below figure). The problem is to transfer the disks to one of the bars empty of disks. We are allowed to move only one disk at a time between the two bars so that no disk are laid above disks with smaller diameters into any bars. Compute the minimum number of moves (disk transfers). [2,6,7].

Solution: Suppose that h_n is the minimum number of moves for n disks. It is clear that $h_1 = 1$. Suppose that you know how to transfer (n-1) disks from one bar using two other bars (figures given below demonstrate a case where n=5). We transfer (n-1) disks to one of the two other bars by *hn*-1 moves (status B in below figure). Then, nth disk is transferred to other empty bar by one move (status C). Next move is to transfer (n-1) disks using the two other bars (without moving the disk with the largest diameter) (status D) which is assumed possible by *hn*-1 moves. Thus,

$$h_n = 2h_{n-1} + 1 \tag{8.1}$$

It would simply be deduced that:

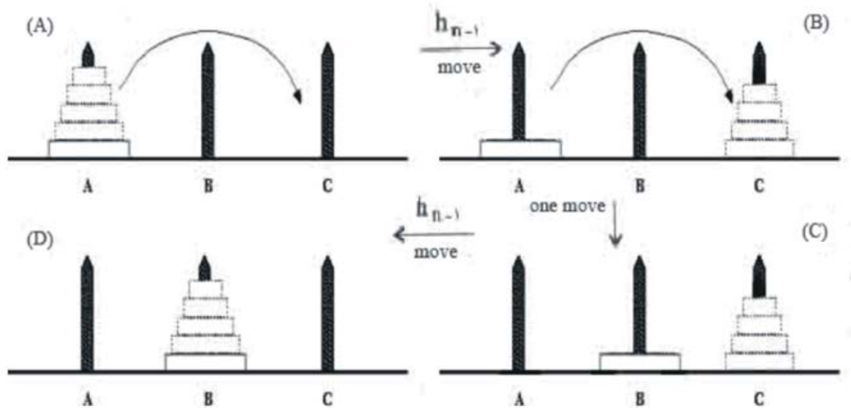
$$h_k = 2^k - 1 \tag{8.2}$$

Since (1) and (2) give:

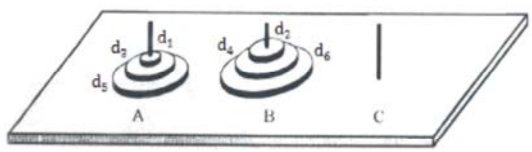
$$h_{k+1} = 2h_k + 1 = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$$

As such, (2) is verified by deduction; that is, $h_n = 2^n$-1.

A second method to achieve a closed formulation for h_n will be offered under section (8.1).



Equation (1) gives: $h_2 = 2h_1 + 1 = 3, h_3 = 2h_2 + 1 = 7.h_4 = 15$



Some Generalizations for Tower of Hanoi: Suppose that we have 2n disks with diameters d_1 to d_{2n}, so that: $d_1<d_2<...<d_{2n}$. There are three bars, one with disks of diameters d_1, d_2,...d_{2n-1} the other of d_2, d_4,...d_{2n}, as shown in figure given below. How many moves, as those used for transferring disks in Tower of Hanoi, are needed for the whole disks to be transferred to the empty bar? [6,3],

Solution: Suppose h'_{2n} as the number of moves for 2n disks. Therefore, disks of diameters d_1, d_{2n-2} can be transferred from A, B and C in h'_{2n-2} moves. Thus, B is left with disk of diameter d_{2n} and A with disk of diameter d_{2n-1} exclusively. Move 2n-2 disks on C using bars A and B to bar A in h_{2n-2} moves. Then, move disk of diameter d_{2n} form B to C, which is free and move 2n-1 disks left on A, just like Tower of Hanoi, to C in h_{2n-1} moves. So,

$$h'_{2n} = h'_{2n-2} + h_{2n-2} + 1 + h_{2n-1}$$

Based on (8.2):

$$h'_{2n} = h'_{2n-2} + 3 \times 2^{2n-2} - 1$$

Integration rule leads to:

$$h'_{2n} - h'_2 = \sum_{k=2}^{n} \left(3 \times 2^{2k-2} - 1\right)$$

$$= \frac{3}{4} \sum_{k=2}^{n} 4^k - (n-1)$$

Based on sum of a geometric progression with ratio 4, the following is concluded:

$$h'_{2n} - h'_2 = \frac{3}{4}\left(\frac{4^{n+1} - 16}{4 - 10}\right) - (n-1)$$

Thus,

$$h'_{2n-h_2=4^n-n-3}$$

If there are two disks, they can be transferred to bar C by a double move; so,

$$h'_{2n} = 4^n - (n+1)$$

Theorem: Odd prime numbers are finite in number. [7]. Arguments [7,4]

Suppose that p is an odd prime number and suppose also that n disks are available so that disks of diameter equal to prime number p and its multiples are put into one bar and others into another one and moves similar to those in Tower of Hanoi are needed for all disks to be transferred to the bar empty of disks.

Problem 8.3 is different from 8.1 and 8.2. Consider Figures (5), (6) and (7) and compare. Number of disks is set to 6 and p equal to 3.

First, put disks 1 and 2 over disk 3 in 2^2 -1 moves; then, put disks 1, 2 and 3 over disk 4 in 2^3 -1 moves and move disks 1, 2, 3, 4 and 5, arranged in a descending order, while disk 6 having already been moved, so:

Since prime numbers are infinite in number, when the disks are adequately increased, we will be able to create new and different games as many as is desirable with the same tools as used for Tower of Hanoi.

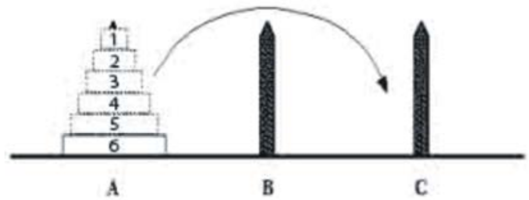


Fig. 5: Bar A containing 6 disks

First status: n=6

Minimum number of moves needed: (2^6 - 1= 63)

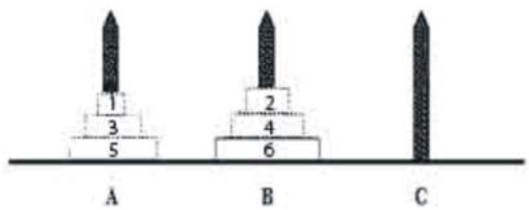


Fig. 6: Bar A and B each containing 3 disks with odd and even diameters respectively

Second status: n=6

Minimum number of moves needed:

(H_{2*3} = 43 - (3+1) = 64 - 4 =60)

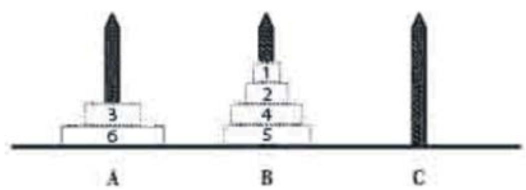


Fig. 7: Bar A containing disks with diameters valued as multiples of 3, i.e. 3 and 6 and bar B containing disks with diameters of other values; i.e. 1, 2, 4 and 5

Third status: n=6

Minimum number of moves needed:

(2^2 -1) + (2^3 -1) + (2^5 -1) + 1= 3 + 7 + 31 + 1= 42

Game Tuesday X: This game is identical with game X-O on a 1×N strip divided into some blocks and played by two parties, each using a cross (×) mark. One who makes a complete row of three consecutive crosses for the first time is the winner.

Of course, soon you will find out that playing in the first or second block coming just after the crossed block is not wise. So, when you put a cross on a block, suppose also the two neighboring blocks as occupied (which may exceed the end of the strip). It means that we change the game in a way that one plays with triminoes (Figure 3) covering three consecutive blocks of the strip while

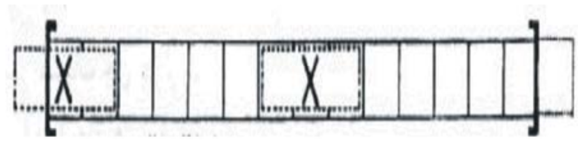


Fig. 4: Triple X cross is in ・・。7 form

putting triminoes over edge of the page is not allowed (excluding the end of the strip where you can move out by a single block). Figure 4 shows that game Tuesday X resembles game Skatel in which move is defined as casting 3 consecutive pins.

A row with n blocks between two crosses is marked by XnX; a row with n blocks between a cross and the end of the strip by [Xn]; a strip with n empty blocks by [n]. As such, Nim value for game Tuesday X and ・。。7 is as follows:

That Nim sequence for Tuesday X is alternating or not is still a matter of question.

CONCLUSION

Taking a quick look at the math games described above, particularly games derived from Tower of Hanoi, one may observe that they are so cheap and cost-effective since all you need to play most of these games are pencil, paper, beads, dices and boards. Another obvious conclusion drawn from games like Tower of Hanoi tells us that, as a test for creativity level of individuals, they can be generalized as shown in 8.1. Most importantly, creativity and mental training, infused indirectly, improve. According to Nikolai Lobachevsky's theory, there is no branch of mathematics, even of pure mathematics, which is not to be used one day for a phenomenon in the real world. As one of the main conclusions come from simple math games, games turned into problems someday will be used in the real world outside our mind.

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