

Factorial Evaluation of the Compressive Strength of Concrete Based on Binder Density and Water-Binder Ratio

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Abstract: This paper presents a factorial evaluation of the compressive strength of concrete based on binder density and water-binder ratio. The range of process parameters used are: 21.35-43.5, 280-420 and 0.48-0.67 for concrete compressive strength, binder density and water-binder ratio respectively, the hydration period is 28 days. An empirical model; $V = 0.078\alpha - 55.39\varepsilon + 37.96$ evaluates the compressive strength of the concrete as a sum of two linear parts involving the binder density and water-binder ratio. Results predicted by the model indicate that compressive strength of the concrete increases with increase in the binder density and decrease in the water-binder ratio, in accordance with previous work. The validity of the model was rooted on the core model expression $V - K = N\alpha - h\varepsilon$ where both sides of the expression are correspondingly almost equal. The standard error incurred in predicting the model-based concrete compressive strength relative to the actual results was 1.45%. Compressive strength of the concrete per unit binder density were evaluated as 0.158 and 0.153 Mpa /Kg m⁻³ as obtained from actual and model-predicted results respectively. The maximum deviation of model-predicted results from those of the actual was < 7%. This implies over 93% operational confidence levels for the derived model as well as 0.93 dependency coefficient of concrete compressive strength on binder density & water-binder ratio. The correlation coefficients between compressive strength of concrete and binder density & water-binder ratio were all > 0.97.

Key words: Evaluation - Concrete - Compressive strength - Binder density - Water - Binder ratio

INTRODUCTION

The need to re-integrate and recycle industrial wastes for use in the field of construction has led to series of research and development to test the physical and chemical properties of by-products of so many processes. Concrete industries for instance have encouraged the use of mineral admixtures which are by-products of other industries. Today, concrete manufacturers in several parts of the world incorporate industrial by products to reduce the negative impact initiated by concrete on environment. One of these materials is the silica fume which consists of silicon or ferrosilicon alloys resulting from amorphous form of silicon tetrachloride combusted by hydrogen-oxygen flame. Several researchers [1-8] have successfully investigated the use of the silica fume in concrete mix and discovered their enhancement to the concrete properties. The researchers submitted that binding materials play an important role in the quality, durability and strength of the cement mortar.

Another researcher [9] investigated the addition of silica fume to concrete mix with the expectation of achieving a high strength concrete. The results of the investigation revealed clearly a high increase in compressive strength and significant reduction in slump even though with the addition of super plasticizer by approximately 0.75% on average. Furthermore, a similar work was carried out [10] with silica fume partially replacing cement in a concrete mix. The essence of the research was to investigate both physical and durability properties such as cyclic freezing and thawing, sulphate attack and alkaline silica reactivity. The results of the investigation show maximized compressive strength after a hydration period of 28 days on replacing 15% of cement with silica fume. In this research, a variable dosage of super plasticizer was added as water - binder ration which equaled 0.35%.

The mineral admixture and its influence on high performance concrete have been studied [11]. In this research, cement was replaced with micro silica while super plasticizer was added to the mix for more

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workability. Results culled from the research revealed that better strengths were achieved at 15% replacement. Further research [12] has been carried out to investigate a high performance concrete using mineral admixtures such as silica fume. In the study, several percentages of cement replacement; 0, 5, 10, 15, 20, and 25% were considered. The water-to-binder ratio considered was equal to 0.29. The hydration periods considered were 7 and 28 days. Results culled from the investigation indicate that the compressive strength increases with increase in the silica fume added. This trend was observed to ensue until 15% replacement reached. Beyond 15% replacement, the compressive strength reduced. It was also noted that workability decreases with increase in the silica fume replacement.

The influence of water/binder ratio on the hardened state properties of cement mortar was investigated [13] after 28 days curing period. Empirical equations were derived from results generated from the investigation to evaluate the strength of cement mortar mixes with various water / binder ratios. Results of the research confirmed that Abram's law is valid for most of cement mortars established. The cement mortar consists of ordinary Portland cement with 15% partially replaced by silica fume, fine aggregate (sand) with varying portions of 1:3, 1:4, 1:5, 1:6 and different water- binder ratios ranged from 0.4 to 0.8. Results of the research [13] led to the establishment of a relationship between split tensile strength and compressive strength of cement mortar. Furthermore, a reduction in compressive and tensile strength of cement mortar was observed while increasing the water-to- binder ratio higher than 0.5 in case of cement: sand of 1:3. The same effect was observed at a water-to binder ratio of 0.7, while using cement: sand of 1:4, 1:5, and 1:6. Based on the foregoing, the researcher concluded that the optimized water -binder ratio required for achieving workable cement mortar was mainly based on cement: sand ratio.

Several researchers [14-26] have clearly derived or reviewed empirical equations to show the effect of water-cement ratio or water-binder ratio on the strength of concrete or cement mortar. Early research [14] in concrete technology, has viewed Abram's formula as the first to describe the dependence of concrete strength on water-cement ratio. Abram's suggested a formula that clearly represents a sterling relationship between concrete strength and water-cement ratio. Another researcher [15] submitted that the formula clearly gives an inverse effect of water-cement ratio on the compressive strength.

The formula [15] is as follow:

$$\text{Strength} = \frac{K_1}{K_2^{w/c}} \quad (1)$$

where K_1 and K_2 are constants, w represents mass of water and c assigned for the mass of cement. The validity of this formula was proven above water-cement ratio ranges from 0.3 to 1.20 for an average Portland cement concrete cured under normal temperature and moisture. The values of both constant coefficients K_1 and K_2 were deduced [16] using Abrams relationship between strength and water-cement ratio following his investigation at different aging period of 7 and 28 days. Equation (2) presents the resultant of the estimated coefficient; K_1 and K_2 .

$$f_{c7} = \frac{63.45}{14^x} \text{ and } f_{c28} = \frac{96.55}{8.2^x} \quad (2)$$

where, f_{c7} and f_{c28} represent the strengths in MPa at 7 and 28 days respectively, while, x represents the water-cement ratio.

An empirical equation which can estimate the compressive and split tensile strength of mortar through determination of the water-cement ratio has been suggested [17]. The equation was based on Abram's law and limited to water-cement ratio greater than 0.4. Another researcher [18] looked at the possible influence of aging on Abram's law. The scientist proved the validity of Abram's law at various ageing ranges from 3 to 365 days. Earlier research [17] had already deduced several other parameters which affect the mechanical properties of cement mortar such as water-cement ratio, cement-sand ratio, types of cement material and the aggregate characteristics.

Studies [16, 19-24] have been carried out on the addition of mineral admixtures into concrete mix. In the mix design, the water-cement ratio was replaced by water-binder ratio instead. Based on the foregoing, other scientists [16, 25] posited that the strength prediction becomes more accurate. Consequently, another scientist [26] formulated the water-to-binder ratio as;

$$x = \frac{w}{c+kf+s} \quad (3)$$

where x assigned for water-to-binder ratio; w , c , and f represented water, cement and fly ash content; in addition, s denotes granulated blast furnace slag (GBFS), and k is an efficiency factor.

Although several researchers [14-26] have formulated the relationship between water-cement ratio or water-binder ratio and compressive strength of concrete, no one has established the relationship between compressive strength of concrete, binder density and water-binder ratio through formulation of empirical equations or models.

This research aims at factorial evaluation of the compressive strength of concrete based on binder density and water-binder ratio.

MATERIALS AND METHODS

The concrete cube size measuring 150x150x150mm in dimension was used. The batching of the concrete cubes was by weight. The concrete was produced using a range of process parameters: water-binder ratio; 0.48-0.67 and binder density; 280-420 Kg/m³. The hydration period is 28 days. The cement used is Portland limestone Cement (PLC). The aggregates used conformed to BS877. The concrete cubes were lubricated with oil before the mixed concrete was placed inside it in order to reduce friction between the concrete and the cubes. When the concrete was properly mixed, the concrete cubes were filled one-third of their height and compacted 150 times. The cubes were later filled to two-third of their height and finally filled completely. In each of the layer, the concrete cubes were compacted 150 times respectively. The concrete cubes were cast and cured for 28 days respectively. At the end of the hydration period, the concrete cubes were crushed to determine, their compressive strength [27].

RESULTS AND DISCUSSION

Table 1: Variation of the compressive strength of concrete V with water-binder ratio ε and binder density α respectively [27]

(α)	(ε)	(V)
280	0.67	21.35
300	0.65	25.10
320	0.63	28.85
340	0.61	32.60
420	0.48	43.50

Computational analysis of the actual results shown in Table 1, gave rise to Table 2 which indicate that;

$$V - K = N\alpha - H\epsilon \tag{4}$$

Introducing the value of K, N and H into equation (4) reduces it to;

$$V - 37.96 = 0.078\alpha - 55.39\epsilon \tag{5}$$

Equation (5) can be re-arranged to give;

$$V = 0.078\alpha - 55.39\epsilon + 37.96 \tag{6}$$

The derived model is equation (6).

where,
 K = 37.96, N = 0.078 and H = 55.39; equalizing constants (determined using C-NIKBRAN [27])

- (ε) = Water-binder ratio
- (α) = Binder density (Kg/m³)
- (V) = Compressive strength of concrete (Mpa)

Boundary and Initial Conditions: A cube sized concrete block 150 x 150 x 150mm produced from a mixture of sand, aggregates and cement was considered and subjected to compressive test using appropriate crushing loads. The concrete is assumed to be unaffected by dissolved gases in the atmosphere.

The range of process parameters used are: 21.35-43.5, 280-420 and 0.48-0.67 for concrete compressive strength, binder density and water-binder ratio respectively. The hydration period is 28 days.

Table 2: Variation of V- K with Nα - Hε

V - K	Nα - Hε
-16.61	-16.12
-12.86	-12.60
-9.11	-9.94
-5.36	-7.27
5.54	6.17

Model Validity: Equation (6) is the derived model. The validity of the model is rooted on the core model equation (4) where both sides of the equation are correspondingly almost equal. Table 2 also agrees with equation (4) considering values of V- K and Nα - Hε precisely evaluated from the actual results in Table 1. Furthermore, the derived model was validated by comparing the compressive strength of concrete predicted by the model and that obtained from the experiment. This was done using various analytical techniques which includes computational, statistical, graphical and deviational analyses.

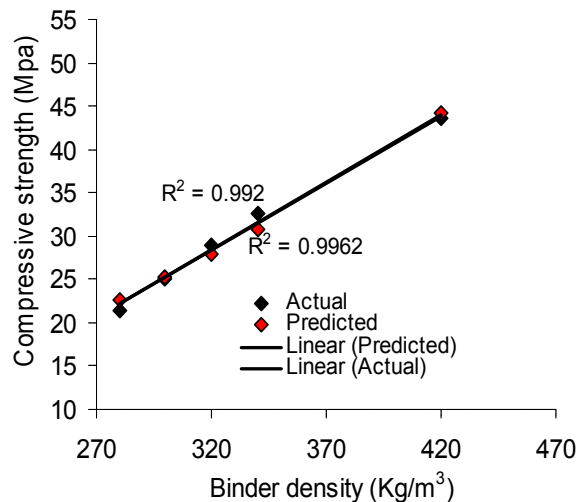


Fig. 1: Coefficient of determination between compressive strength of concrete and binder density as obtained from actual and model-predicted results

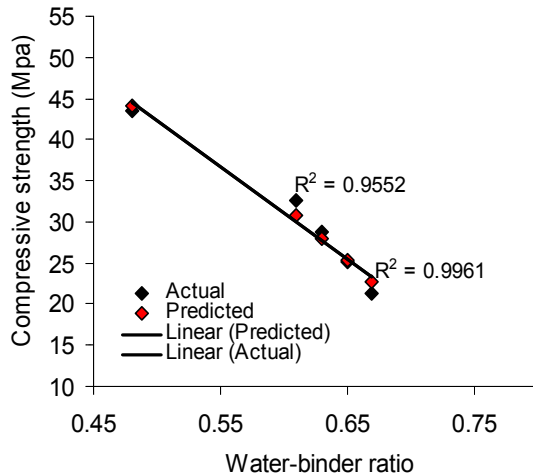


Fig. 2: Coefficient of determination between compressive strength of concrete and water-binder ratio as obtained from actual and model-predicted results

Computational Analysis: Compressive strength of concrete per unit binder density.

The compressive strength of concrete per unit binder density V_α (Mpa)/ Kg m^{-3} was calculated from the equation;

$$V_\alpha = V / \alpha \tag{7}$$

Re-written as

$$V_\alpha = \Delta V / \Delta \alpha \tag{8}$$

Equation (8) is detailed as

$$V_\alpha = V_2 - V_1 / \alpha_2 - \alpha_1 \tag{9}$$

where,

V_α = Change in the compressive strengths V_2, V_1 at Binder densities α_2, α_1

Considering the points (280, 21.35) & (420, 43.5) and (280, 22.69) & (420, 44.13) as shown in Fig. 3, designating them as (α_1, V_1) & (α_2, V_2) for actual and model-predicted results, and then substituting them into equation (9), gives the slopes: 0.158 and 0.153 Mpa/ Kg m^{-3} respectively as compressive strength per unit binder density respectively.

Results predicted by the model indicate that compressive strength of the concrete increases with increase in the binder density and decrease in the water-binder ratio. This is in accordance with previous work [27].

Statistical analysis

Correlation: The correlation coefficients between compressive strength of concrete and binder density & water-binder ratio were evaluated from the coefficients of determination in Figs. 1 and 2 (designated as results of the actual and derived model) using equation (10). These results are 0.9960 and 0.9981 & 0.9773 and 0.9980 respectively.

$$R = \sqrt{R^2} \tag{10}$$

Standard Error (STEYX): The standard error incurred in predicting the model-based compressive strength relative to values of the actual results is 1.45%. The standard error was evaluated using Microsoft Excel version 2003.

Graphical Analysis: The validity of the derived model was further verified by plotting values of the actual, besides the model-predicted results using Microsoft Excel (version 2003) to evaluate the trend of both results. Figs. 3 and 4 indicate very close alignment of curves and shapes which depicted significantly similar trend of data point's distribution for the actual and derived model-predicted compressive strength. This shows proximate agreement between both results.

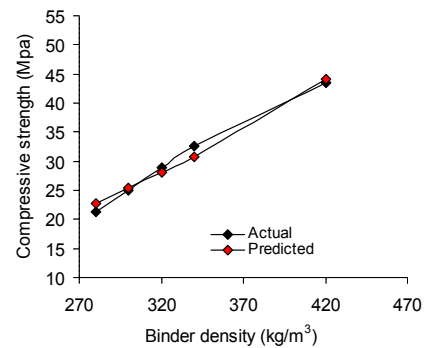


Fig. 3: Variation of compressive strengths of concrete with binder density as obtained from actual and model-predicted results

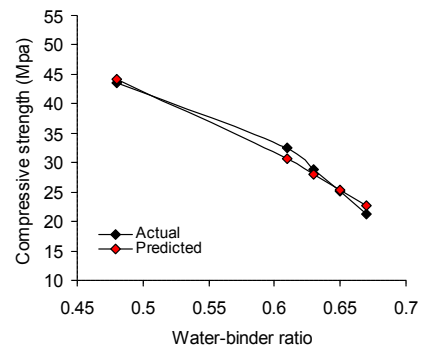


Fig. 4: Variation of compressive strengths of concrete with water-binder ratio as obtained from actual and model-predicted results

Deviational Analysis: Analysis of the compressive strength of concrete obtained from the actual and model-predicted results shows deviation on the part of model-predicted results. This was attributed to the fact that the effects of the surface properties of the cement which played vital roles during the hydration were not considered during the model formulation. This necessitated the introduction of correction factor, to bring the model-predicted compressive strength of concrete to those of the corresponding experimental values.

The deviation D_v , of model-predicted compressive strength of concrete from the corresponding actual result was given by.

$$D_v = \left(\frac{V_P - V_E}{V_E} \right) \times 100 \quad (11)$$

where

V_E and V_P are compressive strengths evaluated from experiment and derived model respectively.

Fig. 5 shows that maximum deviation of model-predicted compressive strength of concrete from the actual results was less than 7%. This translates into over 93% model operational confidence. The figure shows that the least and highest deviations of model-predicted results (from actual results) are 1.04 and 6.28 %.

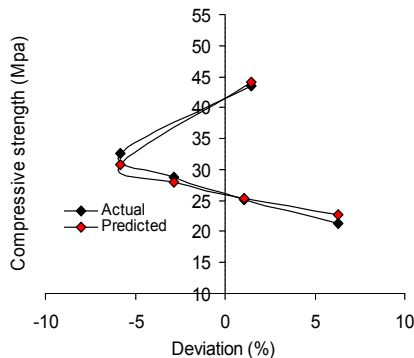


Fig. 5: Deviation of model-predicted results from actual values relative to compressive strength of concrete

These deviations correspond to model-predicted compressive strengths: 25.36 and 22.69 (Mpa); binder densities: 300 and 280(Kg/ m³) and water-binder ratios: 0.65 and 0.67 respectively.

Correction factor, Cf to the model-predicted results was given by;

$$C_f = - \left(\frac{V_P - V_E}{V_E} \right) \times 100 \quad (12)$$

Comparative analysis of Figs. 5 and 6 show that the evaluated correction factors are negative of the deviation as shown in equations (11) and (12).

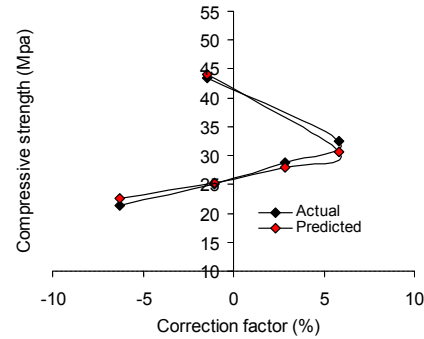


Fig. 6: Correction factor to model-predicted results relative to compressive strength of concrete

The correction factor took care of the negligence of operational contributions of the effects of surface properties of the cement which actually affected the concrete hydration process. Introduction of the corresponding values of Cf from equation (12) into the model gives exactly the corresponding actual compressive strength. Fig. 6 indicates that the maximum correction factor to the model-predicted results was less than 7%. Fig 6 shows that the least and highest correction factors to the model-predicted results are -1.04 and - 6.28%. These correction factors also correspond to model-predicted compressive strengths: 25.36 and 22.69 (Mpa); binder densities: 300 and 280 (Kg/ m³) and water-binder ratios: 0.65 and 0.67 respectively.

The deviation of model predicted results from that of the actual is just the magnitude of the value. The associated sign preceding the value signifies deviation deficit (negative sign) or surplus (positive sign).

CONCLUSION

Factorial evaluation of the compressive strength of concrete based on binder density and water-binder ratio was carried out. A derived empirical model; $V=0.078\alpha-55.39\epsilon + 37.96$ evaluated the compressive strength of the concrete as a sum of two linear parts involving the binder density and water-binder ratio. Compressive strength of the concrete increases with increase in binder density and decrease in the water-binder ratio, in accordance with previous work.

The validity of the model was rooted on the core model expression $V- K = N\alpha - H\epsilon$ where both sides of the expression are correspondingly almost equal. The standard error incurred in predicting the model-based concrete compressive strength relative to the actual results was 1.45%. Compressive strength of the concrete

per unit binder density were 0.158 and 0.153 Mpa /Kg m⁻³ as obtained from actual and model-predicted results respectively. The maximum deviation of model-predicted results from the actuals was < 7%. This implies over 93% operational confidence levels for the derived model as well as 0.93 dependency coefficient of concrete compressive strength on binder density & water-binder ratio. The correlation coefficients between compressive strength of concrete and binder density & water-binder ratio were all > 0.97.

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