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HA Generalization of the Wang-Qin's Identity

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Abstract: We give a generalization of the Wang-Qin's relation involving invariant and inverse invariant sequences.

Key words: Binomial transformation · Wang-Qin's expression · Invariant sequences

INTRODUCTION

Chang-Fu Wang (1992) conjectured that [1]:

$$\sum_{k=0}^{n}\binom{n}{k}(-1)^k a_{k+2} = a_{n+2}, \qquad n \geq 0, \tag{1}$$

for the sequence:

$$a_n = (-1)^n \int_0^1 \binom{x}{n} dx, \tag{2}$$

later, Hou-Rong Qin proved the conjecture.

Here we show the following generalization of (1):

$$Q \equiv \sum_{k=0}^{n}\binom{n}{k}(-1)^k a_{k+m} = (-1)^m a_{n+m}, \qquad a_n = (-1)^n \int_b^c \binom{x}{n} dx, \qquad m = b + c + 1, \tag{3}$$

where b and c are arbitrary integers.

In fact:

$$Q = (-1)^m \int_b^c \sum_{k=0}^{n}\binom{n}{k}\binom{x}{k+m} dx, \tag{4}$$

but we have the properties [2-4]:

$$\sum_{k=0}^{n}\binom{n}{k}\binom{x}{k+m} = \binom{n+x}{n+m} = (-1)^{n+m}\binom{m-1-x}{n+m}, \tag{5}$$

then from (4), (5) and $m = b + c + 1$:

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$$Q=(-1)^n\int_{m-c-1}^{m-b-1}\binom{y}{n+m}dy=(-1)^n\int_{b}^{c}\binom{y}{n+m}dy=(-1)^m a_{n+m}, \qquad q.e.d.$$

Thus, the Wang-Qin's identity (1) is a particular case of (3) for $b = 0$, $c = 1$, $m = 2$.

If $b = -1$, $c = 0$, $m = 0$ we obtain that $a_n=(-1)^n\int_{-1}^{0}\binom{x}{n}dx$ and (3) implies the relation:

$$\sum_{k=0}^{n}\binom{n}{k}(-1)^k a_k = a_n, \tag{6}$$

that is, $\{a_n\}$ is an invariant sequence [1].

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