European Journal of Applied Sciences 10 (3): 84-, 2018
ISSN 2079-2077

DOI: 10.5829/idosi.ejas.2018.84.

On the Janjic's Definition of Stirling Numbers

[1]*J. Yaljá Montiel-Pérez,* [2]*J. López-Bonilla and* [2]*S. Vidal-Beltrán*

[1]Centro de Investigación en Computación, Instituto Politécnico Nacional, CDMX, México
[2]ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4,
1er. Piso, Col. Lindavista CP 07738 CDMX, México

Abstract: We exhibit that the definitions of Janjic for the Stirling numbers are consequences of the formulas of Hoppe and Quaintance-Gould.

Key words: Stirling numbers · Hoppe's formula · Generalized chain rule of differentiation

INTRODUCTION

Janjic [1] employs the relations:

$$\frac{d^n}{dx^n} f(u) = \sum\nolimits_{k=0}^{n} S_n^{[k]} u^k \frac{d^k}{du^k} f(u), \quad u = e^x, \tag{1}$$

$$\frac{d^n}{dx^n} f(v) = \frac{1}{x^n} \sum\nolimits_{k=0}^{n} S_n^{(k)} \frac{d^k}{du^k} f(v), \quad v = \ln x, \tag{2}$$

where $f(t)$ is an arbitrary function, to define the Stirling numbers $S_n^{(k)}$ and $S_n^{(k)}$ [2].

In Sec. 2 we show that (1) and (2) are consequences of the formulas of Hoppe [3, 4] and Quaintance-Gould [2], respectively.

Janjic's Relations: In the Hoppe's expression:

$$\frac{d^n}{dx^n} f(u) = \sum\nolimits_{k=0}^{n} \frac{(-1)^k}{k!} \frac{d^k}{du^k} f(u) \sum\nolimits_{j=0}^{k} (-1)^j \binom{k}{j} u^{k-j} \frac{d^n}{dx^n} u^j, \tag{3}$$

we apply $u = e^x$, thus $\frac{d^n}{dx^n} u^j = j^n u^j,$ then (1) is immediate by the Euler's result [2]:

$$S_n^{[k]} = \frac{(-1)^k}{k!} \sum\nolimits_{j=0}^{k} (-1)^j \binom{k}{j} j^n \tag{4}$$

We know the property [2]:

$$n! \binom{y}{n} = \sum\nolimits_{k=0}^{n} S_n^{(k)} y^k, \tag{5}$$

where we can use $y = \frac{d}{dv}$ to deduce:

$$n! \binom{\frac{d}{dv}}{n} = \sum\nolimits_{k=0}^{n} S_n^{(k)} \frac{d^k}{dv^k}, \tag{6}$$

besides, we have the relation [2]:

$$\frac{d^n}{dx^n} f(v) = \frac{n!}{x^n} \binom{\frac{d}{dv}}{n} f(v), \quad v = \ln x, \tag{7}$$

therefore (6) and (7) imply (2).

The expressions (1) and (2) are the formulas (8.13) and (12.34) in [2], respectively.

REFERENCES

1. Janjic, M., 2008. On a non-combinatorial definition of Stirling numbers, arXiv: 0806.2366v1 [math.CO] 14 Jun 2008.
2. Quaintance, J. and H.W. Gould, 2016. Combinatorial identities for Stirling numbers, World Scientific, Singapore.
3. Gould, H.W., 2002. The generalized chain rule of differentiation with historical notes, Utilitas Mathematica, 61: 97-106.
4. Johnson, W.P., 2002. The curious history of Faà di Bruno's formula, The Math. Assoc. of America, 109: 217-234.

Corresponding Author: J. López-Bonilla, ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso, Col. Lindavista CP 07738 CDMX, México.