European Journal of Applied Sciences 10 (3): 81-83, 2018
ISSN 2079-2077

DOI: 10.5829/idosi.ejas.2018.81.83

Mode Conversion Between S Waves and P Waves

[1]H. Torres-Silva, [2]J. López-Bonilla and [3]J. Yaljá Montiel-Pérez

[1]Escuela de Ingeniería Eléctrica y Electrónica,
Universidad de Tarapacá, Arica, Casilla 6-D, Chile
[2]ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 5, 1er. Piso,
Col. Lindavista CP 07738, CDMX, México
[3]Centro de Investigación en Computación, Instituto Politécnico Nacional

Abstract: In this paper we analyze the conditions to obtain mode conversion between S waves and P waves in a geometrical approach. If an elastic media has simultaneously negative density and shear modulus in a certain frequency regime, it may give rise to negative refraction of transverse waves. Meanwhile, if the material's bulk modulus is positive the longitudinal wave becomes evanescent.

Key words: Chiral material · Seismic body waves · Metamaterials

INTRODUCTION

In connection with seismic events, the Elastodynamics is linked with Electrodynamics through transverse waves. Seismic waves encountering interfaces that separate rocks of different elastic properties also undergo reflection, refraction and scattering phenomena in chiral materials which are noncentrosymmetric due to handedness in their microstructures. The elastic field behavior in a chiral medium is readily described herein using the governing equations and constitutive relations for noncentrosymmetric, isotropic micropolar materials. Accordingly, linearly polarized longitudinal waves and left and right circularly polarized transverse waves are eigenstates for elastic waves in the chiral medium. There are two classes of seismic body waves which travel through the interior of the Earth: P waves and S waves. P waves are the fastest seismic waves and consequently, the first to arrive at any given location. Torsion waves, often called S waves, represent the spiraling motion of particles twisting between inner structures. S waves usually have more height, or amplitude, than P waves [1, 2].

The idea of negative refraction in an electromagnetic wave in metamaterials that are realized in practice are composites with built-in resonance structures that exhibit effective negative permittivity and negative permeability for some frequency ranges [2-6]. In an acoustic wave, the continuity and Newton's second law (with harmonic field dependence exp($i\omega t$) can be expressed respectively as $\nabla \cdot \vec{v} + i(\omega/\kappa)p = 0$ and $\nabla p + i\omega\rho\vec{v} = 0$, where $p(\vec{v})$ is the pressure (velocity) field. The density ρ and bulk modulus κ are position dependent in general. By considering a plane-wave solution with wave vector $\vec{k}$ inside a homogeneous medium of constant density and bulk modulus, the refractive index n should be defined by $k = |n|\omega/c$ with $n^2 = \rho / \kappa$, which can have a simultaneously negative effective bulk modulus and density.

Metamaterials with negative electromagnetic constitutive parameters give new propagation characteristics for electromagnetic waves [1-6]. By analogy between elastodynamics and electrodynamics in this work we use the Born- Fedorov equations as the most suitable for applications of our interest [5, 6]. Here the density of matter is equivalent to the electric permittivity $\rho \leftrightarrow \varepsilon_e$ and the Lamé parameter μ_s is equivalent to the reciprocal of the magnetic permeability $\mu_s \leftrightarrow 1/\mu_m$. The speed of transversal seismic S-waves without chirality is given by $c_S = 1\big/\sqrt{\rho/\mu_S}$. The chiral effect given by the factor $(1 \pm \zeta\, k_0)$ which modify the phase velocity $c_T(k_0 = \omega/c_T)$ having a metamaterial behavior when $v_S = c_S\,(1 - \zeta k_0)$ [7, 8].

Mode Conversion S-P Waves: Mode conversion mathematically can be obtained seeing the configuration shown in (Figure 1). There are two cases of total S-P or

Corresponding Author: H. Torres-Silva, Escuela de Ingeniería Eléctrica y Electrónica, Universidad de Tarapacá, Arica, Casilla 6-D, Chile.

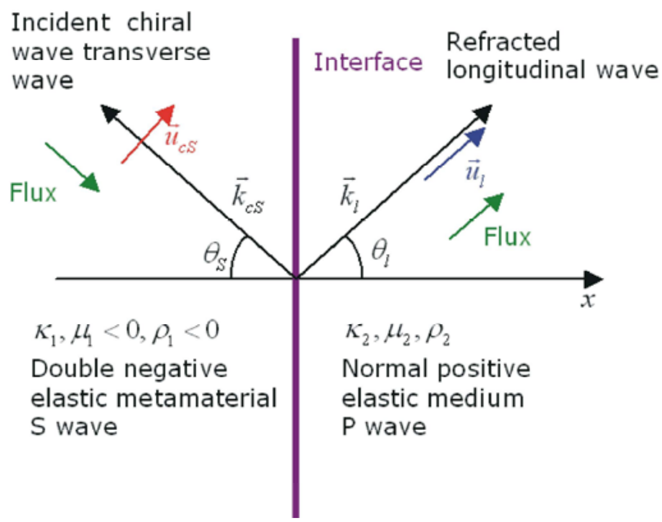

Fig. 1: A schematic graph of S-P mode conversion during the negative refraction on the interface of a negative elastic metamaterial

P-S mode conversions on the interface of a negative metamaterial and a normal solid when we have negative refraction. Next we obtain the conditions of total mode conversions under negative refraction.

For chiral S waves, the vector displacement $\vec{u}_{cS}$ is given by:

$$\vec{u}_{cS} = u_t\left(\sin\theta_S\hat{x} + \cos\theta_S\hat{y}\right)e^{ik_{cS}\left(-\cos\theta_S x + \sin\theta_S y\right)}, \quad (1)$$

for P waves, the vector displacement $\vec{u}_l$ is:

$$\vec{u}_l = u_l\left(\cos\theta_l\hat{x} + \sin\theta_l\hat{y}\right)e^{ik_l\left(\cos\theta_l x + \sin\theta_l y\right)}. \quad (2)$$

The scalar displacements must match on the interface $x = 0$, so we have:

$$\begin{aligned} u_{cS}\sin\theta_S e^{ik_{cS}\sin\theta_S y} &= u_l\cos\theta_l e^{ik_l\sin\theta_l y}, \\ u_{cS}\cos\theta_S e^{ik_{cS}\sin\theta_S y} &= u_l\sin\theta_l e^{ik_l\sin\theta_l y}. \end{aligned} \quad (3)$$

From Eq. (3), we have u_{cS} sin $\theta_S = u_l$ cos θ_l and u_{cS} cos $\theta_S = u_l$ sin θ_l, which means that the conditions are $u_l = u_{cS}$ and $\theta_S + \theta_l = \pi/2$. In the case of normal refraction, for incident angle $0 < \theta_S < \pi/2$ we have $\theta_l < 0$, thus is obtained $\theta_S + \theta_l < \pi/2$, the scalar displacements are not possible to match on the interface, but total conversion is only possible in the case of negative refraction. From Eq. (3), we also have k_{cS} sin $\theta_S = k_l$ sin θ_l, which means the wave vector parallel to the interface must be conserved; when $\theta_S + \theta_l = \pi/2$, we obtain k_{cS} sin $\theta_S = k_l$ cosθ_S.

From $k_{cS} = \left|\frac{\omega}{v_{tS}}\right| = \frac{\omega}{\sqrt{\mu_1/\rho_1}}$ and $k_l = \left|\frac{\omega}{v_l}\right| = \frac{\omega}{\sqrt{(\kappa_2+\mu_2)/\rho_2}}$, we find:

$$\tan\theta_S = \frac{k_l}{k_{cS}} = \frac{\sqrt{\mu_1/\rho_1}}{\sqrt{(\kappa_2+\mu_2)/\rho_2}} \quad (4a)$$

the chiral factor is included in $v_{tS} = c_S\,(1 - \zeta k_0)$ so:

$$k_{cS} = \left|\frac{\omega}{v_{tS}}\right| = \left|\frac{\omega}{c_S\left(1-\zeta k_0\right)}\right| \quad (4b)$$

We also need to consider the matching condition of stresses. For S waves we have,

$$\vec{u}_{cS} = u_{cS}\left(\sin\theta_S\hat{x} + \cos\theta_S\hat{y}\right)e^{ik_{cS}\left(-\cos\theta_S x + \sin\theta_S y\right)}, \quad (5)$$

for p waves:

$$\vec{u}_l = u_l\left(\cos\theta_l\hat{x} + \sin\theta_l\hat{y}\right)e^{ik_l\left(\cos\theta_l x + \sin\theta_l y\right)} \quad (6)$$

By substituting $\theta_S + \theta_l = \pi/2$, $u_l = u_{ct}$ and k_{cS} sin θ_S k_l cos θ_S in Eq. (3), we have:

$$\kappa_2 = \mu_2\cos 2\theta_S - \mu_1\sin 2\theta_S/\tan\theta_S \quad (7)$$

and:

$$-\mu_1 = \mu_2\tan 2\theta_S\tan\theta_S \quad (8)$$

The total conversion matching conditions are:

$$\theta_S + \theta_l = \pi/2,\ \kappa_2 = \mu_2 \cos 2\theta_S - \mu_1 \sin 2\theta_S / \tan\theta_S, \quad (9)$$

$$-\mu_1 = \mu_2 \tan 2\theta_S \tan\theta_S,$$

$$\tan\theta_S = \frac{k_l}{k_{cS}} = \frac{\sqrt{\mu_1/\rho_1}}{\sqrt{(\kappa_2 + \mu_2)/\rho_2}}. \quad (10)$$

Eqs. (4b), (9) and (10) are the main results; angle θ_S is an important input to the simulation work for different cases.

Case 1: We consider a transverse plane wave incident from the left medium of $\mu_1 < 0$, $\rho_1 < 0$ with an incident angle of $0 < \theta_S < \pi/2$ as shown in Fig. 1. First, we can obtain μ_2 via $-\mu_1 = \mu_2 \tan 2\theta_S \tan \theta_S$. Then, we can obtain κ_2 by using $\kappa_2 = \mu_2 \cos 2\theta_S - \mu_1 \sin 2\theta_S/\tan\theta_S = \mu_2/\cos 2\theta_S$. At last, we can obtain ρ_2 employing $\tan\theta_S = \frac{k_l}{k_{cS}} = \frac{\sqrt{\mu_1/\rho_1}}{\sqrt{(\kappa_2 + \mu_2)/\rho_2}}$; therefore, μ_2, κ_2 and ρ_2 are all obtained. It is worth mentioning that when $\alpha > \pi/4$, we have $\mu_2 < 0$, but $\kappa_2 > 0$ and $\kappa_2 + \mu_2 = \mu_2 (1 + \cos 2\alpha)/ \cos 2\alpha > 0$, which indicates a double positive medium for refracted longitudinal waves on the right (together with $\rho_2 > 0$). In this case the medium on the right is not a normal solid.

Case 2: We consider a transverse plane wave incident from the left medium of $\mu_1 > 0$, $\rho_1 > 0$ with an incident angle of $0 < \theta_S < \pi/2$. μ_2, κ_2 and ρ_2 can also be obtained from Eq. (3). Note that we have $\kappa_2 + \mu_2 = \mu_2 (1 + \cos 2\theta_S)/\cos 2\theta_S < 0$ and $\rho_2 < 0$, which indicate a double negative medium for refracted longitudinal waves on the right.

In conclusion, if an elastic media has simultaneously negative density and shear modulus in a certain frequency regime, it may give rise to negative refraction of transverse waves, i.e., $n_S = \sqrt{\rho}\sqrt{1/\mu_S} < 0$. Meanwhile, if the material's bulk modulus is positive so that $\kappa + \mu > 0$, the longitudinal wave becomes evanescent.

REFERENCES

1. Wu, Y., Y. Lai and Z.Q. Zhang, 2011. Elastic metamaterials with simultaneously negative effective shear modulus and mass density, Phys. Rev. Lett. 107, No. 10 (2011) Article ID: 105506.
2. Gioncu, V. and F.M. Mazzolani, 2011. Earthquake engineering for structural design, Spon Press, New York, pp: 223.
3. Torres-Silva, H., D. Torres-Cabezas and J. López-Bonilla, 2017. Parametric instability in Mathieu *equation in earthquake dynamic*s, The Himalayan Physics 6-7, 73-75
4. Torres-Silva, H., 2017., Review about Metamateriales, Dyna, New Technologies, pp: 1-9.
5. Torres-Silva, H., J. López-Bonilla and D. Torres-Cabezas, 2017. Mathieu equation on chiral *membrane actived by electromagnetic field*, Transactions on Maths., 3(1): 6-12.
6. Torres-Silva, H. and D. Torres-Cabezas, 2013. Chiral seismic attenuation with acoustic metamaterials, J. Electromagnetic Analysis and Applications, 5(1): 10-15.
7. Torres-Silva, H. and J. López-Bonilla, 2011. Chiral waves in metamaterial medium, Int. J. of Pure and Applied Sciences and Technology, 2(2): 54-65.
8. Torres-Silva, H. and J. López-Bonilla, 2011. Early prediction and detection of strong earthquakes *through chiral radiation waves*, J. Vectorial Relativity, 6(2): 1-11.