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Metamaterial Bidimensional Transmission Line with Charge Discreteness

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Abstract: In this paper we analyze the behavior of a bidimensional transmission line acting as metamaterial with charge discreteness where the wave number is parallel to group velocity and antiparallel to phase velocity. This paper is an extension of [6] where charge discreteness is not considered. Uncertainty relation makes specific use of the discreteness of electric charge to obtain the condition of metamaterial in quantum electric circuits analogous to two-dimensional electron gas and graphene in metamaterial mode.

Key words: Graphene · Transmission line · Metamaterial · Group velocity · Quantum electric circuits

INTRODUCTION

Nanotechnology is envisioned actually as one of the promoters of technological changes and major innovations [1] in computation (quantum computing and quantum information processes [2, 3]), nanoelectronics, precision medicine (bio-robots) and nanomachines are some of the possible areas of influence of nanotechnology. Quantum theory is the basis for understanding the physics of devices used in integrated circuits of nanoelectronics [4]. In addition, other related transmission lines as chiral systems [5, 6] and also the so-called metamaterials [6, 7] can be modeled by an extended two-dimensional electronic circuit like a sheet of graphene with charge discreteness [8].

Following [6], two-dimensional plasmonic medium can be modeled as a transmission line consisting of distributed kinetic inductance L per unit length and distributed electrical capacitance *C* per unit length (Figure 1) [6]. This plasmonic transmission line differs from the standard electromagnetic transmission line in that the latter employs magnetic inductance instead of kinetic inductance. The plasmonic velocity is then $v_p = \sqrt{1/LC}$, which corresponds to the plasmonic dispersion relation and the network constant a that allow the adjustment with experimental data.

Bi-Dimensional Transmission Line: Fig. 1 of [6] reproduced here, shows the infinite two-dimensional network to be studied in this work, which is a structure formed by infinite LHTL connected in parallel, where all the inductances L are considered equal, as are the capacitances C. The system is translational invariant and therefore there is a specific dispersion relationship.

Using the laws of Kirchhoff, the evolution equation for the cell (j, n), formed by two capacitors and two coils that are shared by neighboring cells, is:

$$L\left(2\dot{I}_{j,n} - \dot{I}_{j-1,n} - \dot{I}_{j+1,n}\right) + \frac{1}{C}\left(2Q_{j,n} - Q_{j,n-1} - Q_{j,n+1}\right) = 0, \tag{1}$$

where the subscripts j and n are integers that vary from -infinity to infinity. The indices of the cells of Fig. 1, or only by exchanging the indices j for n and viceversa.

If we consider the plane wave-type solution for the equation of evolution of the charge in (j, n):

$$Q_{j,n}(t) = Q_0 \cdot e^{i(kj+pn-\omega t)}, \tag{2}$$

where Q_0 it is a constant, by introducing (2) into equation (1), we obtain the dispersion relation:

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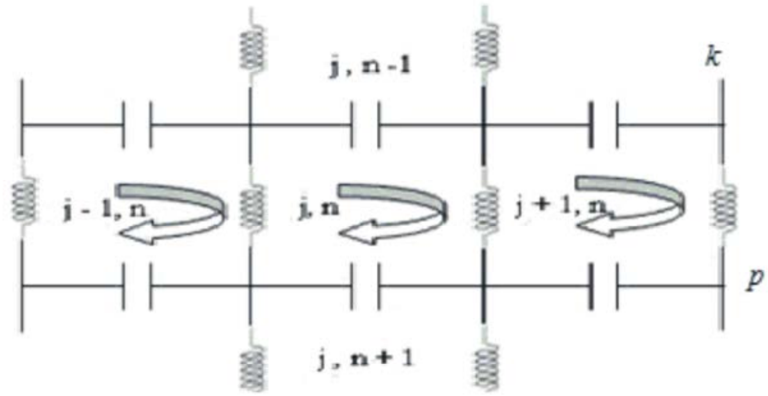

Fig. 1: The infinite two-dimensional network like bidimensional transmission line in space (p, k)

$$\omega_{k,p} = \mathbb{C}\omega_0 \left| \frac{sen\frac{p}{2}}{sen\frac{k}{2}} \right|, \tag{3}$$

where $\mathbb{C}$ is a constant and the wave numbers in each direction of the network are represented by k and p (dimensionless) and the dispersion law (3), is highly dispersive.

Dispersion Relation with Charge Discreteness: We are interested in knowing properties of the nanosized two-dimensional network described by (3), which will allow us to find the spectrum of energies, normal and metamaterial normal modes.

Now by considering charge discreteness following [9-16] and from the Hamiltonian, the usual quantization procedure for flux and charge and the prescription for charge discreteness, we could construct the quantum Hamiltonian for the direct transmission line with charge discreteness (q_e), which may be written as:

$$\widehat{H} = \sum_{m=-\infty}^{\infty} \left\{ \frac{2\hbar^2}{Lq_e^2} \sin^2 \frac{q_e}{2\hbar}\hat{\phi}_m + \frac{1}{2C}(\hat{q}_m - \hat{q}_{m-1})^2 \right\}, \tag{4}$$

where the index m describes the cell (circuit) at position m, containing an inductance L and capacitance C. The conjugate operators, charge $\hat{q}$ and pseudoflux $\hat{\phi}$, satisfy the usual commutation rule:

$$\left[\hat{q}_m, \hat{\phi}_{m'}\right] = i\hbar\delta_{m,m'} \text{ and } \left[\hat{q}_m, \hat{q}_s\right] = \left[\hat{\phi}_m, \hat{\phi}_s\right] = 0. \tag{5}$$

A spatially extended solution of Eq. (4) corresponds to the quantization of the classical electric transmission line with discrete charge (i.e. elementary charge q_e). Note that in the formal limit $q_e \to 0$ the above Hamiltonian gives the well-known dynamics related to the one-band quantum transmission line, similar to the phonon case. The system described by Eq. (5) is very cumbersome since the equations of motion for the operators are highly nonlinear due to charge discreteness. However, this system is invariant under transformation $\hat{q}_k \to \hat{q}_k + q$, that is, the total pseudo flux operator $\hat{\phi} = \sum \hat{\phi}_m$ commutes with the Hamiltonian, simplifying the study of this system:

$$\widehat{H}_m = \frac{2\hbar^2}{Lq_e^2} \sin^2 \frac{q_e}{2\hbar}\hat{\phi}_m - \frac{1}{2C}(\hat{q}_{m+1} - \hat{\phi}_m)^2 \tag{6}$$

where $\widehat{H}_m$ represents the Hamiltonian density operator for the fields. From the above Hamiltonian we find the equations of motion (Heisenberg equations) for the field operators:

$$\frac{\partial}{\partial t}\hat{\phi}_m = \frac{1}{C}(\hat{q}_{m+1} + \hat{q}_{m-1} - 2\hat{q}_m), \tag{7}$$

$$\frac{\partial}{\partial t}\hat{q} = \frac{\hbar}{Lq_e}\sin(\frac{q_e}{\hbar}\hat{\phi}). \tag{8}$$

In general the normal dispersion relation from Eqs. (7) and (8) is:

$$\frac{\omega^2 LC}{k^2} = \cos(q_e\hat{\phi}/\hbar)(\frac{\sin(k/2)}{k/2})^2, \tag{9}$$

where ω(k) is given by:

$$\omega = k\sqrt{\cos(q_e\hat{\phi}/\hbar)}\left|\frac{\sin(k/2)}{k/2}\right| \qquad (10)$$

with p = etc, always v^2 must be positive ($v^2LC > 0$), the two equations (9) and (10) are satisfied simultaneously when ϕ/ϕ_0, $2n\pi$, $n = \pm 1, \pm 2$ that means that $\phi = 2n\pi\phi_0 = nh/q_e$.

For metamaterial regime we have:

$$\omega = -p^{-1}\sqrt{\cos(q_e\hat{\phi}/\hbar)}^{-1}\left|\frac{\sin(p/2)}{p/2}\right|^{-1}, \qquad (11)$$

with k = cte

Phase and Group Velocities: A complete understanding of the system described in this paper requires consideration of some aspects of traditional engineering in this type of systems. That is, the graph of some quantities of relevance for future designs. Fig. 4 shows the behavior of the dispersion law of (3), which is periodic in both directions, where L = C = 1 was assumed by simplification.

This relation presents discontinuities for values of k, $p = 2\pi$ n, with n integer = 0 and it is seen that it is a combination of the dispersion relations of the direct and dual transmission lines [6].

It is known that the group speed is the speed with which the energy is transmitted through the network, as well as the phase velocity is the speed of propagation of the wave profile. Let's look at the details of these quantities using standard numerical simulations.

In the direction of k we have the group velocity:

$$v_{g,k} = \frac{\partial\omega_{k,p}}{\partial k} = -\frac{1}{2}\frac{\omega_{k,p}}{\tan\left(\frac{k}{2}\right)} = -\frac{1}{2}\sqrt{\cos(q_e\hat{\phi}/\hbar)}\frac{\omega_0}{\tan\left(\frac{k}{2}\right)}\left|\text{sen}\frac{k}{2}\right|, \qquad (12)$$

and phase velocity:

$$v_{f,k} = \frac{\omega_{k,p}}{k} = \omega_0\sqrt{\cos(q_e\hat{\phi}/\hbar)}\left|\text{sen}\frac{k}{2}\right|(k)^{-1}, \qquad (13)$$

given the periodicity of both functions, a cut perpendicular to p.

An interesting aspect is that both functions are anti parallel for simultaneous values of k and p in alternating half-periods.

In the direction of p we have the group velocity:

$$v_{g,p} = \frac{\partial\omega_{k,p}}{\partial p} = \frac{1}{2}\frac{\omega_{k,p}}{\tan\left(\frac{p}{2}\right)} = \frac{1}{2}\sqrt{\cos(q_e\hat{\phi}/\hbar)}^{-1}\frac{\omega_0}{\tan\left(\frac{p}{2}\right)}\left|\text{sen}\frac{p}{2}\right|^{-1}, \qquad (14)$$

and the phase velocity:

$$v_{f,p} = \frac{\omega_{k,p}}{p} = \omega_0\sqrt{\cos(q_e\hat{\phi}/\hbar)}^{-1}\left|\text{sen}\frac{p}{2}\right|^{-1}(p)^{-1}, \qquad (15)$$

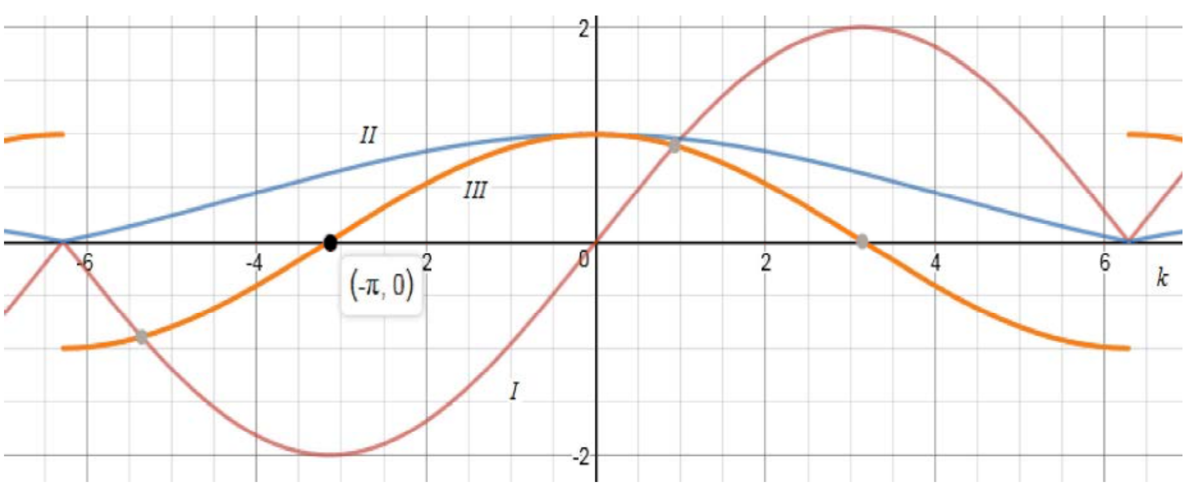

Fig. 2: For $q_e\hat{\phi}/\hbar = \hat{\phi}/\phi_0 = \mp 2n\pi$, curve I represents ω/ω_0, curve II is the phase velocity $v_{f,p}$, curve III is the group velocity $v_{g,p}$. The same values are obtained with $n = 0,1,2,3...$,

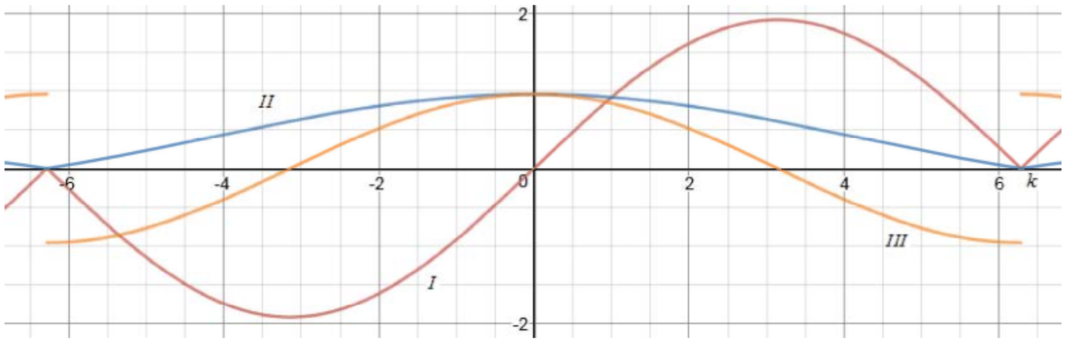

Fig. 3: For $q_e\hat{\phi}/\hbar = \hat{\phi}/\phi_0 = \mp\pi/8$, curve I represents ω/ω_0, curve II is the phase velocity $v_{f,p}$curve III is the group velocity $v_{g,p}$. Here we find a case with fractional charge from $13\pi/8$ to $19\pi/8$.

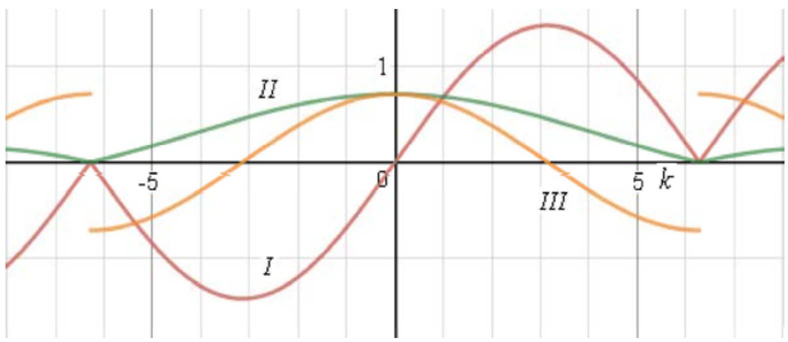

Fig. 4: For $q_e\hat{\phi}/\hbar = \hat{\phi}/\phi_0 = \mp\pi/3$, curve I represents ω/ω_0, curve II is the phase velocity $v_{f,p}$, curve III is the group velocity $v_{g,p}$. Here we find a case with fractional charge, the same values are obtained with $\pi/3, 5\pi/3, 7\pi/3, 11\pi./3, 13\pi/3, 17\pi/3, 19\pi/3$

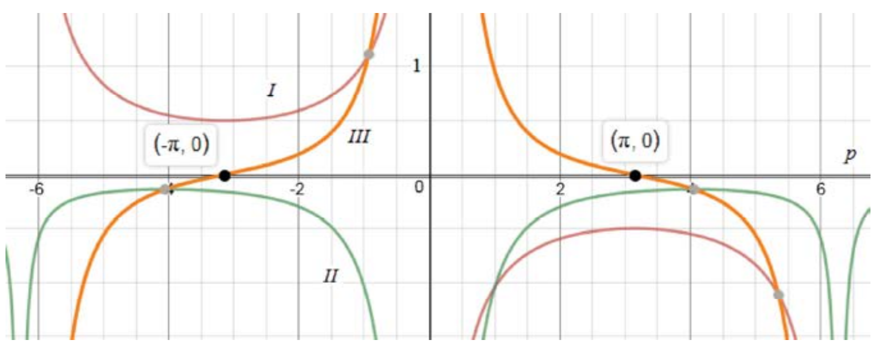

Fig. 5: For $q_e\hat{\phi}/\hbar = \hat{\phi}/\phi_0 = \mp 2n\pi$, curve I represents ω/ω_0, curve II is the phase velocity $v_{f,p}$, curve III is the group velocity $v_{g,p}$. The same values are obtained with $n = 0$,

The two-dimensional network of coupled LC circuits behaves like a network formed by independent harmonic oscillators (similar to that of phonons or plasmons in a crystal, or in general, elemental excitations) in the reciprocal space of wave vectors and whose energy spectrum is given by (10, 11). That is, the dispersion relation (3) corresponds to the definition of the elementary excitations of this nanometric network.

The dispersion ratio (3) shows that the medium is dispersive and is a combination of the corresponding one-dimensional direct (RHTL) and dual (LHTL) transmission lines [4, 6].

Regarding group and phase velocities, in each direction, it is interesting to note that both phase velocities tend to 0 when k and k simultaneously grow, which is to say that the wavelength decreases.

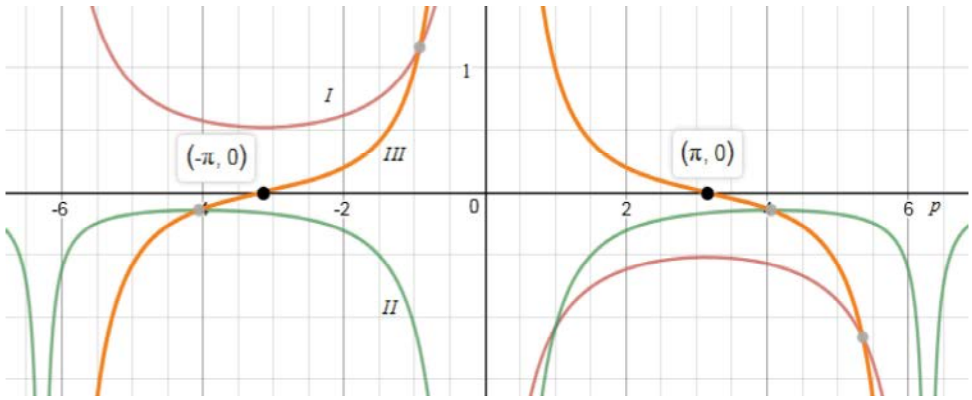

Fig. 6: For $q_e\hat{\phi}/\hbar = \hat{\phi}/\phi_0 = \mp\pi/8$, curve I represents ω/ω_0, curve II is the phase velocity $v_{f,p}$ curve III is the group velocity $v_{g,p}$. The same values are obtained with ∓ $(2n\pi + \pi/8)$. Here we find a case with fractional charge

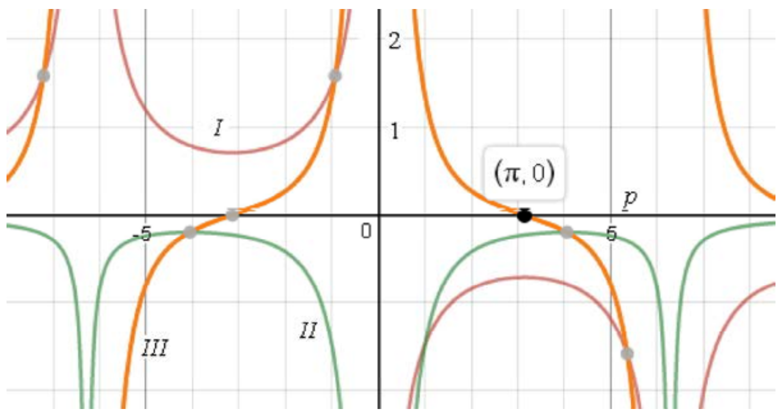

Fig. 7: For $q_e\hat{\phi}/\hbar = \hat{\phi}/\phi_0 = \mp\pi/3$, curve I represents ω/ω_0, curve II is the phase velocity $v_{f,p}$ curve III is the group velocity $v_{g,p}$. Here we find a case with fractional charge, the same values are obtained with $\pi/3$, $5\pi/3$, $7\pi/3$, $11\pi/3$, $13\pi/3$, $17\pi/3$, $19\pi/3$.

The group and phase velocities (Fig. 5), in the direction of union of coils, are antiparallel in the middle of the intervals corresponding to each period. In the direction of union of capacitors (Fig. 6), the group velocity has negative slope and the phase velocity alternately presents positive and negative slopes, except in the section between 0 and 2π. These results can be useful to study fractional quantum Hall effect in graphene [17, 18].

CONCLUSION

The quantum two-dimensional network of coupled circuits given by the relation (3) where the spectrum of the elemental excitations that supports nanometric network is well characterized (10) in metamaterial mode with charge discreteness. On the other hand, a special property observed in the studied network is that, at intervals of half-periods in the direction of k, it behaves similarly to the metamaterials [6-8]. For the quantized electric circuit considered above, the correspondence with the classical theory does not follow from $\hbar = h/2\pi \to 0$ as in usual quantum mechanics, but from $e \to 0$ for a finite value of h. This suggests that a full quantum mechanical treatment of charged particles should be governed by two finite parameters: e and h, instead of only h to study fractional quantum Hall effect in graphene in metamaterial mode.

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