

## Sensitivity of Load Dispatch in an Electric Market of Energy Transactions Using Linear Programming in AMPL

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**Abstract:** This article describes how the three main models of deregulated electric markets in the world operate: the exchange market, the bilateral market and the hybrid market, is focused on the exchange market as it shows how to take out the load dispatch through energy auctions, the second type of market operates through contracts of trade energy directly between generator and consumer, so it is impossible to do a mathematically model the that shows the way the dispatch energy, the last market is a combination of the previous two. Three scenarios are developed in which is shown and explained how to do load dispatch in the exchange market making an optimal dispatch using linear programming in A Mathematical Programming Language (AMPL) according to the conditions established in the Power Electrical System.

**Key words:** Deregulated Market • Electric Power Systems • Electric Exchange Market • Electric Market of Bilateral Transactions • Hybrid Electric Market • AMPL

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### INTRODUCTION

In the world, there are mainly two Electric Markets, Regulated and Deregulated Markets, Regulated Markets are considered monopolies and are characterized by their dependence on all the actions that the state determines for its growth and the way of operation, in the second there is no monopolies, free and fair competition is sought/search between Market Participants and the Consumers can choose who is the best seller to buy the energy they demand. The Deregulated Electric Markets are subdivided into three types; the Market of Exchange, the Market of Bilateral Transactions and the Hybrid Market. Each of these has its particular characteristics and will be described in this document, in order to demonstrate as clearly as possible the load dispatch of the units, we will focus on developing some of the cases that may arise in the Electric Market, these cases fulfil the objective of demonstrating how the optimum dispatching changes as they have limitations and / or contingencies in the Power Electrical System [1, 2].

Is important to mention that in the energy transactions there are Homogeneous products and others recognized as Heterogeneous, in Homogeneous products generators can agree with a common price for that product that will be taken as standard. Heterogeneous products are distinguished by having different qualities among the others, so the price will be different between each generator [6].

**Development:** In energy markets, three different types of deregulated energy markets have been established; the electric exchange market, the electric market of bilateral transactions and the hybrid electric market.

**Electric Market of Bilateral Transactions:** It is characterized by the fact that it does not need an ISO to carry out the contracts for trade energy, however, they must respect the norms that the ISO send out if it exists, for example, for the transmission and distribution needs to inform the amount of energy that travels in the Transmission Lines and in the Distribution Grids so that

it is taken into account and to avoid a congestion, also must be respectful with the concepts of Energy Porting. Their contracts are made only between seller and buyer, the objective is to achieve the maximum satisfaction of both parts.

Although the System Operator does not take part in the definition of prices, a cost per grid usage is established to carry out the energy transactions presented previously, for this there are several methods that determine the associated cost per grid usage, the energy reform implemented in Mexico establishes the postal stamp method to define grid usage costs [7].

**Electric Exchange Market:** This type of market consists of performing energy auctions, these auctions are usually performed by an entity that will function as a system operator, formally known as Independent System Operator (ISO). The ISO receives offers from energy sellers and buyers, which are the prices for which they are willing to sell and buy energy respectively, usually send out a reserve price for each of the parts in the case of sellers are not willing to sell energy below that price and in the case of buyers are not willing to buy energy above the price of the reservation that they send out [1], the mathematical model used to perform energy dispatch optimally is presented in the following sections of this article. There are mainly three cases:

- Offer made by Generators (Sellers).
- Offer made by Consumers (Buyers).
- Both parts offer.

**Then the Three Cases Are Developed**

**Offers Made by Generators (Sellers):** Since the offer in this case are made by the energy sellers, the objective will be to minimize the function, so that the first units to be assigned are the most economical and in the end are assigned the most expensive ones. The mathematical model that describes this function is the following one:

$$\min_{X_{ij}} \sum_{i=1}^m \sum_{j=1}^n C_{vij} X_{ij} \tag{1}$$

Restrictions:

$$\sum_{j=1}^n X_{ij} \leq G_i \quad \forall i$$

$$\sum_{i=1}^m X_{ij} \leq D_j \quad \forall j$$

$$X_{ij} \geq 0 \quad \forall i, j$$

$$X_{ij} \leq R_{ij}$$

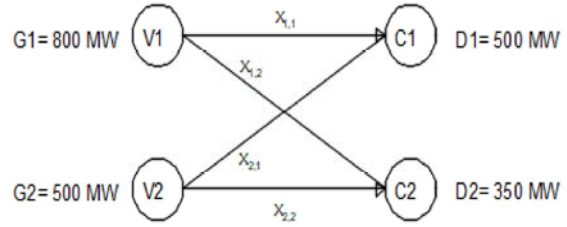


Fig. 1: System to be used in all three scenarios

Table 1: Sales Prices of G1 and G2. Offer form Sellers.

Generators	Consumers	
	2 (\$/MW)Buyer 1	4 (\$/MW)Buyer 2
Seller 1 (G1)	10.0	9.4
Seller 2 (G2)	12.0	8.4

Variables mean:

$G_i$  Quantity of Energy of the Seller  $i$  willing to sell

$D_j$  Quantity of Energy demanded by buyer  $j$ .

$C_{vij}$  Price specified by Seller  $i$  to a buyer  $j$  by a heterogeneous product.

$X_{ij}$  Quantity of Energy Selled to the Seller  $i$  to a buyer  $j$ .

$R_{ij}$  Restrictions from the system

As can be seen from equation 1, the decision variables must be no-negatives. In this equation the first restriction avoids to exceed the amount of energy offered by the seller  $i$ . The second restriction requires not delivering less energy than the buyer  $j$  needs to acquire. The reason why the value of the offer made by the sellers is minimized is to force the product to flow from the seller who offers the lowest price to the buyers until the amount of energy that he puts up for sale is exhausted and successively [2].

In order to illustrate the energy transactions in this market model, we will consider three scenarios, considering the following system in cases where the bidders are the Generators:

**Scenario 1. Unrestricted Power Dispatch in the Power Electrical System:**

It is observed that generator one is willing to sell 800 MW and the second generator has 500 MW available for sale. Considering the prices indicated in table 1. In this system, no transmission line capacities and losses are not considered the reliability of the system is not evaluated.

The problem consist in make an assignment of units, that is to allocate the generation plants that are dispatched to the load centers optimizing the total cost,

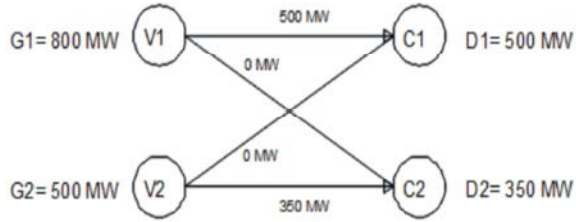


Fig. 2: Energy dispatch for scenario 1.

Table 2: Economic Dispatch for scenario 1..

Transactions	Price [\$]	Dispatch [MW/hr]
$X_{1,1}$	10	500
$X_{1,2}$	9.4	0
$X_{2,1}$	12	0
$X_{2,2}$	8.4	350

we seek to make an economic dispatch by assigning to each of the load centers the optimal combination that guarantees the most economical service satisfying the energy that they demand

In this work the software A Mathematical Programming Language (AMPL) [3] is used in order to solve the problem described previously.

By adequately representing equation 1 and their corresponding restrictions in the AMPL software, the dispatch that minimizes the cost of energy is as:

The results show that the optimal dispatch is \$ 7940. The buyer 1 acquires all its energy from seller 1, at a price of \$ 10 MW, multiplying the energy demanded by the unit price that we have:

$$PC_1 = 500 \text{ MW} * \$10 = \$5,000/\text{MW}[hr] \quad (2)$$

The buyer 2 acquires his energy from seller 2 at a Price of 8.4 per MW, operating:

$$PC_2 = 350 \text{ MW} * \$8.4 = \$2940/\text{MW}[hr] \quad (3)$$

Add up the total costs of energy:

$$Pr = \$5000 + \$2940 = \$7940/\text{MW}[hr] \quad (4)$$

**Scenario 2. Power Dispatch Considering Constrains in the Power Electrical System:** In this case it is considered that the electrical system only allows the energy transactions established in Table 3.

**So Now the Power Dispatch Is as Follows:** Taken the restrictions that are now, the system is forced to change the way it delivers energy, then buyer 1 buys 400MW of

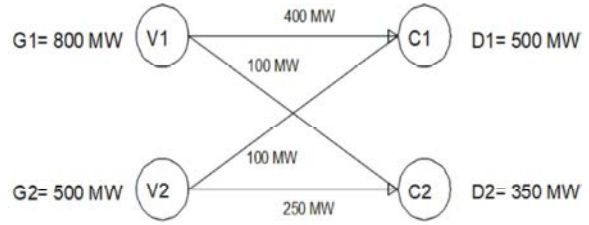


Fig. 3: Energy Dispatch for scenario 2.

Table 3: System restrictions for scenario 2.

Transactions	Restrictions [MW]
$X_{1,1}$	400
$X_{1,2}$	400
$X_{2,1}$	250
$X_{2,2}$	250

Table 4: Economic dispatch for the second scenario

Transactions	Price [\$]	Dispatch [MW/hr]
$X_{1,1}$	10	400
$X_{1,2}$	12	100
$X_{2,1}$	9.4	100
$X_{2,2}$	8.4	250

energy from seller 1 and 100MW from seller 2. Buyer 2 purchases 100 MW from seller 1 and 250 of the seller 2. The optimum point of purchase remaining at \$ 8240. It can also be broken down as in the previous case:

$$PC_1 = (400 \text{ MW} * \$10) + (100 \text{ MW} * \$9.4) = \$4,940/\text{MW}[hr] \quad (5)$$

$$PC_2 = (100 \text{ MW} * \$12) + (250 \text{ MW} * \$8.4) = \$3,300/\text{MW}[hr] \quad (6)$$

$$P_T = \$4,940 + \$3,300 = \$8,240/\text{MW}[hr] \quad (7)$$

**Scenario 3. Energy Dispatch considering constrains and a congestion in the Power Electrical System:** For the third case the conditions of the previous case are taken into account and additionally it is considered that by a contingency in the system the seller 2 cannot dispatch energy to the buyer 2 (Restriction  $X_{2,2}$ )

**The Energy Dispatch Is Made by the Following Form:** As we can see in the Table 5, there is no assignment in the second generator with the second buyer ( $X_{2,2}$ ) due to assumed contingency in the system, in this sense the way of doing the dispatch changes and the new optimization point is in the price of \$ 9140.

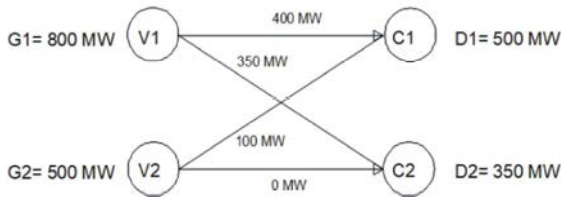


Fig. 4: Energy Dispatch for scenario 3.

Table 5: System restrictions for scenario 3

Transactions	Restrictions [MW]
$X_{1,1}$	400
$X_{1,2}$	400
$X_{2,1}$	250
$X_{2,2}$	0

Table 6: Economic Dispatch for scenario 3.

Transactions	Price [S]	Dispatch [MW/hr]
$X_{1,1}$	10	400
$X_{1,2}$	12	350
$X_{2,1}$	9.4	100
$X_{2,2}$	8.4	0

**Offers made by Consumers (Buyers):** Now we will observe the cases in which the price offer is made by the buyer, in this sense the objective function is reversed and instead of minimizing we must maximize. The mathematical model is the same as the previous one, with the difference that now the function must be maximized.

$$\max_{X_{ij}} \sum_{i=1}^m \sum_{j=1}^n C_{cij} X_{ij} \quad (8)$$

Restrictions:

$$\sum_{j=1}^n X_{ij} \leq G_i \quad \forall i$$

$$\sum_{i=1}^m X_{ij} \leq D_j \quad \forall j$$

$$X_{ij} \geq 0 \quad \forall i, j$$

$$X_{ij} \leq R_{ij}$$

$C_{cij}$  Price specified by buyer  $j$  to seller  $i$  by a heterogeneous product

In these cases, the procedure to obtain the solution is done in reverse, because this is how we will achieve the maximum gain, this is, the first user to be dispatched will be the one who buys the energy at a higher price and the last one will be the one which has offered the lowest price.

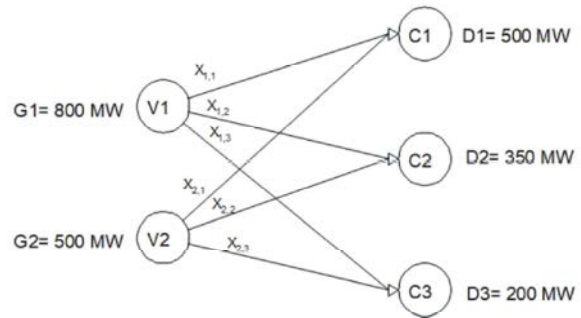


Fig. 5: System to be used in all three scenarios.

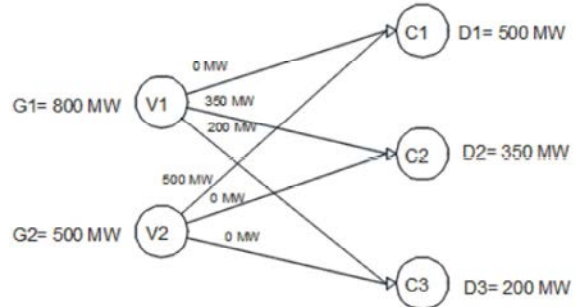


Fig. 6: Energy Dispatch for scenario 1. Offer made by Buyers.

Table 7: Prices of C1, C2 and C3. Offer made by Consumers.

Generator	Consumers		
	(\$/MW)Buyer 1	(\$/MW)Buyer 2	(\$/MW)Buyer 3
Seller 1 (G1)	10.0	9.4	10.2
Seller 2 (G2)	12.0	8.4	11.4

Table 8: Economic Dispatch for scenario 1. Offer made by Buyers.

Transactions	Price [S]	Dispatch [MW/hr]
$X_{1,1}$	10	0
$X_{1,2}$	9.4	350
$X_{1,3}$	10.2	200
$X_{2,1}$	12	500
$X_{2,2}$	8.4	0
$X_{2,3}$	11.4	0

The exercises to work on these cases will be performed with the system of the Figure 5.

**Scenario 1. Unrestricted Power Dispatch in the Power Electrical System:** The prices that we will use are show in Table 7.

As in the case of offers made by sellers, in the first scenario we do not take into account any type of restriction, other than the energy put on sale by our generation units and the demand by the consumer centres, in this way the software will find the optimum

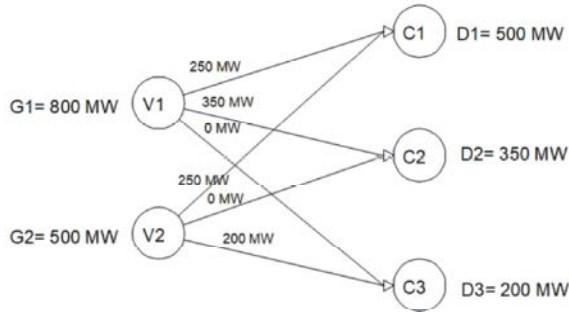


Fig. 7: Units assignment for scenario 2. Offer made by Buyers.

Table 9: System restrictions for scenario 2. Offer made by Buyers.

Transactions	Restrictions [MW]
$X_{1,1}$	400
$X_{1,2}$	400
$X_{1,3}$	400
$X_{2,1}$	250
$X_{2,2}$	250
$X_{2,3}$	250

Table 10: Economic Dispatch for scenario 2. Offer made by Buyers.

Transactions	Price [\\$]	Dispatch [MW/hr]
$X_{1,1}$	10	250
$X_{1,2}$	9.4	350
$X_{1,3}$	10.2	0
$X_{2,1}$	12	250
$X_{2,2}$	8.4	0
$X_{2,3}$	11.4	200

ideal point for the given system, for this case the allocation of units is as:

$$GV_1 = (350 \text{ MW} * \$9.4) + (200 \text{ MW} * \$10.2) = \$5,330/\text{MW}[\text{hr}] \quad (9)$$

$$GV_2 = (500 \text{ MW} * \$12) = \$6,000/\text{MW}[\text{hr}] \quad (10)$$

$$G_T = \$5,330 + \$6,000 = \$11,330/\text{MW}[\text{hr}] \quad (11)$$

As we can see in the price sum the gain for sellers has its optimum point at \$11,330.

**Scenario 2. Energy Dispatch considering limitations in the Electrical Power System:** In this case it is considered that the electrical system only allows the energy transactions established in Table 9.

**Operating in AMPL the Results Obtained Are the Next:** As we can see in Table 10, the system is forced to change his form of energy dispatch. The optimum profit point for

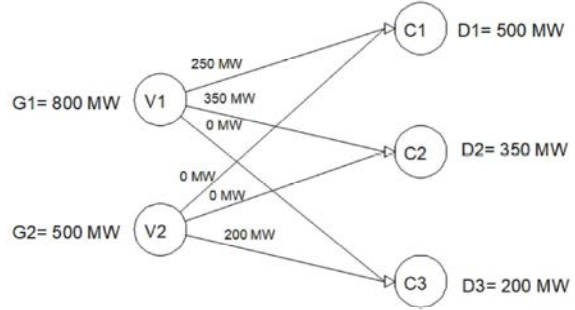


Fig. 8: Energy Dispatch for scenario 3. Offer made by Buyers.

Table 11: System restrictions for scenario 3. Offer made by Buyers.

Transactions	Restrictions [MW]
$X_{1,1}$	400
$X_{1,2}$	400
$X_{1,3}$	400
$X_{2,1}$	0
$X_{2,2}$	250
$X_{2,3}$	250

Table 12: Economic Dispatch for scenario 3. Offer made by Buyers.

Transactions	Price [\\$]	Dispatch [MW/hr]
$X_{1,1}$	10	250
$X_{1,2}$	9.4	350
$X_{1,3}$	10.2	0
$X_{2,1}$	12	0
$X_{2,2}$	8.4	0
$X_{2,3}$	11.4	200

sellers now stands at: \$ 11,070. Remaining the profits for each generator as this:

$$GV_1 = (250 \text{ MW} * \$10) + (350 \text{ MW} * \$9.4) = \$5,790/\text{MW}[\text{hr}] \quad (12)$$

$$GV_2 = (250 \text{ MW} * \$12) + (200 \text{ MW} * \$11.4) = \$5,280/\text{MW}[\text{hr}] \quad (13)$$

$$G_T = \$5,790 + \$5,280 = \$11,070/\text{MW}[\text{hr}] \quad (14)$$

**Scenario 3. Power Dispatch considering constrains and a congestion in the Power Electrical System:** For the third case the conditions of the previous case are taken into account and additionally it is considered that by a contingency in the system the generator 2 cannot sell energy to the buyer 1 (Assignment  $X_{2,1}$ ) being this the most expensive line will see a clear change in the dispatch of energy. The restrictions for this scenario are shown in Table 11.

**Taking into Account the Restrictions in Table 11, the Energy Dispatch Is the Follows:** As we can see in the results obtained, it is impossible to dispatch the 500MW

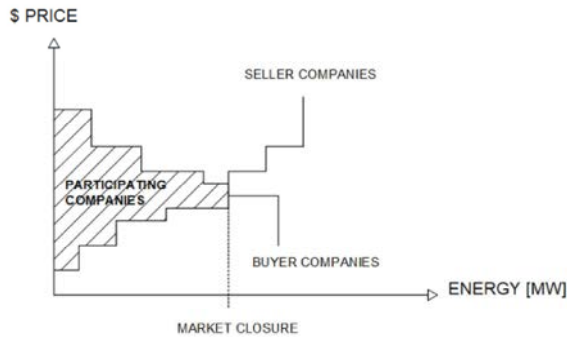


Fig. 9: Market closure in case of offers by Generators and Consumers simultaneously.

demanded by the Buyer one given the conditions for this scenario, in this case it is assumed to leave out of service half of the load demanded by the company, this phenomenon is known as shooting of load and happens with some frequency in the Power Electrical Systems, some of these load shots are programmed to lower costs in critical hours and others arise unexpectedly due to contingencies in the System.

The solution in this case finds the optimum point of maximum profits in \$8,070. Itemise as follows for each seller:

$$GV_1 = (250 \text{ MW} * \$10) + (350 \text{ MW} * \$9.4) = \$5,790/\text{MW}[\text{hr}] \quad (15)$$

$$GV_2 = (200 \text{ MW} * \$11.4) = \$2,280/\text{MW}[\text{hr}] \quad (16)$$

$$P_T = \$5,790 + \$2,280 = \$8,070/\text{MW}[\text{hr}] \quad (17)$$

**Offers Made by Generators and Consumers Simultaneously:** There is a one more scenario that consists in the offer of both parts, this is, when the sellers bid at the same time that the buyers of energy offer; the way to solve this case is by doing a market close [1]. The curves of offers from sellers and buyers should be drawn. For sellers considering the amount of energy they are willing to sell and the price at which they want to sell it, this curve tends to go from less to more in the price. For buyers the amount of energy they demand and the price they are willing to pay for it is considered, this curve will tend to go from more to less than the price. The point where both bidding curves intersect will be the point called Market Close, this means that all companies, both sellers and consumers that will take place in this market will be those that are into that point of intersection and behind or below depending on how it was graphed. Look at Figure 9.

**Hybrid Electric Market:** This type of market is generally established in the world, since it allows both types of transactions in the aforementioned markets, the Electric Exchange Market and the Electric Market of Bilateral Transactions, for example the Wholesale Electricity Market has established in Mexico in which respecting the bases of the exchange market, the ISO that in the case of Mexico is the CENACE receives the offers of the Generators of Energy and the offers of the Consumers; the energy is received through the Transmission Lines and is normally delivered through the General Distribution Grids to the Final Users. On the other hand according to the bases of the Bilateral Transactions Market, the contracts of sale of energy between Qualified Users (name given in Mexico to the consumers that have the requirements to be participants of the Wholesale Electricity Market) and Generators of Energy [1, 4, 5].

## RESULTS

The program has been used for the purpose of facilitate the calculation of optimization through linear programming, the objective functions are the equation that the program seeks to optimize through the cplex method. This algorithm is a way of optimize, but not the only one. The objective of the scenarios is to show different cases that can be presented in the energy markets. The solutions found are those expected and were verified by doing the programs previously and comparing with similar works that did not contemplate all the scenarios shown in this article, these results can also be verified solving the problems by the graphic optimization method.

## CONCLUSIONS

From the results obtained in the different scenarios with the test systems, it was observed that:

In cases where the offers are given by the sellers, the objective is to minimize the function and the optimal point is finding in the first scenario when the buyers pay the lowest possible price for the energy that they demand. At the same time, restrictions are added to the test system, which are reflected in the program AMPL. This optimization point is shifted in such a way that the final price is rising for consumers.

For the scenarios in which the buyers offer the objective is to maximize the function, then the optimum point is when the sellers obtain the highest gain, otherwise the scenarios in which sellers offer by adding restrictions to the test system, this Optimal point is

shifted in such a way that when adding limitations, the profits for the sellers are reduce.

The approach of the offers of sellers and the buyers of electric power presented in this work can be approached as an energy auction problem.

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