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Qi-Guo's Connection Between the Lah Numbers and the Kummer Hypergeometric Function

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Abstract: We give a simple proof of a relation obtained by Qi-Guo for the sum of the Lah numbers in terms of the confluent hypergeometric function.

Key words: Hypergeometric function - Petkovsek-Wilf-Zeilberger's method - Lah numbers

INTRODUCTION

Qi-Guo [1, 2] showed the following relation:

$$A \equiv \sum_{k=1}^{n} L_n^{[k]}\, z^{k-1} = n!\ e^{-z}\ {}_1F_1(n+1; 2; z), \tag{1}$$

Involving the Lah numbers [3-9]:

$$L_n^{[k]} = \frac{n!}{k!}\binom{n-1}{k-1}, \tag{2}$$

And the confluent hypergeometric function [10]; here we shall prove (1) via the algorithm explained in [11-20]. In fact, from (2):

$$A = n!\ \sum_{r=0}^{\infty} t_r\ , \qquad t_r = \frac{(n-1)!}{r!\,(r+1)!\,(n-r-1)!}\, z^r, \tag{3}$$

Therefore $\frac{t_{r+1}}{t_r} = \frac{(r+1-n)}{(r+2)(r+1)}\,(-z)$, then (3) implies the hypergeometric relation:

$$A = n!\ {}_1F_1(1-n; 2;\ -z), \tag{4}$$

But we have the Kummer's identity [10]:

$${}_1F_1(a; b;\ -z) = e^{-z}\ {}_1F_1(b-a; b; z), \tag{5}$$

Whose application in (4) gives (1), q.e.d.

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