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Solving Bernoulli Differential Equations by using Newton's Interpolation and Lagrange Methods

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Abstract: There has been greater attempt to solving differential equations by analytic methods and numerical methods. Most of researchers treated numerical approach to solve first order ordinary differential equations. These methods such as Runge Kutta method, Taylor series method and Euler's method, etc. Faith Chelimo Kosgei studied this problem by combined the newton's interpolation and Lagrange method, Nasr Al Din IDE also studied this problem by Using Newton's Interpolationand Aitken's Method for Solving First Order Differential equation. This study will use Newton's interpolation and Lagrange's methodto solve Bernoulli Differential Equations as a type of first order differential equation.

Key words: Differential equation · Bernoulli Differential Equations · Analytic method · Numerical method · Newton's interpolation method · Lagrange Methods

INTRODUCTION

Many problems can be formulated in theform of ordinary differential equation, specially Bernoulli differential equationsof first order, hence we need to study and solve the Bernoulli differential equations. A numerical method is used to solve numerical problems. The differential equation problem [1-10], consists of at least one differential equation and at least one additional equation such that the system together have one and only one solution called the analytic or exact solution to distinguish it from the approximate numerical solutions that we shall consider In thispaper, to find the solution of differential equation of first order, Faith C.K. [1] studied this problem by using combination of newton's interpolation and Lagrange method, Nasr Al Din Ide [2] studied this problem by using of Newton's Interpolation and Aitken's Method for Solving First Order Differential equations. In this study we will combine of Newton's interpolation and Lagrange's method [4-10]. Finally we verified on a number of examples and numerical results obtained show the efficiency of the methodgiven by present study in comparison with the exact solution.

Let's consider the Bernoulli differential equation which can be written in the following standard form:

$$y' + P(x)y = Q(x)y^n \tag{1}$$

where P and Q are functions of x and n is a constant $n \neq 1$ (the equation is thus nonlinear).
wherey is a known function and the values in the initialconditions are also known numbers.

Combined Newton's Interpolation and Lagrange Method [1, 2]: This study combine both Newton's interpolation methodand Lagrange method.it used newton's interpolation method tofind the second two terms then use the three values for y toform a quadratic equation using Lagrange interpolation method as follows;

Newton's Interpolation Method [1, 2, 10]:

$$f_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \dots + a_n(x-x_0)(x-x_1)\dots a_2(x-x_{n-1}) \tag{2}$$

where,

$$a_0 = y_0,\ a_1 = \frac{f(x_1)-f(x_0)}{(x_1-x_0)},\ a_2 = \frac{\dfrac{f(x_2)-f(x_1)}{(x_2-x_1)} - \dfrac{f(x_1)-f(x_0)}{(x_1-x_0)}}{(x_2-x_0)} \tag{3}$$

etc,

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Lagrang Interpolation Method [1, 8]:

$$y_n = \frac{(x-x_1)-(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)-(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)-(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2 \quad (4)$$

Description of the Method: This method will combine both Newton's interpolation methodand Lagrange method. It used newton's interpolation method tofind the second two terms then use the three values for y toform a linear or quadratic equations using Lagrange interpolation method as follows;

$$f_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + ... + a_n(x-x_0)(x-x_1)...a_2(x-x_{n-1}) \quad (5)$$

where,

$$a_0 = y_0,\ a_1 = \frac{f(x_1)-f(x_0)}{(x_1-x_0)},\ a_2 = \frac{\frac{f(x_2)-f(x_1)}{(x_2-x_1)} - \frac{f(x_1)-f(x_0)}{(x_1-x_0)}}{(x_2-x_0)} \quad (6)$$

etc,

$$y_1 = a_0 + a_1\,(x-x_0) \quad (7)$$

$$y_2 = a_0 + a_1\,(x-x_0) + a_2\,(x-x_0)\,(x-x_1) \quad (8)$$

Forming quadratic interpolation of Lagrange, we have:

$$y_n = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}.y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}.y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}.y_2 \quad (9)$$

Note: We can use Newton's Forward Interpolation Formula instead of Newton's divided Interpolation method in (2.1).

Examples: In this section, we will check the effectiveness of thepresent technique(3). First numerical comparison for the following test examples taken in [3].

Example 1:

Solve, $y' = y + x.y^{\frac{1}{2}}$, the exact solution of this problem is

$y = (c.e^{\frac{x}{2}} - x - 2)^2$

For $c = 1$, *the exact solution of this problem is* $y = (e^{\frac{x}{2}} - x - 2)^2, hence, y(0) = 1$

Now, by taking the step h = 0.01

First by using Newton's interpolation, we have

$a_0 = 1 = y_0$

$a_1 = \frac{f(x_1)-f(x_0)}{(x_1-x_0)} = [\frac{dy}{dx}]_{0,1} = 0$

$y_1 = 1 + 0(0.01-0) = 1$

$a_2 = \frac{\frac{f(x_2)-f(x_1)}{(x_2-x_1)} - \frac{f(x_1)-f(x_0)}{(x_1-x_0)}}{(x_2-x_0)} = \frac{[\frac{dy}{dx}]_{0.01,0.01} - [\frac{dy}{dx}]_{0,0}}{0.02-0} = 0.55$

$y_2 = 1 + 0(0.02-0) + 0.55(0.02-0)(0.02-0.01) = 1.000110000$

Hence, forming quadratic interpolation of Lagrange, we have:

$$y_n = \frac{(x-0.01)(x-0.02)}{(0-0.01)(0-0.02)}.1 + \frac{(x-0)(x-0.02)}{(0.01-0)(0.01-0.02)}.1 + \frac{(x-0)(x-0.01)}{(0.02-0)(0.02-0.01)}.1.00011$$
$$= 0.55x^2 - 0.0055x + 1$$

Table 1 givesthe approximation solution and the exact solution of example 1 with the error for x=0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1

Table 1: Solution of $y' = y + x.y^{\frac{1}{2}}\ \ y(0) = 1$

x	Combined Newton's Interpolation and Lagrange	Exact Values	Absolut error
0	1	1	0
0.01	1	1.009999833	0.009999833
0.02	1.000110000	1.019998665	0.019888665
0.03	1.000330000	1.029995492	0.029665492
0.04	1.000660000	1.039989307	0.039329307
0.05	1.001100000	1.049979102	0.048879102
0.06	1.001650000	1.059963867	0.058313867
0.07	1.002310000	1.069942587	0.067632587
0.08	1.003080000	1.089877829	0.076834247
0.09	1.003960000	1.090000000	0.086040000
0.1	1.004950000	1.100000000	0.095050000

Example 2:

Solve $y' = 2xy + 2x^3.y^2$, *the exact solution of this problems is* $y = 1/(c.e^{-x^2} + 1 - x^2)$

For c = 0, *the exact solution of this problems is* $y = 1/(1-x^2)$, *hence,* $y(0) = 1$

Now, by taking the step h = 0.01
First by using Newton's interpolation, we have

$$a_0 = 1 = y_0$$
$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = [\frac{dy}{dx}]_{0,1} = 0$$
$$y_1 = 1 + 0(0.01 - 0) = 1$$
$$a_2 = \frac{\frac{f(x_2) - (x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = 0.01000001$$
$$y_2 = 1.000002$$

Hence, forming quadratic interpolation of Lagrange, we have:

$$y_n = 0.01x^2 + 0.0001x + 1$$

Table 2 gives the approximation solution and the exact solution of example 2 with the error for x=0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1

Table 2: Solution of $y' = 2xy + 2x^3.y^2, \ \ y(0) = 1$

x	Combined Newton's Interpolation and Lagrange	s	Absolut error
0	1	1	0
0.01	1.000009000	1.000100010	0.000099110
0.02	1.000002000	1.000400610	0.000381600
0.03	1.000006000	1.000900811	0.000394160
0.04	1.000012000	1.001602564	0.001590564
0.05	1.002495000	1.002506266	0.002566760
0.06	1.000093000	1.003613007	0.003583007
0.07	1.000042000	1.004924128	0.004882128
0.08	1.000056000	1.006441224	0.006385224
0.09	1.000072000	1.008166146	0.008094146
0.1	1.000090000	1.010101010	0.010011010

Example 3:

Solve $y' = x^3.y^3 - xy, \text{ the exact solution of this problems is}$
$y = 1/(c.e^{x2} + 1 + x^2)$

For c = 0, $\text{the exact solution of this problems is } y = 1/(1+x^2), \text{ hence, } y(0) = 1$

Now, by taking the step h = 0.01
First by using Newton's interpolation, we have,

$$a_0 = 1 = y_0$$
$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = [\frac{dy}{dx}]_{0,1} = 0$$
$$y_1 = 1 + 0(0.01 - 0) = 1$$
$$a_2 = \frac{\frac{f(x_2) - (x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = -0.005$$
$$y_2 = 0.999999$$

Hence, forming quadratic interpolation of Lagrange, we have:

$$y_n = 0.005x^2 + 0.00005x + 1$$

Table 3 gives the approximation solution and the exact solution of example 3 with the error for x = 0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1

Table 3: Solution of $y' = x^3.y^3 - xy, \ \ y(0) = 1$

x	Combined Newton's Interpolation and Lagrange	Exact Values	Absolut error
0	1	1	0
0.01	1	0.999900010	0.000099990
0.02	0.999999000	0.999600160	0.000398840
0.03	0.999997000	0.999100809	0.000896191
0.04	0.999994000	0.998402556	0.001591444
0.05	0.999990000	0.997506234	0.002483766
0.06	0.999985000	0.996412914	0.003572086
0.07	0.999979000	0.995123893	0.004855107
0.08	0.999972000	0.993640700	0.004855107
0.09	0.999964000	0.991965083	0.007998917
0.1	0.999950500	0.990099010	0.009851490

CONCLUSIONS

In this work, we havebeen applied the combined Newton's interpolation and Lagrange method to solve nonlinear Bernoulli differential equationof first order, we find a good result compared to the exact solution through a number of examples showing that.

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