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Linear First Order ODE to Non-Linear ODE

Beletu Worku Beyene and Wasihun Assefa Woldeyes

Department of Mathematics, College of Natural and Computational Science,
Maddawalabu University, Bale Robe, Ethiopia

Abstract: Transformation of the first order linear ordinary differential equations by introducing a class of non-linear first order ODEs. The authors provide solutions for this transformed ordinary differential equation and also present some examples in to clarify applications of the results.

Key words: First order · Linear and non-linear ODEs

INTRODUCTION

Introducing Preliminary Results: An ordinary differential equation of firs order is an algebraic equation of $f\left(x, y, \frac{dy}{dx}\right) = 0,$, involving derivatives of some unknown function with respect to one independent variable [1, 2, 3, 4, 5]. The first order linear ordinary differential equation on the unknown y can be expressed in normal form as:

$$\frac{dy}{dx} + p(x)y = f(x). \tag{1.1}$$

where P(x) and f(x) are both continuous functions and P(x) is called the coefficient of the first order linear differential equation. The analytic solution of (1.1), can be expressed in the form;

$$y(x) = e^{-\int p(x)dx;}\left(\int e^{\int p(x)dx} f(x)dx + C\right); \forall C \in R. \tag{1.2}$$

In 1695 Jacob Bernoulli [6] first proposed the study of the following nonlinear first order ODE

$$\frac{dy}{dx} + P(x)y = f(x)y^{\alpha}, \alpha \in R \text{ is fixed.} \tag{1.3}$$

If $\alpha = 0$ and $\alpha = 1$, then (1.3) is linear, otherwise it is nonlinear. When $\alpha \neq 0$ and $\alpha \neq 1$, if we set $u = y^{1-\alpha}$, then the equation will be reduced to the linear form of (1.1), which will be easily solved. Thus,

$$\frac{du}{dx} + (1-\alpha)P(x)u = (1-\alpha)f(x). \tag{1.4}$$

Next, in section two we present the main result of this paper that is a general transformation of the first order linear ordinary differential equation (1.1) in to a subclass of non-linear first order ordinary differential equations.

Main Results

Theorem: Suppose $h(y)$ is a differentiable function .then a subclass of non linear ODEs;

$$\frac{d}{dx}\big(h(y)\big) + P(x)h(y) = f(x), \tag{2.1}$$

has a general implicit solutions, which satisfies

$$h(y) = e^{-\int p(x)dx}\left(\int e^{\int p(x)dx} f(x)dx + C\right); \forall C \in R.$$

Proof: Using [7], let $m = h(y)$, so that $\frac{dm}{dx} = \frac{d}{dx}\big(h(y)\big)$.

Then, we get

$$\frac{dm}{dx} + P(x)m = f(x). \tag{2.2}$$

Using (1.2), we obtain

$$h(y) = e^{-\int p(x)dx}\left(\int e^{\int p(x)dx} f(x)dx + C\right); \forall C \in R,$$

which is the required result.

Hence, we complete the proof of (2.1).

Corresponding Author: Beletu Worku Beyene, Department of Mathematics, College of Natural and Computational Science, Maddawalabu University, Bale Robe, Ethiopia.

CONCLUSION

In this paper besides having an important history background, it also has interesting applications. In particular, the ideas of this paper may be a base to obtain generalized version of other first order ordinary differential equations which are on the progress. Furthermore, the approach adopted in this paper was meant to reach not only researchers but also undergraduate students.

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