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A New Method to Investigate the Optimal Solutions of Game Theory Problems

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Abstract: Game theory problems are the most exigent phenomena in various fields of mathematical science, applied mathematics and engineering. In this article, we constitute the solutions of $m \times 2$ game problems by using (a) Alternative simplex method and (b) Duality which is appeared on the iterative procedure and verify graphically these methods by using graphical method. The worked-out results discerned that the suggested method is effective, robust, simple, straightforward and authoritative technique to deal with various kinds of game problems in the territory of game theory.

Key words: Alternative simplex method · Duality · Graphical method · Game problem · Optimum solutions
MSC 2010: 91A30; 91A35; 91A44; 91A80.

INTRODUCTION

Game theory is the science of strategy or at least the optimal decision making of independent and competing actors in a strategic setting. Game must be contemplation, in abroad sense, is the study of human conflict and co-operation within a competitive situation where somebody must win and somebody must lose. It is well-known that, the game theory problems phenomena deals with linear programming problem (LPP) which played an significant role in a wide range of inflictions: especially evolutionary biology, war, politics, psychology, economics, trade and commerce, traffic jam etc. Since the effective effectuation of game theory problems is the real world problems, so the last decades many researchers have been fascinating to attain the solution of game theory problems. The key pointers of game theory were mathematicians Jon von Neumann (1903-1957) and Jon Nash (1928-2015) expression is to solve the problems on the maximum losses. Khobragade *et al*. [1, 2] solve the game theory problems by KKL method. Vaidya *et al*. [3, 4] obtain the solution of game problem by using new approach. Ghadle [5], Baburao *et al*. [6] attain the solution of game theory problem by an alternative simplex method and so on (see for example [7-18]).

The structure of this paper arranged as following sections: In section 2 "Algorithm of proposed method", we briefly describe our suggested method. In section 3 "Numerical example by this method", as illustrations, we attain optimal solutions of game theory problems and verifying these methods by graphical method. Finally, in section 4 we represented our "Conclusions".

Algorithm of Proposed Method: In this section, the algorithm of our proposed method is described step by step as follows

Step 1: For $(m \times n)$ game when m is 3 or more and n is always 2. Player A has m approach of action and player B has n approach of action. We assume the probabilities of two players A and B are $p_1, p_2, \ldots \ldots \ldots, p_m$ and $q_1, q_2 \ldots \ldots \ldots, q_n$ to choose their pure strategies that is $s_A = (p_1, p_2, \ldots \ldots \ldots, p_m)$ and $s_B = (q_1, q_2, \ldots \ldots \ldots, q_n)$. Then we can write,

$$\sum_{i=1}^{m} p_i = 1 \text{ and } \sum_{j=1}^{n} q_j = 1$$

where $p_i, q_j \geq 0$ for all values of i, j.

Now consider the game can be represented linear programming problem is as below;

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Player A
Minimize $Z = 1/V$ or $x_1 + x_2 + ... + x_m$
Subject to:

$a_{11}\, x_1 + a_{12}\, x_2 + ... + a_{m1}\, x_m \geq 1$
$a_{21}\, x_1 + a_{22}\, x_2 + ... + a_{m2}\, x_m \geq 1$
……………………………….
$a_{m1}\, x_1 + a_{m2}\, x_2 + ... + a_{mn}\, x_m \geq 1$
$x_1, x_2, ..., x_m \geq 0$

Player B
Maximize $Z = 1/V$ or $y_1 + y_2 + ... + y_n$
Subject to:

$a_{11}\, y_1 + a_{12}\, y_2 + ... + a_{n1}\, y_n \leq 1$
$a_{21}\, y_1 + a_{22}\, y_2 + ... + a_{n2}\, y_n \leq 1$
………………………………….
$a_{m1}\, y_1 + a_{m2}\, y_2 + ... + a_{mn}\, y_n \leq 1$
$y_1, y_2, ..., y_n \geq 0$

The steps for the computational of the optimal solution for player B are as follows

Step 2: For converting inequalities into equation, check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply by (-1) to the analogous equation of constraints and introduce slack variable.

Step 3: Choose max $\Sigma x_{ij}\ x_{ij} \geq 0$ for entering vector and settle upon the greatest coefficient of decision variables. If greatest coefficient is unique, then element corresponding to this row and column turn into pivot (leading) element otherwise we can use tie breaking technique.

Step 4: For constructing simplex table we use usual simplex method and follow the next step.

Step 5: Disapprove corresponding row and column. Proceed to step 3 for residual elements and repeat the similar procedure until an optimal solution is obtained or there is intimation for unbounded solution.

Step 6: If all rows and columns are disapproved, then the current solution is an optimal solution.

Steps for the computation of the optimal solution for player A are as follows.

Step 7: Introduce surplus variable, the primal problem convert to dual and again we use from step 3 to step 6.

Step 8: Using the condition $X_0^T = C_B^T B^{-1}$ to attain the solution of the given problem.

Step 9: For verifying these methods, draw the graph by using graphical method and obtain the value of the game.

Numerical Example: Under this section, our proposed method has been put to use to investigate the new solutions of (3 × 2) *i.e.* (m × 2) type game problems in game theory.

Example 1: Solve the following (3 × 2) game by linear programming technique,

Player B

$$\text{Player A} \begin{pmatrix} 3 & -1 \\ -3 & 3 \\ -4 & -3 \end{pmatrix}$$

Solution:
For Player B
Maximize $Z = y_1 + y_2$ or $1/V$
Subject to

$3y_1 - y_2 \leq 3$
$-3y_1 + 3y_2 \leq 1$
$-4y_1 + 3y_2 \leq 1$
$y_1, y_2 \geq 0$

LPP is in standard form for player B:
Maximize $Z = y_1 + y_2$
Subject to

$3y_1 - y_2 + s_1 = 1$
$-3y_1 + 3y_2 + s_2 = 1$
$-4y_1 - 3y_2 + s_3 = 1$
$y_1, y_2, s_1, s_2, s_3 \geq 0$

Now we construct the simplex table:

Iteration 1

C_B	Basis	X_B	y_1	y_2	s_1	s_2	s_3	ratio
0	s_1	1	3*	-1	1	0	0	1/3
0	s_2	1	-3	3	0	1	0	-
0	s_3	1	-4	-3	0	0	1	-

Iteration 2

1	y_1	1/3	1	-1/3	1/3	0	0	×
0	s_2	2	0	2*	1	1	0	1
0	s_3	7/3	0	-13/3	4/3	0	1	-

Iteration 3

1	y_1	2/3	1	0	1/2	1/6	0	-
1	y_2	1	0	1	1/2	1/2	0	-
0	s_3	20/3	0	0	21/6	13/6	1	-

Since all rows and columns are ignored, hence an optimum solution has been reached.

Therefore optimum solution

$$y_1 = \frac{2}{3} \quad y_2 = 1$$

Max. $Z = \frac{5}{3}$

The optimal strategies for player B are;

$$q_1 = \frac{y_1}{Z} = \frac{2}{5} \qquad q_2 = \frac{y_z}{z} = \frac{3}{5}$$

And the value of the game $V = \frac{1}{z} = \frac{3}{5}$

For player A:
Minimize $Z = x_1 + x_2 + x_3$ or $1/V$
Subject to
$3x_1 - 3x_2 - 4x_3 \geq 1$
$-x_1 + 3x_2 - 3x_3 \geq 1$
$x_1, x_2, x_3 \geq 0$

Introducing surplus variable $u_1 \geq 0, u_2 \geq 0$
Min $Z = x_1 + x_2 + x_3 + 0u_1 + 0u_2$
Subject to
$3x_1 - 3x_2 - 4x_3 - u_1 = 1$
$-x_1 + 3x_2 - 3x_3 - u_2 = 1$

It's dual problem is
Max $Z = w_1 + w_2$
Subject to
$3w_1 - w_2 \leq 1$
$-3w_1 + 3w_2 \leq 1$
$-4w_1 - 3w_2 \leq 1$
$w_1, w_2 \geq 0$

Introducing slack variable $v_1, v_2, v_3 \geq 0$
We get
Max $Z_D = w_1 + w_2 + 0v_1 + 0v_2 + 0v_3$
Subject to
$3w_1 + w_2 + v_1 = 1$
$-3w_1 + 3w_2 + v_2 = 1$
$-4w_1 - 3w_2 + v_3 = 1$
$w_1, w_2, v_1, v_2, v_3 \geq 0$

The simplex table is:

Iteration 1

C_B	Basis	w_1	w_2	v_1	v_2	v_3	Const. b	Ratio
0	v_1	3*	-1	1	0	0	1	1/3
0	v_2	-3	3	0	1	0	1	-
0	v_3	-4	-3	0	0	1	1	-

Iteration 2

1	w_1	1	-1/3	1/3	0	0	1/3	×
0	v_2	0	2*	1	1	0	2	1
0	v_3	0	-13/3	4/3	0	1	7/3	-

Iteration 3

1	w_1	1	0	2/3	1/6	1/6	2/3	×
1	w_2	0	1	1/2	1/2	0	1	×
0	v_3	0	0	21/6	13/6	1	20/3	-

Since all rows and columns are ignored.

$$w_1 = \frac{2}{3} \quad w_2 = 1$$

Max, $Z^* = \frac{5}{3}$ = Min Z

$$C_B^T = (1 \;\; 1 \;\; 0) \;\; B^{-1} = \begin{pmatrix} 2/3 & 1/6 & 1/6 \\ 1/2 & 1/2 & 0 \\ 21/6 & 13/6 & 1 \end{pmatrix}$$

Hence primal solution, $X_0^T = C_B^T \;\; B^{-1} = \begin{pmatrix} \frac{7}{6} & \frac{2}{3} & \frac{1}{6} \end{pmatrix}$

$$x_1 = \frac{7}{6}, x_2 = \frac{2}{3}, x_3 = \frac{1}{6}$$

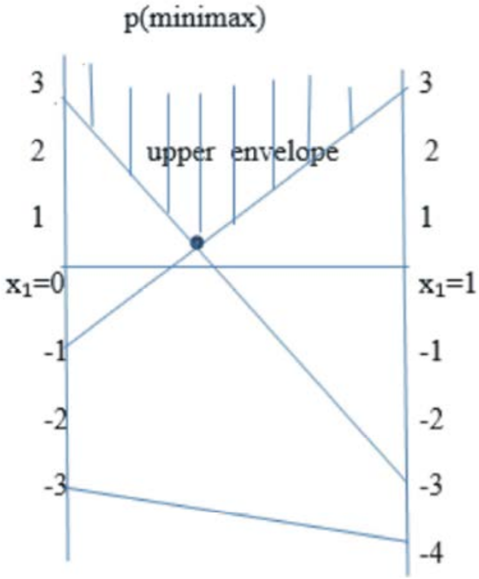
Value of the game $V = \frac{1}{z} = \frac{3}{5}$

Graphical method (verification part):
Given the game is:

B_1 B_2

$$\begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \begin{pmatrix} 3 & -1 \\ -3 & 3 \\ -4 & -3 \end{pmatrix}$$

The graph is;



Hence the matrix can be written as:

$$\begin{matrix} A_1 \\ A_2 \end{matrix}\begin{pmatrix} 3 & -1 \\ -3 & 3 \end{pmatrix}$$

The value of the game

$$V = \frac{(a_{11}a_{22} - a_{21}a_{12})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{3}{5}$$

Example 2: Solve the following 3 × 2 game problem

Player B

$$\text{Player A} \begin{pmatrix} 1 & 2 \\ 2 & 2 \\ 3 & 1 \end{pmatrix}$$

Solution:
For player B
Maximize $Z = y_1 + y_2$ or $1/V$
Subject to
$y_1 + 2y_2 \le 1$
$2y_1 + 2y_2 \le 1$
$3y_1 + y_2 \le 1$
$y_1, y_2 \ge 0$

LPP is in standard form for player B
Max $Z = y_1 + y_2$
Subject to
$y_1 + 2y_2 + s_1 = 1$
$2y_1 + 2y_2 + s_2 = 1$
$3y_1 + y_2 + s_3 = 1$
$y_1, y_2, s_1, s_2, s_3 \ge 0$

Now we construct the simplex table

Iteration 1

C_B	Basis	X_B	y_1	y_2	s_1	s_2	s_3	Ratio
0	s_1	1	1	2	1	0	0	1
0	s_2	1	2	2	0	1	0	½
0	s_3	1	3*	1	0	0	1	1/3

Iteration 2

0	s_1	2/3	0	5/3*	1	0	-1/3	2/5
0	s_2	1/3	0	4/3	0	1	-2/3	¼
1	y_1	1/3	1	1/3	0	0	1/3	×

Iteration 3

1	y_2	2/5	0	1	3/5	0	-1/5	×
0	s_2	-1/5	0	0	-4/5	1	-2/5	-
1	y_1	1/5	1	0	1/5	0	6/15	×

Since all the rows and columns are ignored.
Then from the table we get

$$y_1 = \frac{1}{5} \quad y_2 = \frac{2}{5}$$

Max $Z = \frac{3}{5}$

The optimal strategies for player B are;

$$q_1 = \frac{y_1}{Z} = \frac{1}{3}, \quad q_2 = \frac{y_z}{Z} = \frac{2}{3}$$

The value of the game $V = \frac{5}{3}$

For player A:
Minimize $Z = x_1 + x_2 + x_3$ or $1/V$
Subject to
$x_1 + 2x_2 + 3x_3 \ge 1$
$2x_1 + 2x_2 + x_3 \ge 1$
$x_1, x_2, x_3 \ge 0$

Introducing surplus variable $u_1, u_2 \ge 0$
Min $Z = x_1 + x_2 + x_3 + 0u_1 + 0u_2$
Subject to
$x_1 + 2x_2 + 3x_3 - u_1 = 1$
$2x_1 + 2x_2 + x_3 - u_2 = 1$

It's dual problem is
Max $Z = w_1 + w_2$
Subject to
$w_1 + 2w_2 \leq 1$
$2w_1 + 2w_2 \leq 1$
$3w_1 + w_2 \leq 1$
$w_1, w_2 \geq 0$

Introducing slack variable v_1, v_2, v_3 then
Max $Z_D = w_1 + w_2 + 0w_1 + 0v_2 + 0v_3$
Subject to
$w_1 + 2w_2 + v_1 = 1$
$2w_1 + 2w_2 + v_2 = 1$
$3w_1 + w_2 + v_3 = 1$
$w_1, w_2, v_1, v_2, v_3 \geq 0$

Now we construct the simplex table

Iteration 1

C^B	Basis	w_1	w_2	v_1	v_2	v_3	const b	Ratio
0	V_1	1	2	1	0	0	1	1
0	v_2	2	2	0	1	0	1	½
0	v_3	3*	1	0	0	1	1	1/3

Iteration 2

0	v_1	0	5/3*	1	0	-1/3	2/3	2/5
0	v_2	0	4/3	0	1	-2/3	1/3	¼
1	w_1	1	1/3	0	0	1/3	1/3	×

Iteration 3

1	w_2	0	1	3/5	0	-1/5	2/5	×
0	v_2	0	0	-4/5	1	-2/5	-1/5	-
1	w_1	1	0	-1/5	0	2/5	1/5	×

Since all the rows and columns are ignored.
From the table

$$w_1 = \frac{1}{5}, \quad w_2 = \frac{2}{3}$$

Max $Z^* = \frac{3}{5}$ = Min Z

Here $C_B = (1 \;\; 0 \;\; 1) \quad B^{-1} = \begin{pmatrix} 3/5 & 0 & -1/5 \\ -4/5 & 1 & -2/5 \\ -1/5 & 0 & 2/5 \end{pmatrix}$

Hence primal solution $X_0^T = C_B^T B^{-1}$

$$= \begin{pmatrix} \frac{2}{3} & 0 & \frac{1}{5} \end{pmatrix}$$

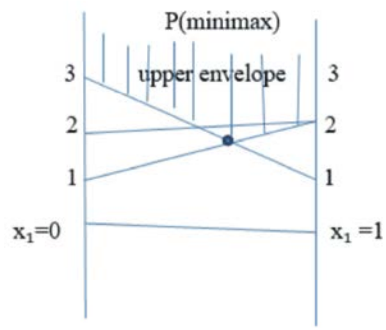
$$x_1 = \frac{2}{5} x_2 = 0, x_3 = \frac{1}{5}$$

Value of the game $V = \frac{5}{3}$

Graphical method (verification part):
Given the game

$$\begin{matrix} & B_1 & B_2 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} & \begin{pmatrix} 1 & 2 \\ 2 & 2 \\ 3 & 1 \end{pmatrix} \end{matrix}$$

The graph is;



Now the matrix can be written as;

$$\begin{matrix} & B_1 & B_2 \\ \begin{matrix} A_1 \\ A_2 \end{matrix} & \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \end{matrix}$$

The value of the game $V = \dfrac{(a_{11}a_{22} - a_{21}a_{12})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \dfrac{5}{3}$

CONCLUSIONS

The main purpose of this work was to constitute the optimal solutions for a large category of game theory problems by effectively implementing the alternative simplex method and duality. We also justify these methods graphically by applying graphical method which helps to obtain the accurate value of the game. The most prepossessing feature of this method is that it comprises very simple arithmetic and logical calculation, that's why it is very easy to understand and use. It is observed that this method alleviates number of iterations and enhances the optimum solutions in most of the cases. Efficiency of this method has also been tested by solving several numbers of game problems and it is noticed that the new suggested method yields comparatively a better result when compared to the solution using traditional existence methods. Moreover, it is also quite capable to achieve the goal for those who investigate to solve the game problems by using linear programming technique.

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