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Orthogonal Derivative for Higher Orders

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Abstract: We give an elementary deduction of the Rangarajan–Purushothaman's formula for the orthogonal derivative introduced by Cioranescu – (Haslam-Jones) – Lanczos.

Key words: Differentiation via integration · Generalized derivative

INTRODUCTION

We know the Cioranescu [1] – (Haslam-Jones) [2] – Lanczos [3] generalized derivative:

$$f'(x_0) = \lim_{\varepsilon \to 0} \frac{3}{2\varepsilon^{\varepsilon}} \int_{-\varepsilon}^{\varepsilon} f(v + x_0)\, v\, dv, \tag{1}$$

which represents differentiation via integration [4-8]. This orthogonal derivative for higher orders was studied by Rangarajan-Purushothaman [9-11] via Legendre polynomials and here we give an elementary deduction of their corresponding formula.

Generalized Derivative: We consider the integral:

$$\int_{-\varepsilon}^{\varepsilon} Q_n(\frac{t}{s}) f(x+t) dt = \varepsilon \int_{-1}^{1} Q_n(u) f(x + \varepsilon u) du, \tag{2}$$

but the Taylor expansion allows write:

$$f(x+\varepsilon u) = f(x) + \varepsilon f'(x) u + \ldots + \frac{\varepsilon^n}{n!} f^{(n)}(x) u^n + \frac{\varepsilon^{n+1}}{(n+1)!} f^{(n+1)}(x) u^{n+1} + \ldots,$$

then from (2):

$$\frac{1}{\varepsilon^{n+1} \int_{-s}^{s}} Q_n(\frac{t}{\varepsilon}) f(x+t) dt = \sum_{k=0}^{n-1} \frac{s^{k-1}}{k!} f^{(k)}(x) \int_{-1}^{1} u^k Q_n(u) du +$$
$$\frac{1}{n!} f^{(n)}(x) \int_{-1}^{1} u^n Q_n(u) du + \sum_{j=n+1}^{\infty} \frac{s^{j=n} j!}{} f^{(j)}(x) \int_{-s}^{s} u^j\, Q_n(u)\, du, \tag{3}$$

which suggesting to select $Q_n(u)$ with the property $\int_{-1}^{1} u^k \; Q_n(u)\, du = 0$ for $k \leq n-1$ and it is evident that the Legendre polynomials satisfy it [10, 12]:

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$$\int_{-1}^{1} u^k \; P_n(u)\; du = 0, \qquad k = 0, \ldots, n-1; \qquad \int_{-1}^{1} u^n P_n(u) du = \frac{2(n!)}{(2n+1)!!}. \tag{4}$$

Thus, from (3) and (4) we obtain the celebrated formula of Rangarajan–Purushothaman [9-11]:

$$f^{(n)}(x) = \lim_{\varepsilon \to 0} \frac{(2n+1)!!}{2\varepsilon^{n+1}} \int_{-s}^{s} P_n\left(\frac{t}{s}\right) f(x+t)\; dt, \tag{5}$$

which reproduces (1) for $n = 1$ because $P_1\left(\frac{t}{s}\right) = \frac{t}{s}$.

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