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Orthogonal Derivative

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Abstract: We use the Kempf et al process of integration by differentiation to obtain the Lanczos generalized derivative.

Key words: Orthogonal derivative · Integration by differentiation · Lanczos derivative

INTRODUCTION

Kempf *et al*. [1, 2] exhibit how to determine a definite integral via differentiation, in fact, they find the interesting expression:

$$\int_a^b F(x)dx = \lim_{t\to 0} F\left(\frac{d}{dt}\right)\left[\frac{a^{bt}-e^{at}}{t}\right]. \tag{1}$$

Here we give a simple proof of (1), and we employ it to obtain the Lanczos generalized derivative [3-9]. In fact:

$$\int_a^b x^n dx = \frac{1}{n+1}(b^{n+1}-a^{n+1}) = [\frac{d^n}{d^{tn}}\sum_{r=0}^{\infty}\frac{b^{r+1}-a^{r+1}}{(r+1)!}t^{\gamma}]t=0,$$

$$= \lim_{t\to 0}\left[\frac{d^n}{dt^n}\frac{1}{t}\sum_{k=1}^{\infty}\frac{b^k-a^k}{k!}t^k\right] = \lim_{t\to 0}\frac{d^n}{dt^n}\frac{e^{bt}-e^{at}}{t},$$

then:

$$\int_a^b F(x)dx = \sum_{n=0}^{\infty}\frac{F^{(n)}(0)}{n!}\int_a^b x^n dx = \lim\ _{t\to 0}\sum_{n=0}^{\infty}\frac{F^{(n)}(0)}{n!}\frac{d^n}{dt^n}\frac{e^{bt}-e^{at}}{t},$$

hence (1) is immediate.

Now we apply (1) for the case $F(x) = x\,f(x + x_0)$ with $a = -b = -\varepsilon$, therefore:

$$\frac{e^{bt}-e^{at}}{t} = 2\varepsilon\left(1+\frac{\varepsilon^2}{3!}t^2+\frac{\varepsilon^4}{5!}t^4+...\right),\ \ F(x) = f(x_0)x + f'(x_0)x^2\frac{1}{2!}f''(x_0)x^3+...,$$

thus from (1):

$$\int_{-\varepsilon}^{\varepsilon} xf(x+x_0)dx = 2\varepsilon\lim_{t\to 0}\left[f(x_0)\frac{d}{dt}+f'(x_0)\frac{d^2}{dt^2}+\frac{1}{2!}f''(x_0)\frac{d^3}{dt^3}+...\right]\left(1+\frac{\varepsilon^2}{3!}t^2+\frac{\varepsilon^4}{5!}t^4+...\right),$$

$$= 2\varepsilon^3\left[\frac{1}{3}f''(x_0)+\frac{\varepsilon^2}{5-3!}f'''(x_0)+\frac{\varepsilon^4}{7-5!}fv(x_0)+...\right],$$

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then it is evident the expression:

$$f'(x_0) = \lim_{\varepsilon \to 0} \frac{3}{2\varepsilon^3} \int_{-\varepsilon}^{\varepsilon} x f(x + x_0) dx, \qquad (2)$$

which coincides with the Lanczos generalized derivative [3-9].

The relation (1) represents integration by differentiation, but (2) expresses the inverse process, that is, differentiation by integration.

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