Computational and Applied Mathematical Sciences 2 (1): 01-02, 2017
ISSN 2222-1328

DOI: 10.5829/idosi.cams.2017.01.02

On the Post-Widder's Formula for the Laplace Inverse Transform

J. López-Bonilla, O. Puente-Navarrete and V.M. Salazar Del Moral

ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso,
Col. Lindavista CP 07738, CDMX, México

Abstract: We employ an expression for the Laplace transform, based in integration by differentiation, to deduce the Post-Widder's formula for the inversion of this transform.

Key words: Inversion of the Laplace transform · Post-Widder's formula

INTRODUCTION

If we know the Laplace transform [1]:

$$F(s)=\int_0^\infty e^{-st} f(t)dt, \tag{1}$$

the aim is to determine $f(t($; in [2] was obtained the following formula to do the integration in (1) via differentiation:

$$F(s)=f\left(-\frac{d}{ds}\right)\frac{1}{s}. \tag{2}$$

Here we show that (2) leads to the Post-Widder's expression [3-6] for the Laplace inverse transform.

Inversion of the Laplace Transform: In (2) we apply the operator $\frac{d^m}{ds^m}$ and $f(t)$ is written in its Taylor's series with respect to $t = b$.

$$F^{(m)}(s)=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}f^{(n)}(b)\sum_{k=0}^{n}\binom{n}{k}b^{n-k}(\frac{d}{ds})^{m+k}\frac{1}{s},$$

but $(\frac{d}{ds})^{m+k}\frac{1}{s}=\frac{(-1)^{m+k}(m+k)!}{s^{m+k+1}}$ and we take $b = t$ with $s=\frac{m}{t}$, hence:

$$F^{(m)}\left(\frac{m}{t}\right)=\frac{(-1)^m t^{m+1}}{m^{m+1}}m!\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}f^{(n)}(t)t^n\sum_{k=0}^{n}\binom{n}{k}\frac{(-1)^k(m+k)!}{m!m^k},$$

$$=(-1)^m(\frac{t}{m})^{m+1}m!\left[f(t)+tf^{(1)}(t)\frac{1}{m}+\frac{t^2}{2!}f^{(2)}(t)\frac{1}{m}\left(1+\frac{2}{m}\right)+\frac{t^3}{3!}f^{(3)}(t)\frac{1}{m^2}\left(5+\frac{6}{m}\right)+...\right],$$

therefore:

$$\lim_{m\to\infty}\frac{(-1)^m}{m!}(\frac{m}{t})^{m+1}F^{(m)}\left(\frac{m}{t}\right)=f(t), \tag{3}$$

Corresponding Author: J. López-Bonilla, ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso, Col. Lindavista CP 07738, CDMX, México.

which coincides with the celebrated formula of Post-Widder [3-6] for the inversion of the Laplace transform.

REFERENCES

1. Debnath, L. and D. Bhatta, 2007. Integral transforms and their applications, CRC Press, Fl-USA.
2. Kempf, A., D.M. Jackson and A.H. Morales, 2015. How to (path-) integrate by differentiating, J. Phys: Conf. Series, 626: 012015.
3. Post, E., 1930. Generalized differentiation, Trans. Amer. Math. Soc., 32(4): 723-781.
4. Widder, D.V., 1934. The inversion of the Laplace integral and the related moment problem, Trans. Amer. Math. Soc., 36(1): 107-200.
5. Widder, D.V., 1946. The Laplace transform, Princeton University Press, New Jersey.
6. Hernández-Galeana, A., J. López-Bonilla, B. Man Tuladhar and J. Rivera-Rebolledo, 2013.On the inverse Laplace transform, Kathmandu Univ. J. Sci. Eng. & Tech., 9(1): 161-164.