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Bernoulli Polynomials

J. López-Bonilla and R. López-Vázquez

ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso,
Col. Lindavista CP 07738, CDMX, México

Abstract: We use the generating function of Bernoulli polynomials to prove the Raabe's theorem and after we show that it gives the Deeba-Rodriguez's identity which implies the Namias expressions involving Bernoulli numbers.

Key words: Bernoulli numbers · Namias identities · Deeba-Rodriguez's identity · Bernoulli polynomials · Raabe's theorem

INTRODUCTION

The Bernoulli polynomials $B_N(x)$ are generated via the expression [1-5]:

$$\sum_{k=0}^{\infty} B_k(x)\frac{t^k}{k!} = \frac{te^{xt}}{e^t - 1}, \qquad (1)$$

therefore:

$$\sum_{k=0}^{\infty} B_k \frac{t^k}{k!} = \frac{t}{e^t - 1}, \qquad (2)$$

involving the Bernoulli numbers $B_n = B_n(0)$ [6-11].

Here we employ (1) and (2) to show the Raabe's theorem [2, 4, 12, 13]:

$$m^{n-1}\sum_{j=0}^{m-1} B_n(\frac{j}{m}) = B_n, \quad m \geq 1, \ n \geq 0, \qquad (3)$$

which allows prove the Deeba-Rodriguez's identity [14]:

$$\sum_{k=0}^{n} m^{k-1}\binom{n}{k} B_k \sum_{j=0}^{m-1} j^{n-k} = B_n, \qquad (4)$$

and for $m = 2,3$ we obtain the Namias relations [15, 16]:

$$\sum_{k=0}^{n-1} 2^k \binom{n}{k} B_k = 2(1-2^n)B_n, \qquad (5)$$

$$\sum_{k=0}^{n-1} 3^k (1+2^{n-k})\binom{n}{k} B_k = 3(1-3^n)B_n. \qquad (6)$$

Raabe's Theorem

From (1) for $x = \frac{j}{m}$:

$$\sum_{k=0}^{\infty}[\sum_{j=0}^{m-1} B_k(\frac{j}{n})]\frac{t^k}{k!} = \frac{t}{e^t - 1}\sum_{j=0}^{m-1}(e^{\frac{t}{m}})^j = \frac{t}{e^{\frac{t}{m}} - 1} = m\frac{t/m}{e^{t/m} - 1},$$

$$= m\sum_{k=0}^{\infty} B_k \frac{(\frac{t}{m})^k}{k!} = \sum_{k=0}^{\infty} \frac{1}{m^{k-1}} B_k \frac{t^k}{k!},$$

then (3) is immediate, q.e.d.

We know the property [2, 4]:

$$B_n(x) = \sum_{k=0}^{n}\binom{n}{k} B_k \ x^{n-k}, \qquad (7)$$

whose application into (3) allows write:

$$B_n = m^{n-1}\sum_{j=0}^{m-1}\sum_{k=0}^{n}\binom{n}{k} B_k \frac{j^{n-k}}{m^{n-k}},$$

which implies (4), q.e.d. Thus, the Deeba-Rodriguez's identity is deducible from the Raabe's theorem.

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Corresponding Author: J. López-Bonilla, ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso, Col. Lindavista CP 07738, CDMX, México.

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