

Energy Efficient Digital Filter to Suppress Awgn in Images

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Abstract: All traditional filters like Mean and Wiener filters control additive white Gaussian noise (AWGN) from an image very efficiently under low and moderate noise conditions. But, these filters alter and distort the edges unreasonably. Lee filter and non-local means (NL- Means) filter work well under very low noise condition. The method noise for these filters is low as compared to other spatial-domain filters. The computational complexity of simple mean filter is low whereas that of NL-Means filter is very high. Mean, Wiener, Lee and NL-means filters are incapable of suppressing the Gaussian noise quite efficiently under high noise conditions. Consequently, some energy efficient digital filters should be intended with the ultimate characteristics like suppress Gaussian noise very well beneath low, moderate and high noise conditions without distorting the edges and intricate details of an image, Having low method noise and Having less computational difficulty. In this paper a novel spatial-domain image filtering scheme is proposed. The Adaptive Window Wiener filter (AWWF) developed here is a very good scheme to suppress Gaussian noise under moderate noise conditions. It also retains the edges and textures of an image very well.

Key words: Image denoising • Gaussian noise • Digital filters

INTRODUCTION

Digital Image Processing usually refers to the processing of a 2-dimensional (2-D) picture signal by a digital hardware. The 2-D image signal might be a photographic image, text image, graphic image (including synthetic image), biomedical image (X-ray, ultrasound, etc.), satellite image, etc. In a broader context, it implies processing of any 2-D signal using a dedicated hardware, e.g. an application specific integrated circuit (ASIC) or using a general-purpose computer implementing some algorithms developed for the purpose. An image is a 2-D function (signal), $f(x, y)$, where x, y are the spatial (plane) coordinates. The magnitude of f at any pair of coordinates (a, b) is the intensity or gray level of the image at that point. In a digital image, a, b and the magnitude of f are all finite and discrete quantities. Each element of this matrix (2-D array) is called a picture element or pixel. Image processing refers to some algorithms for processing a 2-D image signal, i.e. To operate on the pixels directly (spatial-domain processing) or indirectly (transform-domain processing). Such a processing may yield another image or some attributes of the input image at the output.

It is a hard task to distinguish between the domains of image processing and any other related areas such as computer vision. Though, essentially not correct, image processing may be defined as a process where both input and output are images. At the high level of processing and after some preliminary processing, it is very common to perform some analysis, judgment or decision making or Perform some mechanical operation (robot motion). These areas are the domains of artificial intelligence (AI), computer vision, robotics, etc. Digital image processing has a broad spectrum of applications, such as digital television, photo-phone, remote sensing, image transmission and storage for business applications, medical processing, radar, sonar and acoustic image processing, robotics and computer aided manufacturing (CAM) and automated quality control in industries. Most of the image-processing functions are implemented in software. A significant amount of basic image processing software is obtained commercially.

Literature Review: Image processing may be performed in the spatial-domain or in a transform-domain. To perform a meaningful and useful task, a suitable transform, e.g. discrete Fourier transform (DFT) [1],

discrete cosine transform (DCT) [2-3], discrete Hartley transform (DHT), discrete wavelet transform (DWT) [4-11], etc., may be employed. Depending on the application, a suitable transform is used.

Image enhancement techniques are used to highlight certain features of interest in an image. Two important examples of image enhancement are: (i) increasing the contrast and (ii) changing the brightness level of an image so that the image looks better. It is a subjective area of image processing. On the other hand, image restoration is very much objective. The restoration techniques are based on mathematical and statistical models of image degradation. Denoising (filtering) [12, 13] and deblurring [14, 15] tasks come under this category. Image processing is characterized by specific solutions; hence a technique that works well in one area may totally be inadequate in another. The actual solution to a specific problem still requires a significant research and development. Image restoration [16, 17] is one of the prime areas of image processing and its objective is to recover the images from degraded observations. The techniques involved in image restoration are oriented towards modeling the degradations and then applying an inverse procedure to obtain an approximation of the original image. Hence, it may be treated as a deconvolution operation. Depending on applications, there are various types of imaging systems. X-ray, Gamma ray, ultraviolet and ultrasonic imaging systems are used in biomedical instrumentation. In astronomy, the ultraviolet, infrared and radio imaging systems are used. Sonic imaging is performed for geological exploration. Microwave imaging is employed for radar applications. But, the most commonly known imaging systems are visible light imaging. Such systems are employed for applications like remote sensing, microscopy, measurements, consumer electronics, entertainment electronics, etc.

The images acquired by optical, electro-optical or electronic means are likely to be degraded by the sensing environment. The degradation may be in the form of sensor noise, blur due to camera misfocus, relative object camera motion, random atmospheric turbulence and so on. The noise in an image may be due to a noisy channel if the image is transmitted through a medium. It may also be due to electronic noise associated with a storage-retrieval system. Noise in an image is a very common problem. An image gets corrupted with different types of noise during the processes of acquisition, transmission/reception and storage/ retrieval. Noise may be classified as substitutive noise (impulsive noise: e.g., salt & pepper

noise, random-valued impulse noise, etc.) and additive noise (e.g., additive white Gaussian noise). The impulse noise of low and moderate noise densities can be removed easily by simple denoising schemes available in the literature. The simple median filter [18, 19] works very nicely for suppressing impulse noise of low density. However, now-a-days, many denoising schemes [18-31] are proposed which are efficient in suppressing impulse noise of moderate and high noise densities. In many occasions, noise in digital images is found to be additive in nature with uniform power in the whole bandwidth and with Gaussian probability distribution. Such a noise is referred to as Additive White Gaussian Noise (AWGN). It is difficult to suppress AWGN since it corrupts almost all pixels in an image. The arithmetic mean filter, commonly known as Mean filter can be employed to suppress AWGN but it introduces a blurring effect. Efficient suppression of noise in an image is a very important issue.

Denoising finds extensive applications in many fields of image processing. Image denoising is usually required to be performed before display or further processing like texture analysis [32-38], object recognition [39-42], image segmentation [43-45], etc. Conventional techniques of image denoising using linear and nonlinear techniques have already been reported and sufficient literature is available in this area. Recently, various nonlinear and adaptive filters have been suggested for the purpose. The objectives of these schemes are to reduce noise as well as to retain the edges and fine details of the original image in the restored image as much as possible. However, both the objectives conflict each other and the reported schemes are not able to perform satisfactorily in both aspects. Hence, still various research workers are actively engaged in developing better filtering schemes using latest signal processing techniques. In this research work, efforts have been made in developing some novel filters to suppress AWGN quite efficiently.

Proposed Work: As the edges in an image are specially taken care of in this work, the proposed filter is found to be good in edge preservation. It is a fact that a noise-free sample can be estimated with better accuracy from a large number of noisy samples. Similarly, in order to estimate a true pixel in a particular region from a noisy 2-D image, a large number of pixels in the neighborhood surrounding the noisy pixel are required. In other words, a larger-sized window, surrounding the pixel to be filtered, can be considered for better estimation.

In the proposed adaptive window Wiener filter, the window is made adaptive i.e. the size of the window varies from region to region. In a flat or homogenous region, the size of the window taken is large enough. The size of window is small in the regions containing edges. The problem here is to distinguish the edge and smooth regions. The edges and smooth regions are easily distinguished if the edge extraction operators are used. Many edge extraction operators such as Sobel, Canny, Roberts, Prewitt etc. are proposed in [46-51]. But, finding the true edges in a noisy environment is not so easy. The edge extraction operator works well on noise free images. So, it is important to make the noisy image a little bit blurred before edge extraction.

In the proposed filter, the mean filter of window size 5×5 is used when the noise level is low and moderate to get the blurred version of the noisy image, whereas a 7×7 window is taken for high-noise AWGN. The Sobel operator is then used on the resulted blurred image to find the edges. A small amount of noise still remains in different regions even after passing the noisy image through the mean filter. The Sobel operator is less sensitive to isolated high intensity point variations since the local averaging over sets of three pixels tends to reduce this. In effect, it is a “little bar” detector, more readily than a point detector. Secondly, it gives an estimate of edge direction as well as edge magnitude at a point which is more informative. Also, the Sobel operator is still relatively easy to implement in hardware form, most obviously by a pipeline approach.

The Proposed Algorithm: The proposed algorithm is given below.

Step 1: The noisy image is passed through a mean filter, as shown in Fig. 1, to get a blurred version of the image.

Step 2: Edge operator (Sobel operator) is applied on the blurred image, obtained in Step-1 to get the edge image. The pixels belong to smooth region and edge region are identified as ‘p’ and ‘q’, respectively. This operation is shown in Fig. 2.

Step 3: Adaptive window Wiener filter is applied on the noisy image. The size of the window is varied with the following concepts.

A-i) If the center pixel is an **edge** pixel, then the size of the window is **small**;

A-ii) If the center pixel belongs to **smooth region**, the size of the window is **large**.

B-i) If the noise power is **low** ($\sigma_n \leq 10$), then the size of the window is **small**;

B-ii) If the noise power is **moderate** ($10 < \sigma_n \leq 30$), then the size of the window is.

Medium: B-iii) If the noise power is **high** ($30 < \sigma_n \leq 50$), then the size of the window is **large**.

This adaptive filtering concept is depicted in Fig.3.

Step 4: All the filtered pixels are united together to obtain the denoised (filtered) image as shown in Fig. 4.

The exact window sizes taken for various conditions are presented in the next sub-section.

The window selection is based on the level of noise present in the noisy image. If the noise level is unknown, a robust median estimator [55] may be applied to predict the level of noise.

When the noise level is **low** ($\sigma_n \leq 10$),

i) a 3×3 window is selected for filtering the noisy pixels belonging to homogenous regions;

ii) the pixel is unaltered if the noisy pixels belong to edges.

When the noise level is **moderate** ($10 < \sigma_n \leq 30$),

i) a 5×5 window is chosen for filtration of noisy pixels of flat regions;

ii) The window size is 3×3 if the noisy pixels to be filtered are identified as edge pixels.

When the noise level is **high** ($30 < \sigma_n \leq 50$),

i) a 7×7 window is used for filtration of noisy pixels of flat regions;

ii) If the noisy pixels to be filtered are identified as edge pixels the window size used is 5×5 .

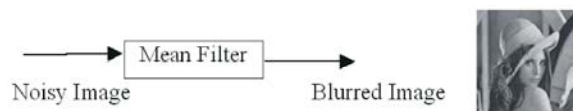


Fig. 1: Blurred image from mean filter (Step 1)

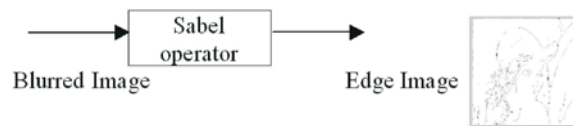


Fig. 2: Edge image from Blurred image (Step 2)

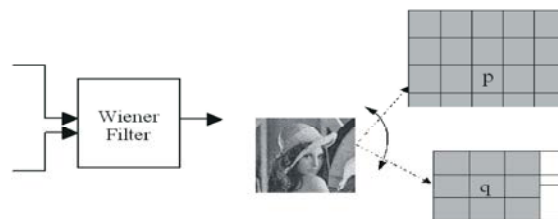


Fig. 3: Filtering operation of the proposed filter (Step 3)

The above Figure 3 shows the filtering operation of the proposed filter. Here 'p' and 'q' are the pixels in which 'p' belongs to the smooth region and 'q' belongs to the edge region of the image taken for experiment. The pixels 'p1' and 'q1' are the filtered pixels corresponding to the pixels 'p' and 'q'. $p1 \cup q1 \cup \dots$ means the filtered pixels image.



Fig. 4: Filtered image of the proposed image Lena

RESULTS AND DISCUSSIONS

Simulation is done in the MATLAB application for the proposed filter for the test image of Lena which was added with the AWGN of different standard deviation values and the results were taken. From the table of results we get the performance of the different traditional filters as well as our proposed filter. Here we have chosen four performance measure characteristics. They are peak noise to signal ratio (PSNR), Root mean square error (RMSE), Execution time and Universal Quality index (UQI). From all the performance measures our proposed filter was observed as the best filter for the moderate noise conditions.

Table 1: Filtering performance of various filters, in terms of PSNR (dB), operated on Lena image under various noise conditions (σ_n varies from 5 to 50)

		Peak Signal to Noise Ratio (PSNR)									
		Standard Deviation of AWGN									
Sl.No	Denoising Filters	5	10	15	20	25	30	35	40	45	50
1	Mean	29.77	29.59	29.31	28.92	28.48	28.05	27.48	26.96	26.49	25.93
2	ATM	29.77	29.59	29.32	28.93	28.50	28.07	27.52	27.03	26.47	25.86
3	Wiener	36.57	32.94	31.47	30.19	29.00	27.96	26.99	26.13	25.43	24.69
4	Lee	37.55	33.47	31.29	29.74	28.66	27.70	27.07	26.41	25.78	25.34
5	Bilateral	32.76	29.14	28.88	28.55	28.13	27.72	27.27	26.71	26.22	25.73
6	Proposed	34.88	33.36	31.78	30.56	29.26	28.13	27.41	26.88	26.01	25.85

Table 2: Filtering performance various filters, in terms of RMSE, operated on Lena image under various noise conditions (σ_n varies from 5 to 50)

		Root Mean Square Error (RMSE)									
		Standard Deviation of AWGN									
Sl.No	Denoising Filters	5	10	15	20	25	30	35	40	45	50
1	Mean	8.28	8.45	8.73	9.13	9.60	10.09	10.77	11.44	12.07	12.88
2	ATM	8.28	8.44	8.69	9.10	9.56	10.04	10.71	11.34	12.08	12.98
3	Wiener	3.78	5.73	6.78	7.87	9.02	10.17	11.39	12.57	13.64	14.84
4	Lee	3.38	5.40	6.93	8.28	9.38	10.50	11.29	12.18	13.10	13.77
5	Bilateral	5.86	8.89	9.15	9.51	9.99	10.48	11.04	11.75	12.44	12.95
6	Proposed	4.59	5.45	6.55	7.54	8.74	9.99	10.86	11.52	12.75	12.95

Table 3: Filtering performance of various filters, in terms of UQI, operated on a Lena image under various noise conditions (σ_n varies from 5 to 50)

		Universal Quality Index (UQI)									
		Standard Deviation of AWGN									
Sl.No	Denoising Filters	5	10	15	20	25	30	35	40	45	50
1	Mean	0.985	0.9838	0.9827	0.9811	0.9791	0.9769	0.9736	0.9700	0.9665	0.9613
2	ATM	0.9878	0.9838	0.9828	0.9812	0.9792	0.9772	0.9740	0.9710	0.9671	0.9621
3	Wiener	0.9958	0.9926	0.9897	0.9861	0.9818	0.9768	0.9709	0.9646	0.9581	0.9503
4	Lee	0.9955	0.9936	0.9893	0.9847	0.9805	0.9755	0.9715	0.9666	0.9610	0.9566
5	Bilateral	0.9616	0.9828	0.9816	0.9802	0.9781	0.9757	0.9728	0.9686	0.9646	0.9598
6	Proposed	0.9953	0.9934	0.9898	0.9986	0.9820	0.9765	0.9721	0.9690	0.9620	0.9568

Table 4: Execution time (Seconds) taken by various filters for Lena image

Sl.No	Denoising filters	Execution time in three different Systems		
		S1	S2	S3
1	Mean	3.23	7.12	17.49
2	ATM	8.09	17.23	30.56
3	Wiener	8.07	15.98	28.76
4	Lee	8.87	14.46	35.45
5	Bilateral	5.83	13.53	35.20
6	Proposed	3.52	7.29	19.30

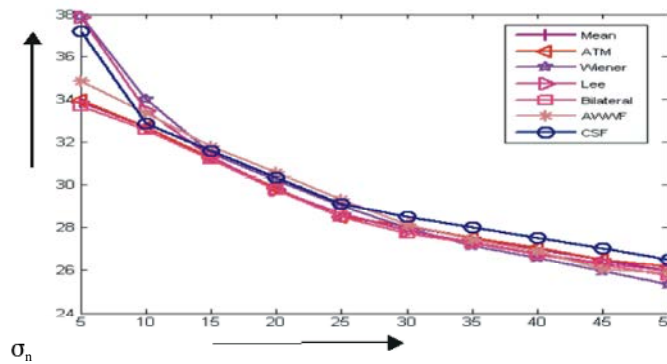


Fig. 5: Filtering performance of various filters, in terms of PSNR (dB), operated on Lena image under various noise conditions (σ_n varies from 5 to 50)

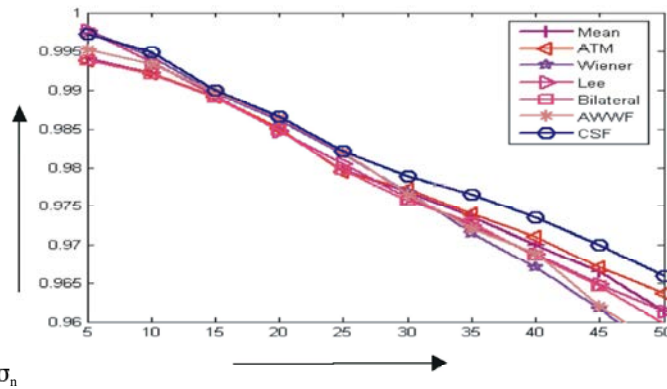


Fig. 6: Filtering performance various filters, in terms of RMSE, operated on Lena image under various Noise conditions (σ_n varies from 5 to 50)

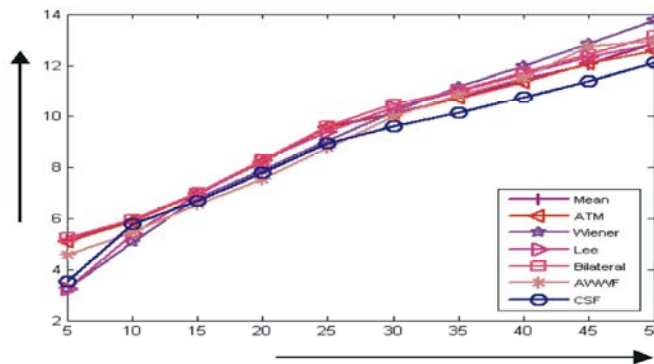


Fig. 7: Filtering performance of various filters, in terms of UQI, operated on a Lena image under various noise conditions (σ_n varies from 5 to 50)



Fig. 8: Performance of Various Filters for Lena Image with AWGN $\sigma_n = 15$ (a) Original image (b) Noisy image (c) – (h): Results of various filtering schemes (c) Mean (d) ATM (e) Wiener (f) Lee (g) Bilateral (h) Proposed filter

CONCLUSION

It is observed that the proposed filter is very capable in suppressing AWGN from images. It is seen that the proposed filter performs well when compared with the other existing transform domain, spatial-domain filters in suppressing additive noise under moderate noise conditions. Under moderate noise conditions the values of PSNR, UQI values are relatively high when compared with other filters. It may be concluded that the proposed filter is best spatial-domain filter in suppressing additive noise under moderate noise conditions because it shows the better performance under less noise conditions also. It conserves the edges and fine details very well, as experimental compare to other filters. The filter performance is low under high noise conditions as per the observations from the experiments. In the view of computational complexity proposed filter is observed to possess moderate computational complexity. And in real time applications in the modern digital world there may be possibility of moderate noise images are more, so the proposed filter will be a very useful one to all. The proposed filter may be used in many real-time applications. Because the filter has got relatively moderate computational complexity as compared to other efficient spatial-domain filters. It suppresses additive white Gaussian noise very effectively from images. This filter maintains the detailed information very well as compared to other spatial-domain filters. As the method noise is moderately less it causes little deformation to the original image. Thus, the proposed filtering method is to be very good spatial-domain image denoising filter under moderate noise conditions.

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