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Performance Modeling and Mechanical Behaviour of Blood Vessel in the Presence of Magnetic Effects

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Abstract: In this investigation, a mathematical modeling of the arterial blood flow with magnetic effects which have been derived from the Navier-Stokes equations and some assumptions. The governing equations are solved by standard finite difference method. Even though the model does not include viscoelastic effect, the results obtained is considered valid since we are able to make a conclusion that from this model, we observe that the size of the blood vessel does influence the blood flow. A little change on the cross-sectional value makes vast change on the blood flow rate. The result obtained is very sensitive to the values of the initial conditions and this helps to explain the condition of hypertension.

Key words: Mathematical modeling · Arterial flow · Standard finite difference method · Magnetic field

INTRODUCTION

Many cardiovascular diseases, particularly atherosclerosis, have been found to be responsible for deaths in both developed and developing countries. The study of blood flow through a stenotic artery is very important because the nature of blood movement and mechanical behaviour of vessel walls are causes of many cardiovascular diseases. Blood flow is a study of measuring the blood pressure and finding the flow through the blood vessel. This study is important for human health. Most of the researchers are study the blood flow in the arteries and veins. One of the motivations to study the blood flow has to understand the conditions that may contribute to high blood pressure. Past studies indicated that one of the reasons a person having hypertension is when the blood vessel becomes narrow. Human body experiences magnetic fields of moderate to high intensity in many situations of day to day life. In recent times, many medical diagnostic devices especially those used in diagnosing cardiovascular disease make use of magnetic fields. It is known from the magneto-hydrodynamics that when a stationary, transverse magnetic field is applied externally to a moving electrically conducting fluid, electrical currents are induced in the fluid. The interaction between these induced currents and the applied magnetic field produces a body forces (known as the Lorentz force) which tends to retard the movement of blood [1].

Jauchem [2] studied the effects of low frequency electromagnetic energy on the peripheral blood circulation and concluded that low frequency, low intensity magnetic field increased blood flow in the great majority. Hypertension is one flight mechanics presumably exposed to radio frequency radiation at a level of 38 times above the permissible exposure limit. There are a number of emerging technologies involving the use of Electro-Magnetic frequencies including new types of cellular telephones, magnetically levitated trains and superconducting magnetic energy storage. The possible effects of these particular Electro-Magnetic frequencies on health have not been studied directly.

Magnetic Resonance Imaging is a tool to study the blood flow phenomena in which magnetic field of large intensity is applied on the body. Although existing guidelines on Magnetic Resonance Imaging magnetic fields have been adequate to preclude any known biological problem to date, the Magnetic Resonance Imaging industry would like to have greater flexibility in developing future designs. Mathematical model used a model to simulate exposure of the human torso to switched magnetic field that would be present during

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Magnetic Resonance Imaging [2]. Kuipers *et al*. [3] investigated the influence of static magnetic fields on cardiovascular and sympathetic function at rest and during physiological stress and also investigated the influence of static magnetic field on pain perception during noxious stimuli.

The biological effects of Magnetic fields have often been linked to nitric oxide (NO), which is responsible for the changes in vessel diameter following magnetic field exposure. Recently magnetic fields have been shown to have positive effects on numerous human systems. For instance, it is documented that magnetic field exposure can provide analgesia, decrease healing time for fractures, increase the speed of nerve regeneration, act as a treatment for depression and provide other medical benefits [4].

Mathematical Model: I have adopted Yang, Zhang and Asada's [5] local arterial flow model. The application of magneto- hydrodynamics in physiological problems is of growing interest. The flow of blood can be controlled by applying sufficient quantity of magnetic field. This includes the assumptions that the arterial vessel is rectilinear, deformable, thick shell of isotropic, incompressible material with circular section and without longitudinal movements. Meanwhile blood is considered as an incompressible Newtonian fluid and the flow is axially symmetric. The model approach is to use the two-dimensional Navier-Stokes equations and continuity equation for a Newtonian and incompressible fluid in cylindrical coordinate (r, z, t):

$$\frac{\partial u}{\partial t}+w\frac{\partial u}{\partial r}+u\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\sigma}{\rho}B^2 u \quad (1)$$

$$\frac{\partial w}{\partial t}+w\frac{\partial w}{\partial r}+u\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}+\frac{w}{r^2}\right) \quad (2)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rw)+\frac{\partial u}{\partial z}=0 \quad (3)$$

Where P is the pressure and ρ is the density v is the kinematic viscosity, u (r, z, t) is the components of velocity in axial (z) directions, w (r, z, t) is the components of velocity in radial (r) directions, B is the magnetic field parameter.

For convenience we define a new variable, which is the radial coordinate, η:

$$\eta=\frac{r}{R(z,t)} \quad (4)$$

Where R(z, t) denotes the inner radius of the blood vessel. Assuming that P is independent of the radial coordinate, η, then the pressure P is uniform within the cross section (P = P(z, t)).

Hence

$$\frac{\partial^2 u}{\partial z^2}\le 1;\ \frac{\partial^2 w}{\partial z^2}\le 1;\ \frac{\partial P}{\partial r}\le 1;$$

Using simple algebra to change the variable such as

$$\frac{\partial u(r,z,t)}{\partial t}=\frac{\partial u(\eta,t)}{\partial t}\frac{\partial \eta}{\partial t}+\frac{\partial u(\eta,t)}{\partial t}\frac{\partial \eta}{\partial t},$$
$$=-\frac{\eta}{R}\frac{\partial u(\eta,t)}{\partial t}\frac{\partial R}{\partial t}+\frac{\partial u(\eta,t)}{\partial t},$$

equations (1), (2) and (3) can be written in the new coordinate (η, z, t) as:

$$\frac{\partial u}{\partial t}+\frac{1}{R}(\eta(u\frac{\partial u}{\partial z}+\frac{\partial R}{\partial t})-w)\frac{\partial u}{\partial \eta}+u\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\frac{\nu}{R^2}(\frac{\partial^2 u}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial u}{\partial \eta})-Mu \quad (5)$$

$$\frac{\partial w}{\partial t}+\frac{1}{R}(\eta(u\frac{\partial u}{\partial z}+\frac{\partial R}{\partial t})-w)\frac{\partial w}{\partial \eta}+u\frac{\partial w}{\partial z}=\frac{\nu}{R^2}(\frac{\partial^2 w}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial w}{\partial \eta}+\frac{w}{\eta^2}) \quad (6)$$

$$\frac{1}{R}\frac{\partial w}{\partial \eta}+\frac{w}{\eta R}+\frac{\partial u}{\partial z}-\frac{\eta}{R}\frac{\partial u}{\partial \eta}\frac{\partial R}{\partial z}=0 \quad (7)$$

Where M is the Hartmann number. The system of equations above is a hemodynamic type of model. [5] stated that according to Belardinelli and Cavalcanti in 1991, the velocity profile in the axial direction, u(η,z,t), is assumed to have the expression in the polynomial form below:

$$u(\eta, z, t) = \sum_{k=1}^{N} q_k (\eta^{2k} - 1) \tag{8}$$

While the velocity profile in the radial direction is

$$w(\eta, z, t) = \frac{\partial R}{\partial z} w\eta + \frac{\partial R}{\partial t}\eta - \frac{\partial R}{\partial t}\frac{1}{N}\sum_{k=1}^{N} q_k (\eta^{2k} - 1) \tag{9}$$

[5] choose N = 1 to simplify (8) and (9), so that

$$u(\eta, z, t) = q(z,t)(\eta^2 - 1) \tag{10}$$

$$w(\eta, z, t) = \frac{\partial R}{\partial z} w\eta + \frac{\partial R}{\partial t}\eta - \frac{\partial R}{\partial t}\eta(\eta^2 - 1) \tag{11}$$

Then, when equations (10) and (11) are substituted into equations (5) and (7), we get the dynamic equations of q(z, t) and R(z, t), which are:

$$\frac{\partial Q}{\partial t} - \frac{3Q}{S}\frac{\partial S}{\partial t} - \frac{2Q^2}{S}\frac{\partial S}{\partial z} + \frac{4\pi\nu}{S}Q + \frac{S}{2\rho}\frac{\partial P}{\partial z} + Mu = 0 \tag{12}$$

$$2R\frac{\partial R}{\partial t} + \frac{R^2}{2}\frac{\partial q}{\partial z} + q\frac{\partial R}{\partial z} = 0 \tag{13}$$

Now, the cross-sectional area S(z, t) and blood flow Q(z, t) are defined as

$$S = \pi R^2, \; Q = \iint_S u d\eta = \frac{1}{2}\pi R^2 q$$

We can use these definitions to express equations (12) and (13) in terms of Q(z,t) and S(z,t):

$$\frac{\partial Q_i}{\partial t} + \frac{4\pi\nu}{S_0}Q_i + \frac{S_0}{2\rho}\frac{\partial P}{\partial z} + \frac{S_i}{2\rho}\frac{\partial P}{\partial z} = 0 \tag{14}$$

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial z} = 0 \tag{15}$$

Numerical Method: The solutions for the cross-sectional area of the artery and its corresponding blood flow can now be obtained by solving the governing equations (14) and (15). The system of equations (14)-(15) is nonlinear partial differential equations. Finite difference method is used to solve such problem. First, the equations have been discretized using the following difference formula in first order accuracy:

$$\frac{\partial Q_i}{\partial z} = \frac{Q_i - Q_{i-1}}{\Delta z} \text{ and } \frac{\partial S_i}{\partial z} = \frac{S_i - S_{i-1}}{\Delta z}$$

Where Δz=L/(N-1), so that the equations becomes difference equations:

$$\frac{\partial Q_i}{\partial t} - \frac{3Q_i}{S_i}\frac{Q_i - Q_{i-1}}{\Delta z} - \frac{2Q_i^2}{S_i}\frac{S_i - S_{i-1}}{\Delta z} + \frac{4\pi\nu}{S_i}Q_i + \frac{S_i}{2\rho}\frac{\partial P}{\partial z} + Mu_i = 0 \tag{16}$$

$$\frac{\partial S_i}{\partial t} = -\frac{Q_i - Q_{i-1}}{\Delta z} \tag{17}$$

Where i= 1, 2, K N. Here, the pressure gradient $\frac{\partial P}{\partial z}$ is kept constant and the value is prescribed.

The discretization of the artery model is shown in Figure 1 below:

Since we are considering local arterial segment, we can simplify the governing equations by linearizing equation (16):

$$\frac{\partial Q_i}{\partial t}+\frac{4\pi v}{S_0}Q_i+\frac{S_0}{2\rho}\frac{\partial P}{\partial z}+\frac{S_i}{2\rho}\frac{\partial P}{\partial z}=0 \qquad (18)$$

We notice that the difference equations (17) – (18) can be written in the form, $\frac{\partial y}{\partial t}=f(y)$ where

$$y=(Q_1,Q_2,Q_3.......Q_N,S_1,S_2.........S_N)$$

and f(y)=
$$\begin{bmatrix} -\frac{4\pi v}{S_0}y(1)+\frac{S_0}{2\rho}\frac{\partial P}{\partial z}+\frac{y(N+1)}{2\rho}\frac{\partial P}{\partial z} \\ -\frac{4\pi v}{S_0}y(2)+\frac{S_0}{2\rho}\frac{\partial P}{\partial z}+\frac{y(N+2)}{2\rho}\frac{\partial P}{\partial z}-M \\ -\frac{y(1)-Q_0}{\Delta z} \\ -\frac{y(1)-Q_0}{\Delta z} \\ .... \\ .... \\ -\frac{y(N-1)-y(N-2)}{\Delta z} \\ -\frac{y(N)-y(N-1)}{\Delta z} \end{bmatrix}$$

The simplest and fast way to solve such problem is by using Mat Lab built-in function ODE45, which is based on Runge-Kutta method. The values of parameters that are required are the initial value of the blood flow, Q_0, the initial cross-sectional area, S0, the axial pressure gradient $\frac{\partial P}{\partial z}$, the kinematic viscosity v and density ρ for blood. The required values in normal condition can be obtained from past works in the field such as:

Initial value of Q and Q_0 = 1 to 5.4 liter/minute [6]
Initial value of S and S_0 = 1.5 to 2.0 cm^3 [7] $\frac{\partial P}{\partial z}$ = 100 to 40 mmHg [8], v = 0.035 cm^2/s [9] ρ = 1.05 g/cm^3 [9].

RESULT AND DISCUSSIONS

In order to simulate how the cross-sectional area of the artery affects the blood flow within the artery, the values of parameters mentioned in the previous section are chosen to be: ρ = 1.05 g/cm^3, M=1.0, ν = 0.035 cm^2/sec, Q_0 = 16.7 cm^3/sec and S_0 = 1.5 cm^2. For simplicity, we chose the length of the artery model, L = 15 cm and the number of nodes of the system, N = 3. Since we consider arteries in a diastole condition only, the chosen time span is 0.2 seconds.

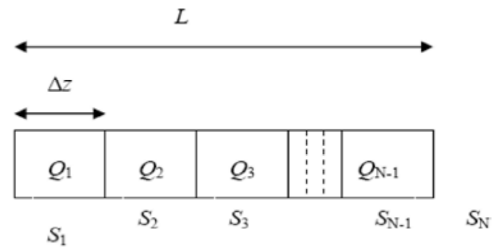

Fig. 1: Discretization of the arterial flow model

Figure 2 shows the blood flow rate and cross-sectional areas for each node. It is observed that the results for Q1, Q2 and Q3 are almost the same as depicted in Figure 2(a). Similarly, the values of S1, S2 and S3 in Figure 2(b) are very close and it is almost a constant. This shows that the values of the blood flow rate and the cross-sectional area are almost the same through out in the small section of arteries. This could be due to the absence of viscoelastic effect in the model. Now since there is not much difference in the blood flow rate between the sections, we will consider only one section which is S2 to make the comparison of the different values of the cross-sectional area. As we can see, the value for the blood flow is decreasing from its initial value. This is also the case for the cross-sectional area, although it decreases in smaller range as shown in Figure 3.

This shows that without changing the value of the pressure gradient and the cross-sectional area of the arteries, the blood flow rate through the arteries is decreasing significantly as time increasing. It also shows that the blood flow is linearly decreasing. We assume this condition is valid in the diastole condition only. Next, we compare this result with smaller crosssectional area. From Figure 4 depicts that if the value of cross-sectional area is smaller, the blood flow rate is decreasing slower than the blood flow rate at a normal condition which implies that when the cross-sectional area is decreased, the blood flow is increased. This condition occurs when the value of cross-sectional area is in the range between 1.5 cm^2 to 0.9 cm^2. Due to the fact that larger amount of blood flows through the arteries in a smaller cross-sectional area, may cause the increasing of pressure in the artery's wall. Thus, blood pressure increases and contributes to high blood pressure.

As shown in Figure 5 above, when the value of cross-sectional area is below 0.8 cm^2, the blood flow rate decreases faster that the normal rate. Figure 6 shows the value of blood flow rate when the cross-sectional area is in range of between 0.1 cm^2 to 0.8cm^2. Clearly, if the cross-sectional area continues to decrease below 0.8 cm^2, the

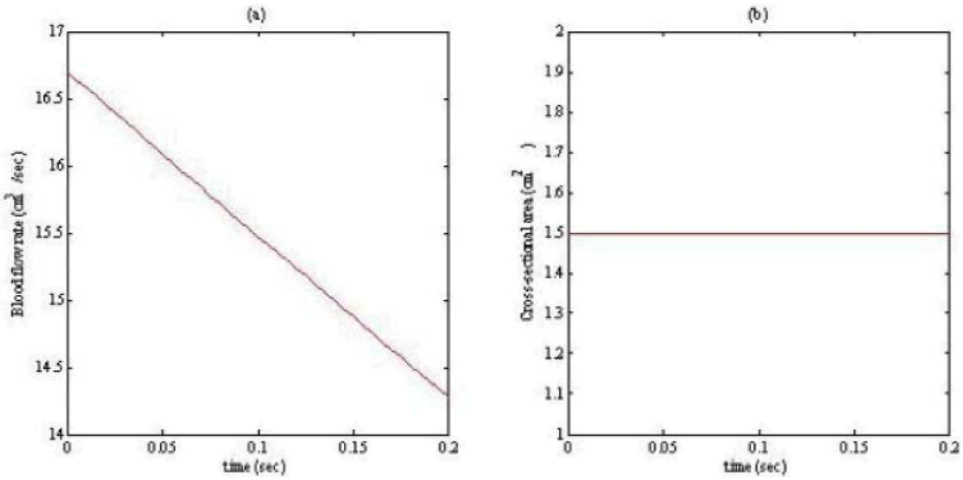

Fig. 2: (a) is the blood flow rate against time and (b) is the cross-sectional area against time M=1.0.

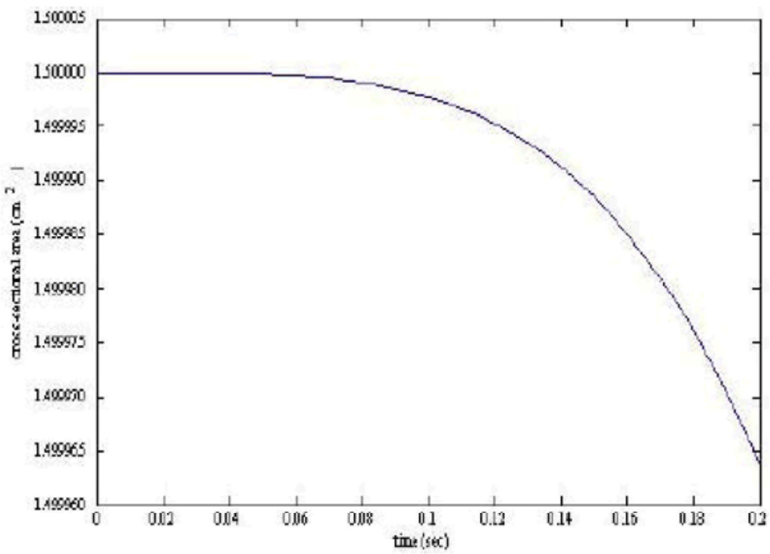

Fig. 3: The cross-sectional area against time.

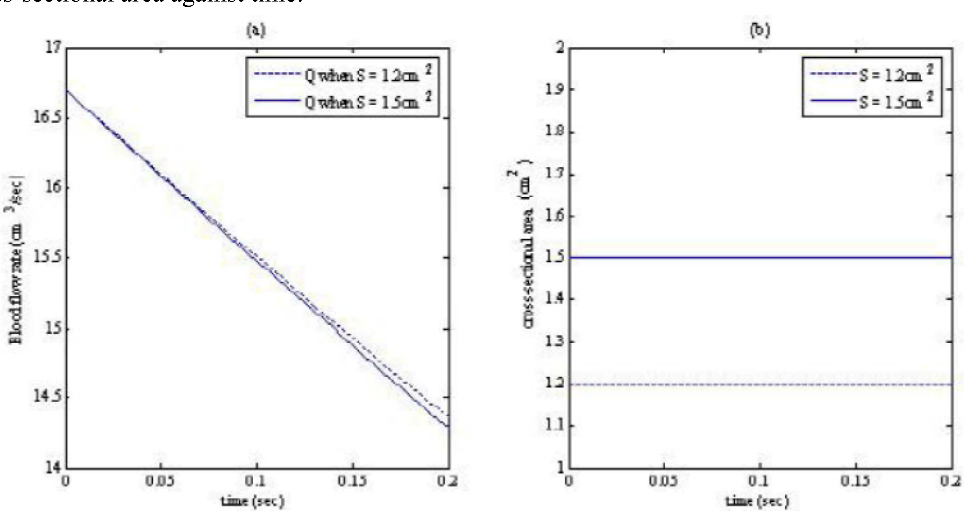

Fig. 4: Comparison graph for blood flow with different value of cross-sectional area with magnetic effects.

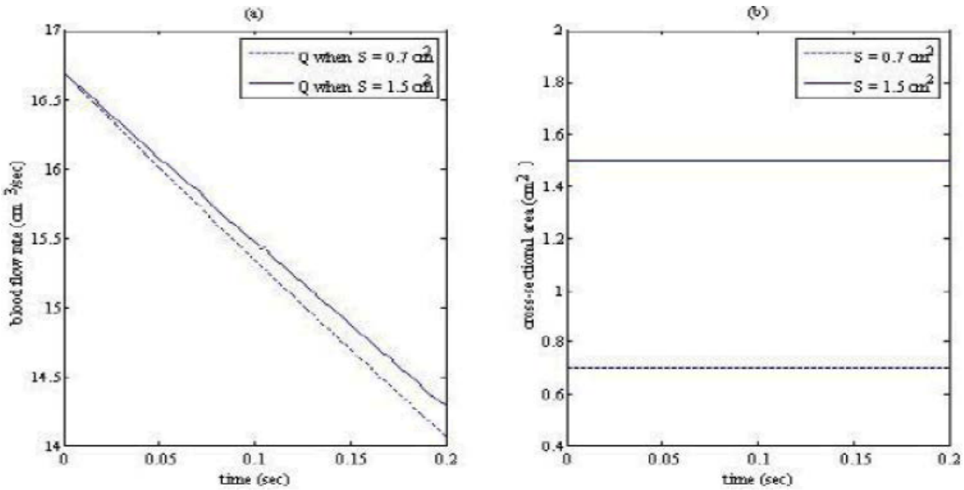

Fig. 5: Comparison of Q at normal cross-sectional area and much smaller cross-sectional Area.

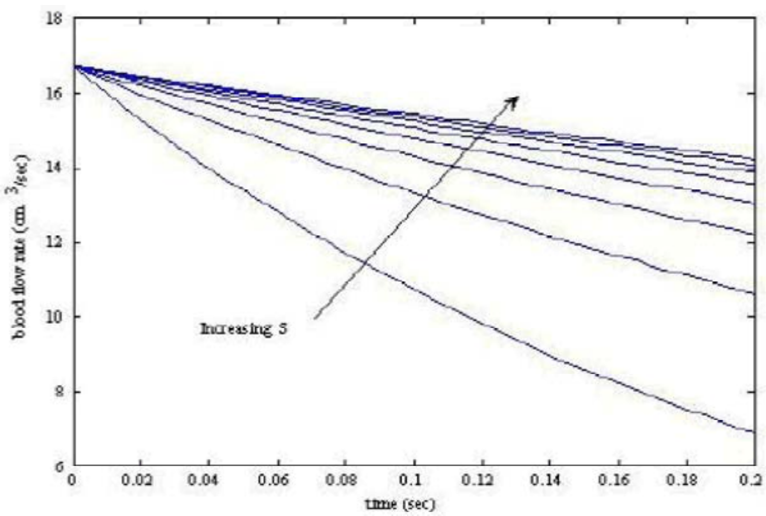

Fig. 6: Blood flow rate when the cross-sectional area is in range of between 0.1 cm^2 to 0.8cm^2 $^{M= 1.0}$

blood flow rate also decreased drastically. From this observation, we can say that this condition occur because the cross sectional area is too small for the blood to get through it. This is also a dangerous condition for human.

From the results obtained, we can conclude that cross-sectional area plays an important part in order for the blood to flow smoothly through the blood vessel. A small change in the value for the cross-sectional area may affect the amount of blood flow rate through the arteries which also may affect the blood pressure. In other words, smaller cross-sectional area from normal size may contribute to hypertension or high blood pressure. When a large amount of fluid flows through a small vessel, it may cause the pressure in the vessel to increase.

CONCLUSION

The present algorithm is economic and efficient having a sharp convergence. In this paper, we have derived a simple mathematical model that can represent the blood flow in the arteries. Even though the model does not include viscoelastic effect, the results obtained is considered valid since we are able to make a conclusion that from this model, we observe that the size of the blood vessel does influence the blood flow. A little change on the cross-sectional value makes vast change on the blood flow rate. I hope that our investigation may be helpful for the medical practitioners and other persons in the area of bio fluid dynamics to understand the flow of blood in the presence of magnetic field. The effects of a magnetic field have been used to control the flow, which may be useful in certain hypertension cases, etc.

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