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On a Combinatorial Relation for Stirling Numbers

R. Cruz-Santiago and J. López-Bonilla

ESIME-Zacatenco, Instituto Politécnico Nacional,
Edif. 5, 1er. Piso, Col. Lindavista CP 07738, CDMX, México

Abstract: We obtain an identity involving Stirling numbers of the first and second kind.

Key words: Stirling numbers · Combinatorial identities

INTRODUCTION

In [1] we find the following identity:

$$\sum_{r=k}^{n} S_r^{(k)}\, S_{n+1}^{[r+1]} = \binom{n}{k}, \quad n \geq k \geq 1, \tag{1}$$

involving the Stirling numbers of the first and second kind [2-6]. Then from (1):

$$\begin{aligned}\binom{n}{k-1} &= \sum_{r=k-1}^{n} S_r^{(k-1)}\, S_{n+1}^{[r+1]} = \sum_{r=k-1}^{n} (S_{r+1}^{(k)} + r\, S_r^{(k)})\, S_{n+1}^{[r+1]}, \\ &= \sum_{r=k}^{n} r\, S_r^{(k)}\, S_{n+1}^{[r+1]} + \sum_{j=k}^{n+1} S_{n+1}^{[j]}\, S_j^{(k)},\end{aligned}$$

therefore:

$$\sum_{r=k}^{n} r\, S_r^{(k)}\, S_{n+1}^{[r+1]} = \binom{n}{k-1}, \tag{2}$$

because $\sum_{j=k}^{n+1} S_{n+1}^{[j]}\, S_j^{(k)} = \delta_{k,n+1} = 0, \quad n \geq k.$

The Fubini polynomials are defined by [7]:

$$F_n(u) = \sum_{k=0}^{n} k!\, S_n^{[k]}\, u^k, \quad n \geq 0, \tag{3}$$

that is:

$$\begin{aligned} &F_0(u) = 1, \qquad F_1(u) = u, \qquad F_2(u) = u + 2u^2, \\ &F_3(u) = u + 6u^2 + 6u^3, \qquad F_4(u) = u + 14u^2 + 36u^3 + 24u^4, \ \ldots \end{aligned} \tag{4}$$

Corresponding Author: J. López-Bonilla, ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 5, 1er. Piso, Col. Lindavista CP 07738, CDMX, México.

Hence from (2) it is immediate the property:

$$F_n(-1) = \sum\nolimits_{k=0}^{n} (-1)^k \; k! \; S_n^{[k]} \; \underset{=}{[2]} \; (-1)^n, \tag{5}$$

besides:

$$F_n\left(-\frac{1}{2}\right) = \frac{2(1-2^{n+1})}{n+1} B_{n+1}, \tag{6}$$

in terms of the Bernoulli numbers [2, 6]; thus:

$$F_j\left(-\frac{1}{2}\right) = 0, \;\; j = 2,4,6, \; ... \tag{7}$$

The relations (5) and (7) can be verified with the expressions (4).

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