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Number of Physical Degrees of Freedom in Constrained Hamiltonian Systems

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Abstract: We give a simple deduction of the Díaz-Higuita-Montesinos formula for the number of physical degrees of freedom in terms of Lagrangian parameters.

Key words: Hamiltonian systems · Singular Lagrangians · First class constraints · Gauge identities

INTRODUCTION

Díaz-Higuita-Montesinos [1, 2] deduced the following expression to obtain the number of physical degrees of freedom (NPDF) in systems with singular Lagrangians:

$$NPDF = N - \frac{1}{2}(l + g + e), \tag{1}$$

in terms of Lagrangian parameters, in fact, N, e, l and g are the total number of generalized coordinates $q_j(t)$, effective gauge parameters [1, 3, 4], genuine constraints and gauge identities [5-10], respectively.

This same calculation can be realized via the Hamiltonian formula [3]:

$$NPDF = N - N_1 - \frac{1}{2}N_2, \tag{2}$$

employing only concepts from the Rosenfeld-Dirac-Bergmann approach [11-19] (also see Bronstein [20] and Haag [21]), where N_1 and N_2 are the total number of first-and second-class constraints [3, 5, 15, 22-24], respectively; let's remember that N_2 is an even number [3, 15, 25] and that the number of degrees of freedom is the same for Hamiltonian and Lagrangian formalisms [26].

In [26, 27] was established the following relation:

$$l = N_1 + N_2 - N_1^{(p)}, \tag{3}$$

being $N_1^{(p)}$ the total number of first-class primary constraints; besides [3]:

$$g = N_1^{(p)} = M - rank([\varphi_j, \varphi_m]), \quad j = 1,\ldots,N_1 + N_2; \quad m = 1,\ldots,M, \tag{4}$$

where M is the amount of independent primary constraints.

If we accept that the Dirac's conjecture [3, 5, 16, 18, 26, 28-30] is valid, then the N_1 first-class constraints (primary and secondary) generate gauge symmetries into the Hamiltonian formalism in according with the number of effective gauge parameters in the Lagrangian process [3]:

$$e = N_1, \tag{5}$$

thus, from (3), (4) and (5) we have that $N_1 = e$, $N_2 = l + g - e$ & $N_1^{(p)} = g$, therefore (2) implies the formula (1) obtained by Díaz-Higuita-Montesinos [1].

In [4] was showed how to apply (1) to several Lagrangians studied in [5, 31-33].

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