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Some Relations for Harmonic Numbers

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Abstract: We employ differentiation of combinatorial relations to generate identities involving harmonic numbers.

Key words: Harmonic numbers · Combinatorial identities

INTRODUCTION

The following combinatorial relations can be found in [1]:

$$\sum_{k=0}^{n} k\binom{n}{k}\binom{x}{k} = n\binom{x+n-1}{n}, \tag{1}$$

$$\sum_{k=0}^{n} \binom{x}{2k}\binom{x+n-k-1}{n-k} = \binom{x+2n-1}{2n}, \tag{2}$$

$$\sum_{k=0}^{n} (-1)^k \binom{x}{k}\binom{x}{n-k} = \frac{1}{2}(-1)^{n/2}\binom{x}{n/2}[1+(-1)^n], \tag{3}$$

$$\sum_{k=0}^{n} (-1)^k \binom{x}{k}\binom{x}{2n-k} = \frac{1}{2}(-1)^n\binom{x}{n}\left[1+\binom{x}{n}\right], \tag{4}$$

besides, we know the properties:

$$[\frac{d}{dx}\binom{x+m}{n}]_{x=n-m} = H_n, \quad [\frac{d}{dx}\binom{x}{n}]_{x=1} = (-1)^{n+1}H_n, \quad [\frac{d}{dx}\binom{x}{k}]_{x=1} = \frac{(-1)^k}{k(k-1)}, \; k \geq 2, \tag{5}$$

whose application to (1) - (4) allows to deduce identities involving harmonic numbers [2-6].

In fact, $[\frac{d}{dx}(1)]_{x=1}$ and (5) imply the property:

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$$\sum_{k=2}^{n}\binom{n}{k}\frac{(-1)^k}{k-1}=n(H_n-1), \tag{6}$$

and $[\frac{d}{dx}(2)]_{x=1}$ produces the expression:

$$\sum_{k=1}^{n}\frac{1}{2k-1}=H_{2n}-\frac{1}{2}H_n. \tag{7}$$

From (5) and $[\frac{d}{dx}(3)]_{x=1}$:

$$\sum_{k=1}^{2m}(-1)^k H_k=\frac{1}{2}H_m,\ \ m\geq 1, \tag{8}$$

and $[\frac{d}{dx}(4)]_{x=-1}$ gives the identity:

$$\sum_{k=0}^{n}(-1)^k(H_{2n-k}+H_k)=\left[(-1)^n+\frac{1}{2}\right]H_n.$$

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