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Agoh-Dilcher Identities for Stirling Numbers of the First Kind

[1]*H.N. Núñez-Yépez and* [2]*J. López-Bonilla*

[1]Departamento de Física, UAM-I, Apdo. Postal 55-534, Iztapalapa CP 09340, CDMX, México
[2]ESIME-Zacatenco, Instituto Politécnico Nacional,
Edif. 4, 1er. Piso, Col. Lindavista CP 07738, CDMX, México

Abstract: We study one implication of two Agoh-Dilcher's relations for Stirling numbers of the first kind.

Key words: Agoh-Dilcher's identities · Stirling numbers

INTRODUCTION

Agoh-Dilcher [1, 2] obtained the following identities involving Stirling numbers of the first kind [3-5]:

$$n!\sum_{r=n,}^{m}\frac{(r)}{}S_{n+m}^{(q+m)} = \sum_{r=n}^{m}(-1)^r S_{n+m}^{(q+r)} S_{m+1}^{(m+1-r)}, \tag{1}$$

$k, m, n, q \geq 0, \quad n \geq m,$

$$\sum_{i=n}^{n-k+1}(k-1+j)m^j\ S_n^{(k-1+j)} = \sum_{r=n}^{k}(-1)^{m+1-k+r} S_{m+1}^{(k-r)} S_{n+m}^{(r)}, \tag{2}$$

then if we employ $q= 0$ and $k = m + 1$ in (1) and (2), respectively, we obtain the relation:

$$\sum_{i=n}^{n-m}(m+j)m^j\ S_n^{(m+j)} = n!\sum_{r=n}^{m}\frac{(r)}{}S_{n+n}^{(m)} \quad n \geq m \geq 0, \tag{3}$$

which it was showed in [6] via another approach. For example, (3) with $m = 1$ generates the identity:

$$\sum_{r=2}^{n} rS_n^{(r)} = \frac{(-1)^n n!}{n-1}, \quad n \geq 2, \tag{4}$$

where it was used the known property $S_{n-1}^{(1)} = (-1)^n (n-2)!$

On the other hand, in [7] it was deduced via arithmetical experimentation, the property:

$$\sum_{r=k}^{n} r\ S_n^{(r)}\ S_r^{[k]} = (-1)^{n+k+1}\ (\mathrm{n-k-1})! \begin{pmatrix} \mathrm{n} \\ \mathrm{k-1} \end{pmatrix}, \quad 1 \leq 1+k \leq n, \tag{5}$$

which for $k - 1$ implies (4) because $S_r^{[2]} = 1,\ \mathrm{r} \geq 1.$ If we remember the values [4, 8]:

$$S_r^{[2]} = 2^{r-1} - 1, \qquad S_r^{[3]} = \frac{1}{2}(3^{r-1} + 1 - 2^r), \tag{6}$$

Corresponding Author: J. López-Bonilla, ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso, Col. Lindavista CP 07738, CDMX, México.

then (5) for k = 2, 3 gives the expressions:

$$\sum_{r=2}^{n} r\, 2^r\, S_n^{(r)} = (-1)^n\, 2(n-3)!\, n(n-3), \quad n \geq 3, \tag{7}$$

$$\sum_{r=3}^{n} r\, 3^r\, S_n^{(r)} = (-1)^n\, 3(n-4)!\, \{n(n-1) + (n-3)\, [n(n-4) - 6(n-1)(n-2)\, H_{n-1}]\}, \quad n \geq 4, \tag{8}$$

with the presence of the harmonic numbers $H_m = \sum_{j=1}^{m} \frac{1}{j}$, [5].

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