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Differentiation of a Tabulated Function

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Abstract: The Gregory-Newton expansion for equidistant interpolation allows obtain the derivative of a tabulated function at the data points. Here we apply this fact to several functions to construct identities involving the digamma function, Laguerre polynomials, harmonic and Stirling numbers.

Key words: Stirling numbers · Laguerre polynomials · Binomial coefficients · Harmonic numbers · Gregory-Newton interpolation

INTRODUCTION

The operation $\frac{d}{dx}$ is not directly applicable to a function which is merely given in discrete equidistant data points. After the interpolation, however, we possess $f(x)$ everywhere and can now perform the differentiation. We are usually interested in the slope of the curve at the same points in which the observations were made. Hence we can differentiate the interpolation formulas and thus obtain a relation between the $\frac{d}{dx}$ and the $\frac{\Delta}{\Delta x}$ operations, in fact, with the Gregory-Newton infinite expansion [1-3] is possible to prove the following expressions:

$$f'(0)=\sum_{r=1}^{\infty}\frac{(-1)^{r-1}}{r}\Delta^r f(0),\quad f'(-1)=-\sum_{r=1}^{\infty}(-1)^r\Delta^r f(0).H_r, \tag{1}$$

accepting its convergence, with the presence of the harmonic numbers [4-6]:

$$H_r=\sum_{k=1}^{r}\frac{1}{k}. \tag{2}$$

In Sec. 2 we realize applications of (1) for the digamma function [4], Stirling numbers [4, 5, 7] and Laguerre polynomials [8-15].

Differentiation via Gregory-Newton Infinite Formula:

- $f(x) = x^n$, therfore from [3]: $\Delta^r f(0) = r!\ S_n^{[r]}$. with the participation of Stirling numbers of the second kind [4, 5, 7]. Then (1) implies the following relations for $n \geq 1$:

$$\sum_{r=1}^{n}(-1)^r(r-1)!S_n^{[r]}=-\delta_{n1},\quad \sum_{r=1}^{n}(-1)^r r!S_n^{[r]}.H_r=(-1)^n n. \tag{3}$$

- $f(x)=\frac{t^x}{x!}$, then from [3]: $\Delta^r f(0)=(-1)^r L_r(t)$ involving the Laguerre polynomials [8-15], besides, $f'(0) = \gamma + Ln\ t$, being γ the Euler-Mascheroni's constant [4, 16-18], hence from (1):

$$\gamma+Ln\ t=-\sum_{r=1}^{\infty}\frac{1}{r}L_r(t), \tag{4}$$

thus for $t = 1$ we obtain that [2] $\gamma=-\sum_{r=1}^{\infty}\frac{1}{r}L_r(1)$, which has convergence very slow.

- $f(x)=\frac{(x+y+p)!}{(x+p)!(y+p)!}\ y,p\geq 0$, therefore [3]:

$$\Delta^r f(0)=\frac{r!}{(p+r)!}\binom{y}{r},\ f'(0)=y\binom{y+p}{p}\sum_{k=1}^{\infty}\frac{(y+k-1)!}{k(y+p+k)!},$$

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then (1) gives the following relation for the digamma function:

$$\psi(y+p+1)-\psi(p+1)=p!\sum_{r=1}^{\infty}\frac{(-1)^{r-1}(r-1)!}{(p+r)!}\binom{y}{r}=\sum_{r=1}^{\infty}\frac{(-1)^{r-1}}{r}\frac{\binom{n}{r}}{\binom{p+r}{p}}, \quad (5)$$

that for $y = n = 0,1,2,...$ implies the expression [3, 5, 19]:

$$\sum_{r=1}^{n}\frac{1}{p+r}=\sum_{r=1}^{n}\frac{(-1)^{r-1}}{r}\frac{\binom{n}{t}}{\binom{p+r}{p}}, \quad (6)$$

which for $p = 0$ generates the Euler's formula for the harmonic numbers (2) [5]:

$$H_n=\sum_{k=1}^{n}\binom{n}{r}\frac{(-1)^{r-1}}{r}. \quad (7)$$

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