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On Lanczos Conformal Lagrangian

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Abstract: In simple manner, we show that the Lagrangian $\sqrt{-g}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}$ gives the same field equations as $\sqrt{-g}\left(R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}R^2\right)$ in four dimensions.

Key words: Conformal tensor · Lanczos scalar · Kretschmann invariant

INTRODUCTION

Straub [1] studies the following Lagrangian involving the Weyl tensor in four dimensions [2]:

$$\sqrt{-g}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}=\sqrt{-g}\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-2\frac{1}{3}R_{\mu\nu}R^{\mu\nu}+\frac{1}{3}R^2\right), \tag{1}$$

and he shows certain procedure to eliminate the Kretschmann invariant $R_{\gamma\lambda\rho\sigma}R^{\gamma\lambda\rho\sigma}$ [3, 4]. Here we exhibit an alternative method to remove this invariant into (1).

In fact, we may use the double dual of Riemann tensor $^*R^*_{\mu\nu\alpha\beta}$ [2] to construct the Lanczos Lagrangian [5-7]:

$$\sqrt{-g}R^{\mu\nu\alpha\beta}\,{}^*R^*_{\mu\nu\alpha\beta}=\sqrt{-g}\left(-R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}+4R_{\mu\nu}R^{\mu\nu}-R^2\right), \tag{2}$$

but it is very known that (2) is an exact ordinary divergence [8-13], then from (2):

$$\sqrt{-g}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}=(\sqrt{-g}A^{\nu})_{\nu}+\sqrt{-g}\left(4R_{\mu\nu}R^{\mu\nu}-R^2\right), \tag{3}$$

hence (1) gives the same field equations as the Lagrangian:

$$\sqrt{-g}\left(R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}R^2\right), \tag{4}$$

which coincides with the result (5) of Strub [1], q.e.d.

The importance of (2) in Riemannian 4-spaces is indicated in [7], in particular, it is fundamental for the existence of the Lanczos potential [6, 14] for the Weyl tensor.

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