African Journal of Basic & Applied Sciences 10 (3): 75-76, 2018
ISSN 2079-2034

DOI: 10.5829/idosi.ajbas.2018.75.76

An Alternative Deduction of the Lanczos Orthogonal Derivative

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Abstract: We show that the Kempf-Jackson-Morales formula to integrate by differentiating implies the Lanczos derivative.

Key words: Generalized derivative · Integration by differentiation

INTRODUCTION

Kempf *et al.* [1-3] introduced the following expression to calculate a definite integral via differentiation:

$$\int_a^b g(x)dx = \lim_{t\to 0} g\left(\frac{d}{dt}\right)\left[\frac{e^{bt}-e^{at}}{t}\right]. \tag{1}$$

In Sec. 2 we employ (1) to deduce the Cioranescu [4] - (Haslam-Jones) [5] - Lanczos [6] generalized derivative:

$$f'(x_0) = \lim_{\varepsilon\to 0} \frac{3}{2\varepsilon^3}\int_{-\varepsilon}^{\varepsilon} f(v+x_0)v\, dv. \tag{2}$$

Orthogonal Derivative:

We consider the integral:

$$\int_{-\varepsilon}^{\varepsilon} f(v+x_0)v\, dv = \int_{x_0-\varepsilon}^{x_0+\varepsilon} f(u)(u-x_0)du \overset{(1)}{=} \lim_{t\to 0} f\left(\frac{d}{dt}\right)\left(\frac{d}{dt}-x_0\right)\frac{ex_0t}{t}(e^{\varepsilon t}-e^{-\varepsilon t}),$$

$$= 2\lim_{t\to 0} f\left(\frac{d}{dt}\right)\varepsilon\left(\frac{d}{dt}-x_0\right)e^{x_0 t}\left(1+\frac{\varepsilon^2 t^2}{3!}+\frac{\varepsilon^4 t^4}{5!}+\dots\right), \tag{3}$$

but we have the properties:

$$\left(\frac{d}{dt}-x_0\right)e^{x_0 t} = 0, \quad \left(\frac{d}{dt}-x_0\right)(e^{x_0 t}\ t^2) = 2e^{x_0 t}\ t,$$

$$\left(\frac{d}{dt}-x_0\right)^k (e^{x_0 t}\ t) = \begin{cases} e^{x_0 t}, & k=1, \\ 0, & k\ge 2, \end{cases} \tag{4}$$

then from (3) and (4):

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$$\lim_{t\to 0}\frac{3}{2\varepsilon^3}\int_{-\varepsilon}^{\varepsilon} f(v+x_0)v\,dv=\frac{1}{2}\lim_{t\to 0} f\left(\frac{d}{dt}\right)\left(\frac{d}{dt}-x_0\right)(e^{x_0 t}t^2)=\lim_{t\to 0} f\left(\frac{d}{dt}\right)(e^{x_0 t}t),$$

$$=\lim_{t\to 0}\left[f(x_0)+f'(x_0)\left(\frac{d}{dt}-x_0\right)+\frac{1}{2!}f''(x_0)\left(\frac{d}{dt}-x_0\right)^2+\dots\right](e^{x_0 t}t),$$

$$\overset{(4)}{=}\lim_{t\to 0} f'(x_0)e^{x_0 t}=f'(x_0),$$

in according with the generalized derivative [7-10] indicated in (2).

The orthogonal derivative for higher orders was obtained by Rangarajan-Purushothaman [11-13] via Legendre polynomials.

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