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Prediction of Soil Sinkage by Multiple Loadings Using Finite Element Method

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Abstract: It is usual practice to use the same wheel tractor for different agricultural field operations. As the agricultural soil is exposed to multiple loadings of the same magnitude in this situation, it is valuable to predict soil sinkage by multiple loadings so as to utilize the tractor power effectively with minimum compaction effects. For this purpose, the finite element method (FEM) was used to predict soil sinkage by multiple loadings (ten loadings) of a rectangular plate and a two-dimensional FEM program entitled PRESSINK was modified and employed to perform required numerical calculations. The agricultural soil was considered as an elastoplastic material and the Drucker-Prager elastoplastic material model was adopted with the flow rule of associated plasticity. Also, to deal with material non-linearity, incremental method was adopted and to allow for the geometric non-linearity, the total Lagrangian formulation was used. The FEM analysis was finally verified through laboratory test. Results of the laboratory test proved that the FEM is a relatively accurate and powerful technique to predict soil sinkage by multiple loadings. Results of the study also indicated that the number of loadings noticeably affected soil sinkage. Moreover, the first three loadings caused critical soil sinkage and the amount of soil sinkage owing to the first three loadings was about 89% and 82% of the total soil sinkage based on the FEM analysis and laboratory test results, respectively.

Key words: Finite element method · Modeling · Prediction · Soil sinkage · Soil compaction · Multiple loadings

INTRODUCTION

Agronomists are concerned about the effects of soil compaction that impedes root growth [1]. Soil compaction is a process through which pore spaces are decreased. It alters the structure of cultivated soil, i.e. the spatial arrangement, the size and shape of clods and aggregates and consequently the pore spaces inside and between these units [2]. Soil compaction can be caused by natural phenomena such as rainfall impact, soaking, internal water tension and the like. On the other hand, artificial soil compaction occurs by tractors and agricultural machines [3]. Soil compaction under tractors and agricultural machines is of special concern because weights of these machines have been increased dramatically in the last decades [4, 5].

One of the most important causes of soil compaction is soil sinkage imposed by wheels or tracks. Therefore, prediction of soil sinkage under wheels or tracks is very important for determining the level of soil compaction [5]. For the last five decades, prediction of soil sinkage has been of great interest to researchers in both agriculture and cross-country mobility and transport [2, 6-16].

Agricultural field operations of different levels of mechanization are greatly dependent on wheel tractors as a source of traction power. Also, it is usual practice to use the same tractor for different operational requirements such as planting, spraying and harvesting. Hence a significant part of the field is exposed to multiple passes of wheels [17]. However, nearly all studies dealing with soil sinkage due to multiple passes of wheels (multiple loadings) have been experimental [17-20]. One disadvantage with the experimental procedure is that it is expensive, laborious and time consuming.

An alternative approach is to make use of finite element method (FEM). The FEM is now confidently recognized as the most powerful general technique for the numerical solution of a variety of problems

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subjected to known boundary and/or initial value conditions encountered in engineering [21-23]. Also, for almost last 40 years this method has been touted as a powerful method to solve soil mechanics problems [2, 5, 13, 14, 16, 24-26].

The non-linear nature of agricultural soils is a complicating factor because they do not comply with linear elastic theory and they demonstrate elastoplastic behavior [24, 25]. Agricultural soils also experience much larger strain than other engineering materials that have usually been modeled by civil and mechanical engineers. Thus, further work is required to improve the FEM before it can be utilized to exactly predict soil behavior. Certainly, latest progresses in improvement of constitutive equations (stress-strain relationships) and theory of plasticity can make the FEM a much more successful method for modeling soil behavior. The objectives of this study were: (a) to develop a FEM model to predict soil sinkage by multiple loadings and (b) to verify the FEM model by comparing its results with those of laboratory tests.

MATERIALS AND METHODS

Material Model Development: Two sources of non-linearity are to be expected when an agricultural soil is under external loads, namely material and geometrical non-linearity [5, 23, 26]. The earlier can be fully described by the stress-strain relationship. In this study, the elastoplastic material model was used to represent non-linear stress-strain relationship of soil. For an elastoplastic material the incremental stress tensor can be related to the incremental strain tensor as [26]:

$$d\sigma_{ij} = D_{ep}\, d\varepsilon_{ij} \tag{1}$$

where:
$d\sigma_{ij}$ = Incremental stress tensor
D_{ep} = Elastoplastic constitutive matrix
$d\varepsilon_{ij}$ = Incremental strain tensor which is the summation of the incremental elastic strain tensor and incremental plastic strain tensor as [27]:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p} \tag{2}$$

The incremental elastic strain tensor $d\varepsilon_{ij}^{e}$ can be expressed by Hooke's law as [27]:

$$d\varepsilon_{ij}^{e} = \frac{(1+\upsilon)}{E} d\sigma_{ij} - \frac{\upsilon}{E} d\sigma_{kk}\delta_{ij} \tag{3}$$

where:
υ = Poisson's ratio
E = Modulus of elasticity
$d\sigma_{kk}$ = Incremental volumetric stress tensor
δ_{ij} = Kronecker delta

The incremental plastic strain tensor $d\varepsilon_{ij}^{p}$ can be expressed by the classical theory of plasticity as [26]:

$$d\varepsilon_{ij}^{p} = d\lambda \frac{\partial F}{\partial \sigma_{ij}} \tag{4}$$

where:
$d\lambda$ = Plastic multiplier
F = Yield function

The incremental plastic strain tensor is actually a vector perpendicular to the tangent of the yield surface. This definition of the plastic strain is usually designated as associated plasticity [26].

The yield function of the Drucker-Prager for an elastoplastic material can be expressed as (Mouazen and Nemenyi, 1999) [26]:

$$F = aJ_1 + J_{2D}^{1/2} - k = 0 \tag{5}$$

where:
J_1 = The first invariant of the stress tensor
J_{2D} = The second invariant of the deviatoric stress tensor
a,k = Soil parameters which can be defined as:

$$a = \frac{2\sin\varphi}{\sqrt{3}(3-\sin\varphi)} \tag{6}$$

$$k = \frac{6c\cos\varphi}{\sqrt{3}(3-\sin\varphi)} \tag{7}$$

where:
c = Soil cohesion
φ = Angle of soil internal friction

From equation (5) it can be concluded that the Drucker-Prager yield criterion accounts for both volumetric and shear behavior.

Governing Equations Development: The governing equations were be obtained by using the principle of virtual work. Consider a solid, in which the internal stresses σ, the distributed loads/unit volume b and external applied forces f form an equilibrium field, to undergo an arbitrary virtual displacement pattern δd^* which result in compatible strains $\delta\varepsilon^*$ and internal displacement δu^*. Then the principle of virtual work requires that [22]:

$$\int_\Omega (\delta\varepsilon^{*T}\sigma - \delta u^{*T} b)d\Omega - \delta d^{*T} f = 0 \tag{8}$$

where:
Ω = The domain of interest

Then the normal finite element discretising procedure leads to the following expressions for the displacement and strains within any element [27]:

$$\delta u^* = N\delta d^* \tag{9}$$

$$\delta\varepsilon^* = B\delta d^* \tag{10}$$

where:
N = Matrix of the shape function
B = Sum of the geometric linear and geometric non-linear strain-displacement matrix

Then the element assembly process gives [22]:

$$\int_\Omega \delta d^{*T}(B^T\sigma - N^T b)d\Omega - \delta d^{*T} f = 0 \tag{11}$$

where, the volume integration over the solid is the sum of the individual element contributions. Since this expression must be true for any arbitrary δd^* value [22]:

$$\int_\Omega B^T\sigma d\Omega - f - \int_\Omega N^T b d\Omega = 0 \tag{12}$$

For solution of nonlinear problems, equation (12) will not generally be satisfied at any stage of the computation and [22]:

$$\psi = \int_\Omega B^T\sigma d\Omega - (f + \int_\Omega N^T b d\Omega) \neq 0 \tag{13}$$

where:

Ψ = The residual force vector

For an elastoplastic situation the material stiffness is continually varying and instantaneously the incremental stress-strain relationship is given by equation (1). For purpose of evaluating the element tangential stiffness matrix at any stage, the incremental form of the equation (13) must be employed. Thus, within an increment of load we have [22]:

$$\Delta\psi = \int_\Omega B^T\Delta\sigma d\Omega - (\Delta f + \int_\Omega N^T \Delta b d\Omega) \tag{14}$$

Substituting for $\Delta\sigma$ from equation (1) result in [22]:

$$\Delta\psi = K_T d - (\Delta f + \int_\Omega N^T \Delta b d\Omega) \tag{15}$$

where:
K_T = Element stiffness matrix associated with the geometric linear and geometric non-linear strain-displacement matrix and can be expressed as:

$$K_T = \int_\Omega B^T D_{ep} B d\Omega \tag{16}$$

FEM Program Development: A plane-stress, plane-strain and axisymmetric FEM program (PRESSINK) written by Owen and Hinton [22] was modified and a new FEM program entitled PRESSINK was developed using the material model, governing equations and assumptions previously discussed to take into account the material and geometrical non-linearity of soil. The FEM program was written in COMPAQ VISUAL FORTRAN 6.5 owing to it abilities to employ the principles of object-oriented programming. Additional required subroutines were also formulated and assembled to form a working program for two-dimensional elastoplastic geometrically non-linear analysis of plane-stress, plane-strain and axisymmetric problems. A modular approach was adopted for the program, in that separate subroutines were employed to perform the various operations required in a non-linear FEM analysis. To deal with material non-linearity and obtain stress and strain information at different steps of loading process, incremental method was adopted and to allow for the geometric non-linearity of the soil, total Lagrangian formulation was used [13, 14, 16].

Test Unit Development: A test unit was constructed to study soil sinkage by multiple loadings. A self-explanatory schematic picture of the test unit is presented in Fig. 1. The test unit contains a soil bin and a rectangular sinkage plate. The soil bin used in the test unit was 250 mm long, 250 mm wide and 250 mm high.

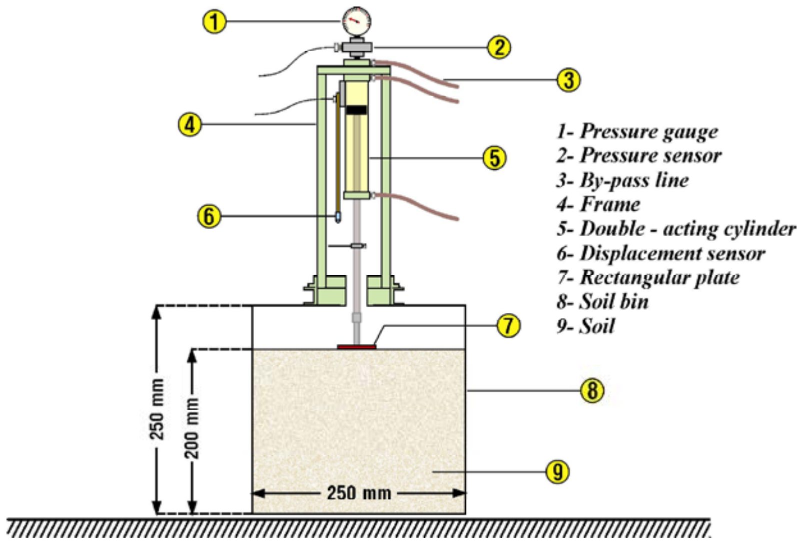

Fig. 1: Test unit

Table 1: Dimensions of the rectangular sinkage plate

Width (mm)	Length (mm)	Aspect ratio (Length / Width)
40	60	1.5

Dimensions of the rectangular sinkage plate are listed in Table 1. Note that the aspect ratio (length/width) of the rectangular plate was 1.5, which is similar to the ones expected for the wheel-soil contact areas (for tracks long narrow rectangular sinkage plates are recommended). The aspect ratio of a wheel/track-soil contact area can be defined as the length of the contact area divided by the width.

FEM Analysis: The FEM analysis was based on the assumptions that the wheel-soil contact area can be approximated by a rectangular region and the wheel contact pressure is uniformly distributed over the rectangular region. These assumptions helped to reduce the elaborations of the problem by allowing it to be analyzed as a plane-stress (two-dimensional) problem rather than a three-dimensional problem [21, 22]. Also, the FEM analysis was performed to simulate the same conditions of the soil-rectangular plate system illustrated in the test unit (Fig. 1). In order to predict soil sinkage due to multiple loadings of the rectangular plate, a two-dimensional FEM mesh (Fig. 2) was generated within a rectangle 200 mm long and 125 mm wide to model the plane stress geometry of the soil-rectangular plate system. The total number of nodal points and elements were 367 and 108, respectively. In this study, the eight-node serendipity elements were used to represent the soil material. These elements were chosen because they give a more accurate answer for larger mesh sizes [28]. Since the problem was symmetric about the vertical axis AB, only one half of the soil-rectangular plate system was meshed and considered during the analysis. It can be seen from Fig. 2 that the left-side boundary line AB was considered as a reflected boundary and the nodes on the bottom boundary line BC were constrained in both horizontal and vertical direction. The nodes on the right-side boundary line CD were constrained in horizontal direction and the nodes on the top boundary line AD were free of any constrains. The rectangular plate was assumed to be a rigid body and the loading was distributed evenly over the left-side three elements at the top of the FEM mesh. Soil parameters used for the FEM analysis of soil-rectangular plate system are shown in Table 2. For the FEM analysis, appropriate boundary conditions information, soil mechanical properties and nodal and elemental data were input as required. The load application on the FEM model was simulated in an incremental manner. For each increment, the displacement of each nodal point was computed. This process was continued until the total pressure of 200 kPa was applied monotonically in increments of 40 kPa. At this point, the soil was unloaded in one step to complete the simulation of the first loading and unloading cycle. Successive loading and unloading cycles were simulated by reloading and unloading in one step. Loading and unloading was done ten times and at the end of each loading and unloading cycle, the total displacement of each nodal point was obtained.

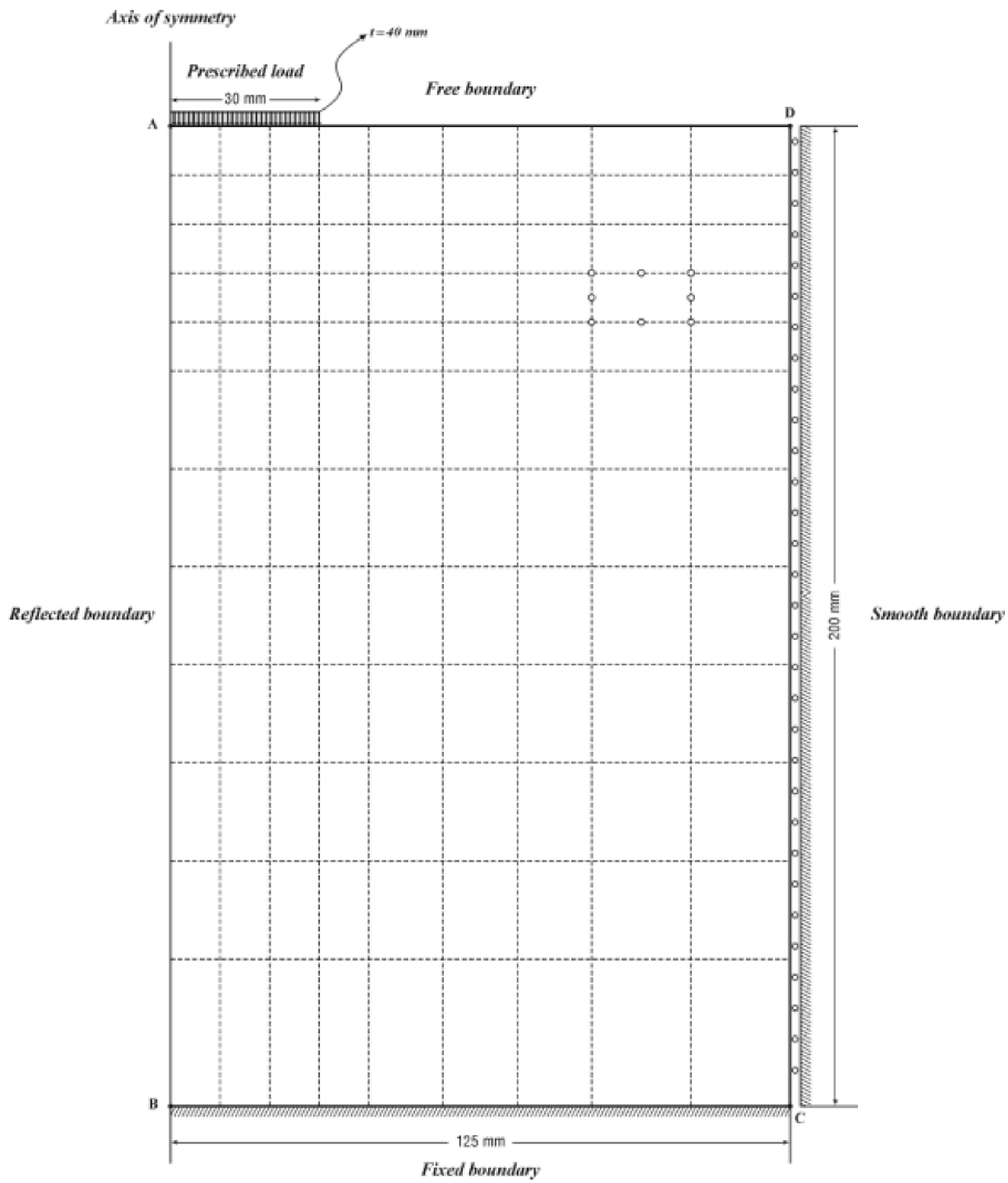

Fig. 2: Two-dimensional FEM mesh of the soil-rectangular plate system

Table 2: Soil properties used for the FEM analysis of the soil-rectangular plate system

Properties	Symbol	Unit	Amount
Modulus of elasticity	E	MPa	150
Poisson's ratio	ν	---	0.3
Cohesion	c	kPa	80
Angle of internal friction	φ	deg	30

Laboratory Test: Laboratory test was performed to verify the prediction of soil sinkage by multiple loadings using the FEM. A sandy-loam soil was chosen for characterizing the agricultural soil. The sandy-loam soil was consisted of 33% sand, 45% silt and 22% clay. To prepare soil bin, as a first step, soil was sieved through a 4-mm mesh sieve. Then, the soil was watered and covered with a sheet of

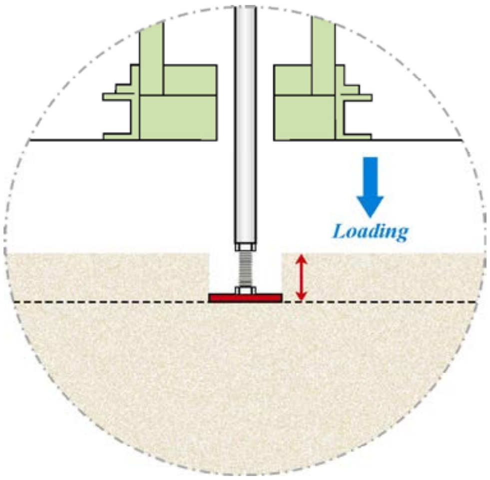

Fig. 3: Loading process

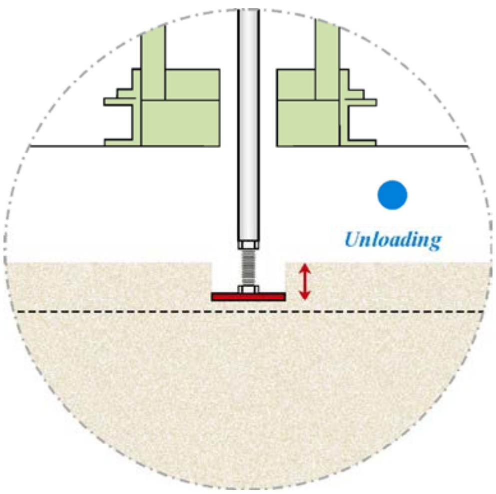

Fig. 4: Unloading process

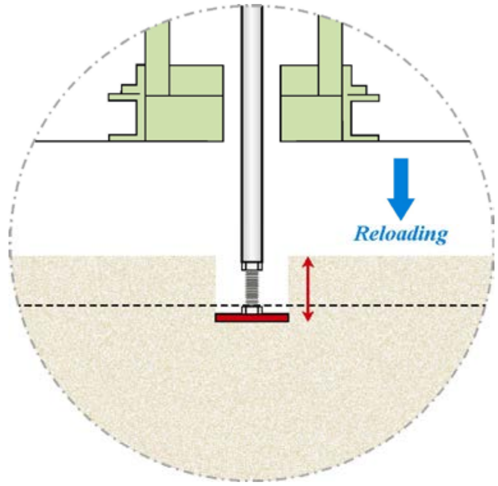

Fig. 5: Reloading process

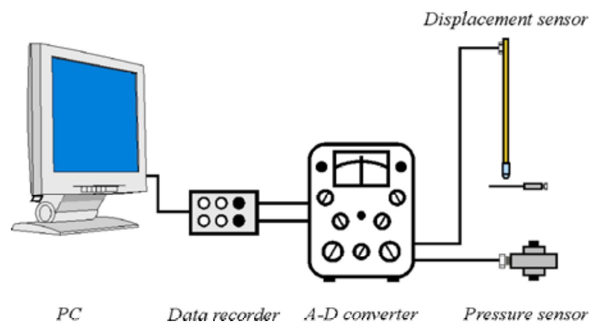

Fig. 6: Data acquisition system

plastic during the night in order to achieve a uniform moisture distribution. The measured soil moisture content on dry basis was about 18%, which made the soil to be in an arable condition as in the field. The soil was then fitted to the soil bin in five layers of 60 mm and each layer was compacted 20 mm using a wooden packer piston with the aid of a hydraulic press until the soil bin became full up to 200 mm. The soil bulk density of 1.70 g cm^{-3} (on wet basis) was determined before multiple loadings tests. Then, for each test run, the rectangular sinkage plate was loaded incrementally up to about 200 kPa in increments of 40 kPa. This process was continued until the total pressure of 200 kPa was applied monotonically (Fig. 3). After that, the soil was unloaded (Fig. 4) in one step to complete the first loading and unloading cycle and at the same time the sinkage depth of the rectangular plate was measured using the displacement sensor. Successive reloading (Fig. 5) and unloading cycles were repeated ten times and at the end of each loading and unloading cycle, the sinkage depth was measured. Applied loads were measured by HBM-Q3 model load cell and at the same time downwards displacements (soil sinkage values) were measured with HBM-W100 model LVDT (Linear Variable Differential Transducer). Both instruments were connected to an amplifier and to a personal computer equipped with an AD card to amplify and record each test outputs (Fig. 6). Also, multiple loadings test was replicated three times and mean of the measured soil sinkage values was used for statistical analyses.

Statistical Analysis: A linear regression with zero intercept was performed to verify the validity of the FEM analysis. Also, to check the discrepancies between the predicted results using the FEM analysis and those measured through the laboratory test, root mean squared error (RMSE) and mean relative percentage deviation (MRPD) were calculated as [13, 14, 16]:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(z_i - z_i^*)^2}{n}} \quad (17)$$

where:
$RMSE$ = Root mean squared error, mm
z_i = Total soil sinkage due to i[th] loading measured through laboratory test, mm
z_i^* = Total soil sinkage due to i[th] loading predicted using the FEM analysis, mm

$$MRPD = \frac{100 \times \sum_{i=1}^{n} \frac{\left| z_i - z_i^* \right|}{z_i}}{n} \quad (18)$$

where:
$MRPD$ = Mean relative percentage deviation,%

RESULTS AND DISCUSSION

Fig. 7 shows the soil sinkage values under the rectangular plate as related to number of loadings which were predicted using the FEM analysis. Results of the FEM analysis indicated that the soil sinkage value due to the first loading was greater than the soil sinkage values caused by other loadings. These results also showed that the total soil sinkage owing to the ten loadings was chiefly affected by the first loading which caused almost 60% of it. Moreover, second and third loadings caused nearly 22% and 7% of the total soil sinkage, respectively. Based on the FEM analysis results, the first three loadings were critical and the amount of soil sinkage due to the first three loadings was about 89% of the total soil sinkage. According to the FEM analysis results, remaining loadings, i.e. forth to tenth loadings altogether caused only 11% of the total soil sinkage.

Fig. 7 also demonstrates the soil sinkage values under the rectangular plate as related to number of loadings which were measured using through the laboratory test. Results of the laboratory test confirmed that the soil sinkage value owing to the first loading was larger than the soil sinkage values caused by other loadings. These results also proved that the total soil sinkage due to the ten loadings was mainly affected by the first loading which caused approximately 57% of it. Furthermore, second and third loadings caused just about 19% and 6% of the total soil sinkage, respectively. According to the laboratory test results, the first three

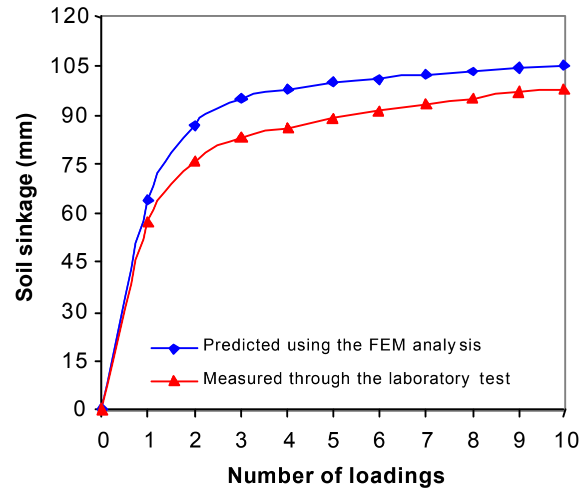

Fig. 7: Soil sinkage values under the rectangular plate as related to number of loadings predicted using the FEM analysis in compared with those measured through the laboratory test

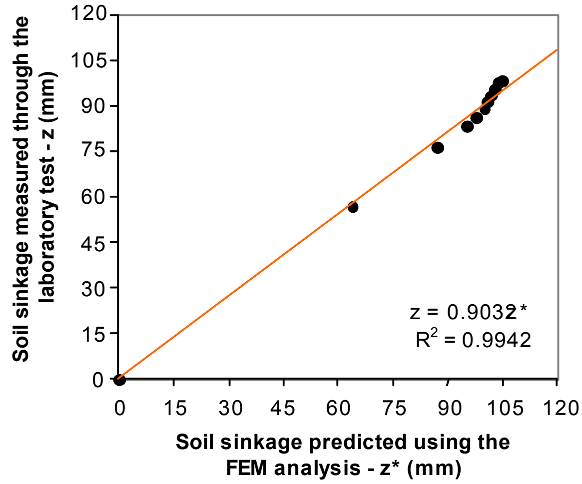

Fig. 8: Soil sinkage values predicted using the FEM analysis and soil sinkage values measured through the laboratory test are plotted against each other and fitted with a linear equation with zero intercept

loadings were critical too and the amount of soil sinkage due to the first three loadings was about 82% of the total soil sinkage. Based on the laboratory test results, remaining loadings, i.e. forth to tenth loadings in total caused only 18% of the total soil sinkage.

From comparison of two curves, it could be concluded that the FEM analysis and the laboratory test gave identical results. A linear regression with zero intercept was performed to verify the validity of the FEM analysis. Fig. 8 shows that the soil sinkage values under the rectangular plate as related to number of loadings predicted using the FEM analysis and those measured through the laboratory test were plotted against each other and fitted with a linear equation with zero intercept. The slope of the line of best fit and its coefficient of determination (R^2) were 0.9032 and 0.9942, respectively. Moreover, to check the discrepancies between the predicted results using the FEM analysis and those measured through the laboratory test, RMSE and MRPD were calculated. The amounts of RMSE and MRPD were 9.6 mm and 11.1%, respectively.

More likely reason for such negligible discrepancies between the predicted results using the FEM analysis and those measured through the laboratory test probably stem from precision modeling of soil behavior. These results are in line with those of Abu-Hamdeh and Reeder [5], Naylor and Pande [23] and Mouazen and Nemenyi [26] who concluded that soil deformations are governed by material and geometrical non-linearity. These results are also in agreement with those of Rashidi *et al*. [13, 14, 16] who concluded that to reasonably predict soil pressure-sinkage behavior, both material and geometrical non-linearity should be accounted for the entire soil volume being modeled. They also concluded that the FEM suggests significant assure for accurate modeling soil behavior and complicated loading geometries and the analysis can be carried out without difficulty on a personal computer.

CONCLUSION

Prediction of soil sinkage by multiple loadings using the FEM analysis and evaluation of the FEM analysis results through laboratory test proved that the FEM is a relatively accurate and powerful technique to predict soil sinkage by multiple loadings. Also, the first three loadings caused critical soil sinkage and the amount of soil sinkage due to the first three loadings was about 89% and 82% of the total soil sinkage based on the FEM analysis and laboratory test results, respectively. Moreover, to rationally predict agricultural soils behavior using the FEM, accounting both material and geometrical non-linearity seems necessary.

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