

Prediction of Soil Infiltration Rate Based on Particle Size Distribution, Bulk Density, Organic Matter and Moisture Content of Soil

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Abstract: Soil infiltration rate is often determined using laborious and time consuming field tests, but it may be more suitable and economical to develop a method which predicts soil infiltration rate based on available physical properties of soil. Therefore, a relation between soil infiltration rate and some physical properties of soil is needed. In this study, for predicting soil infiltration rate (IN) based on particle size distribution, viz. silt content (SI) and clay content (CL) of soil, bulk density (BD) of soil, organic matter (OM) of soil and moisture content (MC) of soil, a five-variable linear regression model was suggested. The statistical results of study indicated that in order to predict soil infiltration rate based on particle size distribution, bulk density, organic matter and moisture content of soil the five-variable linear regression model $IN = 28.13 - 0.220 SI - 0.518 CL + 4.592 BD - 1.440 OM + 0.022 MC$ with $R^2 = 0.9092$ can be strongly recommended.

Key words: Soil • Infiltration rate • Modeling • Prediction • Silt content • Clay content • Bulk density • Organic matter • Moisture content

INTRODUCTION

Surface irrigation methods are widely used throughout the world [1, 2]. Recent advances in the theoretical description and model simulation of surface irrigation methods permit the evaluation of existing procedures and the development of new technologies of irrigation systems and their management. Free water at the soil-atmosphere interface is a source of great importance to man. Efficient management of this water will require greater control of infiltration. Increased infiltration control would help to solve such wide ranging problems as upland flooding, pollution of surface and ground-waters, declining water tables and inefficient irrigation of agricultural lands [3]. For these reasons, soil infiltration rate is perhaps the most crucial process affecting surface irrigation uniformity and efficiency as it is the mechanism that transfers and distributes water from the surface to the soil profile. It is essential to predict the cumulative infiltration in order to estimate the amount of water entering the soil and its distribution. Infiltration also affects both the advance and recession processes and thus is important in estimating the optimal discharge that should be directed to the field [4]. The infiltration process depends on the physical, chemical and biological

properties of the soil surface, the initial distribution of water in the soil prior to irrigation, the movement of water over the surface and the depth of water on the soil surface. These properties and conditions vary over a field and collectively cause infiltration itself to exhibit large variation at the field scale. Therefore, infiltration is difficult to characterize on a field scale because of a large number of measurements is necessary [5].

In the engineering evaluation and design of surface irrigation systems, it has been useful to predict the soil infiltration rate [4]. In general, prediction of the soil infiltration rate involves the adoption of a functional form to be used and the determination of the value of the numerical constants in the adopted equation. Prediction of soil infiltration rate is a major problem in irrigation studies due to proper selection of the technique used to determine the parameters of the empirical infiltration models, the use of empirical infiltration models and its dependence on soil moisture, soil characteristics and surface roughness. Thus, the technique used to determine the soil infiltration characteristics must be appropriate for the purpose of the study [6-8].

Despite the considerable amount of research done, which shows the relationship between soil infiltration rate and soil properties, very limited work has been

conducted to predict soil infiltration rate based on particle size distribution, bulk density, organic matter and moisture content of soil. Therefore, the main objectives of this research were to determine soil infiltration rate model based on particle size distribution (silt and clay content), bulk density, organic matter and moisture content of soil and to verify the model by comparing its results with those of the field tests.

MATERIALS AND METHODS

Experimental Site: Field experiments were carried out at the agricultural fields of Karaj, Alborz Province, Iran. This site is located at latitude of 35° 59' N, longitude of 51° 6' E and altitude of 1300 m above mean sea level in semi-arid climate (345 mm rainfall annually) in the center of Iran.

Experimental Procedure: Eighty-five soil samples were taken at random from different fields of the experimental site. In order to obtain required parameters for determining soil infiltration rate model, silt content (SI), clay content (CL), bulk density (BD), organic matter (OM) and moisture content (MC) of the soil samples were measured using laboratory tests as described by the Soil Survey Laboratory Staff [9]. Also, infiltration rate of the soil in all treatments was measured using a double ring infiltrometer. The infiltrometer was installed in the position of each treatment, filled with water and the initial reading was noted. The depth of water in the infiltrometer was noted after frequent intervals until the rate of infiltration became constant. Table 1 shows infiltration rate, silt content, clay content, bulk density, organic matter and moisture content of the eighty-five soil samples used to determine soil infiltration rate model.

Table 1: Infiltration rate, silt content, clay content, bulk density, organic matter and moisture content of the eighty-five soil samples used to determine soil infiltration rate model

| Sample No. | Infiltration rate (mm/h) | Silt content (%) | Clay content (%) | Bulk density (g/cm ³) | Organic matter (%) | Moisture content (%) |
|------------|--------------------------|------------------|------------------|-----------------------------------|--------------------|----------------------|
| 1 | 8.82 | 34 | 38 | 1.826 | 1.002 | 7.376 |
| 2 | 3.63 | 38 | 42 | 1.610 | 1.030 | 7.897 |
| 3 | 9.14 | 40 | 38 | 1.667 | 0.960 | 8.181 |
| 4 | 4.61 | 40 | 40 | 1.685 | 1.310 | 7.446 |
| 5 | 7.45 | 34 | 38 | 1.528 | 1.010 | 10.47 |
| 6 | 3.15 | 40 | 42 | 1.527 | 1.320 | 7.442 |
| 7 | 4.12 | 38 | 42 | 1.538 | 1.070 | 11.88 |
| 8 | 3.28 | 36 | 40 | 1.619 | 1.190 | 9.048 |
| 9 | 2.03 | 36 | 42 | 1.595 | 1.320 | 10.45 |
| 10 | 6.79 | 40 | 36 | 1.546 | 0.940 | 7.076 |
| 11 | 3.60 | 36 | 42 | 1.626 | 1.100 | 12.22 |
| 12 | 2.50 | 38 | 46 | 1.535 | 1.040 | 10.53 |
| 13 | 2.20 | 34 | 40 | 1.526 | 1.190 | 11.25 |
| 14 | 1.70 | 36 | 40 | 1.606 | 1.160 | 2.620 |
| 15 | 7.46 | 40 | 38 | 1.557 | 1.010 | 4.272 |
| 16 | 3.30 | 38 | 44 | 1.688 | 1.050 | 6.683 |
| 17 | 2.90 | 36 | 44 | 1.437 | 1.003 | 8.904 |
| 18 | 3.10 | 32 | 42 | 1.685 | 1.040 | 9.040 |
| 19 | 9.32 | 34 | 38 | 1.561 | 1.006 | 7.874 |
| 20 | 7.06 | 34 | 38 | 1.677 | 1.003 | 9.738 |
| 21 | 4.30 | 38 | 42 | 1.495 | 1.020 | 6.193 |
| 22 | 14.8 | 36 | 28 | 1.670 | 0.600 | 8.770 |
| 23 | 2.50 | 40 | 40 | 1.677 | 1.350 | 8.082 |
| 24 | 1.70 | 38 | 42 | 1.546 | 1.040 | 6.971 |
| 25 | 6.90 | 36 | 34 | 1.628 | 1.060 | 5.175 |
| 26 | 6.50 | 38 | 34 | 1.481 | 1.130 | 11.88 |
| 27 | 11.9 | 36 | 26 | 1.698 | 0.560 | 5.453 |
| 28 | 9.60 | 40 | 38 | 1.596 | 1.020 | 5.537 |
| 29 | 6.10 | 38 | 36 | 1.594 | 1.060 | 3.433 |
| 30 | 7.11 | 34 | 38 | 1.574 | 1.140 | 7.540 |
| 31 | 2.30 | 36 | 42 | 1.690 | 1.240 | 6.477 |
| 32 | 3.20 | 36 | 38 | 1.693 | 1.110 | 5.022 |
| 33 | 9.60 | 40 | 36 | 1.743 | 1.050 | 6.518 |
| 34 | 8.90 | 36 | 36 | 1.555 | 1.004 | 4.602 |

Table I: Continue

| Sample No. | Infiltration rate (mm/h) | Silt content (%) | Clay content (%) | Bulk density (g/cm ³) | Organic matter (%) | Moisture content (%) |
|------------|--------------------------|------------------|------------------|-----------------------------------|--------------------|----------------------|
| 35 | 2.27 | 36 | 38 | 1.583 | 1.210 | 4.731 |
| 36 | 23.5 | 7 | 22 | 1.843 | 0.390 | 3.243 |
| 37 | 26.0 | 5 | 17 | 1.845 | 0.320 | 4.607 |
| 38 | 22.1 | 10 | 21 | 1.923 | 0.360 | 6.392 |
| 39 | 25.5 | 12 | 20 | 2.032 | 0.350 | 5.923 |
| 40 | 25.2 | 15 | 20 | 1.832 | 0.290 | 4.044 |
| 41 | 22.5 | 15 | 20 | 1.839 | 0.310 | 3.524 |
| 42 | 24.5 | 15 | 20 | 1.935 | 0.300 | 2.349 |
| 43 | 22.8 | 11 | 19 | 1.919 | 0.390 | 7.240 |
| 44 | 26.3 | 9 | 24 | 1.980 | 0.320 | 1.913 |
| 45 | 23.6 | 9 | 12 | 2.070 | 0.360 | 3.538 |
| 46 | 24.3 | 10 | 25 | 2.181 | 0.340 | 3.899 |
| 47 | 27.2 | 9 | 19 | 2.181 | 0.320 | 3.925 |
| 48 | 28.5 | 10 | 20 | 2.038 | 0.340 | 1.858 |
| 49 | 26.2 | 13 | 11 | 2.016 | 0.390 | 4.008 |
| 50 | 27.5 | 14 | 18 | 1.904 | 0.300 | 3.315 |
| 51 | 22.1 | 5 | 16 | 1.872 | 0.310 | 1.438 |
| 52 | 23.5 | 16 | 30 | 1.718 | 0.320 | 6.320 |
| 53 | 6.90 | 46 | 32 | 1.722 | 1.800 | 12.67 |
| 54 | 7.30 | 44 | 32 | 1.572 | 1.780 | 13.82 |
| 55 | 7.10 | 38 | 28 | 1.698 | 1.340 | 14.33 |
| 56 | 9.10 | 48 | 32 | 1.463 | 1.680 | 13.58 |
| 57 | 9.30 | 48 | 30 | 1.500 | 1.830 | 15.43 |
| 58 | 6.00 | 42 | 32 | 1.678 | 1.850 | 7.225 |
| 59 | 7.10 | 44 | 32 | 1.524 | 1.800 | 8.895 |
| 60 | 7.90 | 52 | 32 | 1.639 | 1.630 | 12.53 |
| 61 | 5.40 | 48 | 32 | 1.473 | 1.800 | 15.71 |
| 62 | 6.00 | 50 | 26 | 1.424 | 1.670 | 16.49 |
| 63 | 8.70 | 46 | 30 | 1.423 | 1.280 | 13.90 |
| 64 | 8.90 | 44 | 32 | 1.336 | 1.780 | 13.74 |
| 65 | 9.00 | 38 | 30 | 1.700 | 1.880 | 14.51 |
| 66 | 6.60 | 42 | 30 | 1.452 | 1.780 | 16.70 |
| 67 | 8.22 | 48 | 32 | 1.472 | 1.880 | 14.41 |
| 68 | 6.80 | 48 | 32 | 1.684 | 1.280 | 11.40 |
| 69 | 7.10 | 48 | 32 | 1.575 | 1.360 | 3.634 |
| 70 | 7.70 | 42 | 30 | 1.671 | 0.750 | 5.037 |
| 71 | 7.90 | 40 | 32 | 1.593 | 0.860 | 7.399 |
| 72 | 6.84 | 38 | 30 | 1.612 | 0.830 | 6.266 |
| 73 | 7.90 | 44 | 26 | 1.727 | 0.740 | 8.629 |
| 74 | 6.50 | 42 | 30 | 1.628 | 1.840 | 5.894 |
| 75 | 9.00 | 40 | 32 | 1.652 | 1.860 | 5.963 |
| 76 | 8.80 | 40 | 28 | 1.594 | 1.740 | 4.595 |
| 77 | 8.30 | 48 | 32 | 1.567 | 1.910 | 7.307 |
| 78 | 7.10 | 46 | 30 | 1.621 | 1.880 | 5.970 |
| 79 | 8.00 | 44 | 26 | 1.599 | 1.820 | 3.159 |
| 80 | 9.70 | 46 | 32 | 1.619 | 1.780 | 4.628 |
| 81 | 7.60 | 38 | 28 | 1.476 | 1.580 | 3.317 |
| 82 | 7.30 | 46 | 32 | 1.486 | 1.800 | 5.325 |
| 83 | 8.40 | 44 | 32 | 1.650 | 1.880 | 3.724 |
| 84 | 7.60 | 48 | 32 | 1.608 | 1.880 | 3.664 |
| 85 | 6.90 | 44 | 32 | 1.551 | 1.800 | 3.960 |

Table 2: Infiltration rate, silt content, clay content, bulk density, organic matter and moisture content of the fifteen soil samples used to verify soil infiltration rate model

| Sample No. | Infiltration rate (mm/h) | Silt content (%) | Clay content (%) | Bulk density (g/cm ³) | Organic matter (%) | Moisture content (%) |
|------------|--------------------------|------------------|------------------|-----------------------------------|--------------------|----------------------|
| 1 | 9.25 | 36 | 32 | 1.666 | 1.050 | 7.175 |
| 2 | 3.79 | 32 | 42 | 1.473 | 1.170 | 8.114 |
| 3 | 8.50 | 32 | 40 | 1.732 | 1.003 | 6.611 |
| 4 | 3.95 | 38 | 40 | 1.541 | 1.010 | 11.37 |
| 5 | 6.01 | 38 | 40 | 1.441 | 1.080 | 5.270 |
| 6 | 26.8 | 10 | 14 | 1.986 | 0.390 | 6.316 |
| 7 | 28.4 | 12 | 16 | 2.065 | 0.320 | 3.995 |
| 8 | 21.5 | 13 | 19 | 1.836 | 0.370 | 3.392 |
| 9 | 6.34 | 46 | 34 | 1.587 | 1.880 | 13.16 |
| 10 | 7.61 | 46 | 32 | 1.659 | 1.710 | 13.09 |
| 11 | 6.58 | 48 | 32 | 1.544 | 1.800 | 14.18 |
| 12 | 8.73 | 44 | 28 | 1.535 | 1.270 | 5.101 |
| 13 | 8.50 | 44 | 28 | 1.578 | 1.880 | 6.560 |
| 14 | 6.09 | 44 | 32 | 1.554 | 1.860 | 4.875 |
| 15 | 10.0 | 38 | 26 | 1.557 | 1.580 | 4.108 |

Also, in order to verify soil infiltration rate model, fifteen soil samples were taken at random from different fields of the experimental site. Again, infiltration rate, silt content, clay content, bulk density, organic matter and moisture content of the soil samples were measured as described before. Table 2 shows infiltration rate, silt content, clay content, bulk density, organic matter and moisture content of the fifteen soil samples used to verify soil infiltration rate model.

Regression Model: A typical five-variable linear regression model is shown in equation 1:

$$Y = k_0 + k_1X_1 + k_2X_2 + k_3X_3 + k_4X_4 + k_5X_5 \quad (1)$$

Where:

Y = Dependent variable, for example soil infiltration rate (mm/h)

X₁, X₂, X₃, X₄, X₅= Independent variables, for example silt content (%), clay content (%), bulk density (g/cm³), organic matter (%) and moisture content (%) of soil

k₀, k₁, k₂, k₃, k₄, k₅= Regression coefficients

In order to predict soil infiltration rate from silt content, clay content, bulk density, organic matter and moisture content of soil, a five-variable linear regression model was suggested and all the data (Table 1) were subjected to regression analysis using the Microsoft Excel 2007.

Statistical Analysis: A paired samples t-test and the mean difference confidence interval approach were used to compare the soil infiltration rate values predicted by model with the soil infiltration rate values measured by field tests. The Bland-Altman approach [10] was also used to plot the agreement between the soil infiltration rate values measured by field tests with the soil infiltration rate values predicted by model. The statistical analyses were also performed using Microsoft Excel 2007.

RESULTS AND DISCUSSION

The five-variable linear regression model, p-value of independent variables and coefficient of determination (R²) of the model are shown in Table 3. In this model, soil infiltration rate (IN) can be predicted as a function of silt content (SI), clay content (CL), bulk density (BD), organic matter (OM) and moisture content (MC) of soil. The p-value of independent variables (SI, CL, BD, OM and MC) and R² of the model were 9.42E-05, 2.36E-18, 0.111719, 0.157132, 0.786337 and 0.9092, respectively. Thus, based on the statistical results, the five-variable linear regression model was initially accepted, which is given by equation 2.

$$IN = 28.13 - 0.220 SI - 0.518 CL + 4.592 BD - 1.440 OM + 0.022 MC \quad (2)$$

A paired samples t-test and the mean difference confidence interval approach were used to compare the soil infiltration rate values predicted using the model with the soil infiltration rate values measured by field tests.

Table 3: Five-variable linear regression model, p-value of independent variables and coefficient of determination (R^2) of the model

| Model | p-value | | | | | R^2 |
|---|----------|----------|----------|----------|----------|--------|
| | SI | CL | BD | OM | MC | |
| IN = 28.13 - 0.220 SI - 0.518 CL + 4.592 BD - 1.440 OM + 0.022 MC | 9.42E-05 | 2.36E-18 | 0.111719 | 0.157132 | 0.786337 | 0.9092 |

Table 4: Silt content, clay content, bulk density, organic matter and moisture content of the fifteen soil samples used to verify soil infiltration rate model

| Sample No. | Silt content (%) | Clay content (%) | Bulk density (g/cm^3) | Organic matter (%) | Moisture content (%) | Infiltration rate (mm/h) | |
|------------|------------------|------------------|---------------------------|--------------------|----------------------|--------------------------|-------|
| | | | | | | Field tests | Model |
| 1 | 36 | 32 | 1.666 | 1.050 | 7.175 | 9.25 | 9.93 |
| 2 | 32 | 42 | 1.473 | 1.170 | 8.114 | 3.79 | 4.58 |
| 3 | 32 | 40 | 1.732 | 1.003 | 6.611 | 8.50 | 7.02 |
| 4 | 38 | 40 | 1.541 | 1.010 | 11.37 | 3.95 | 4.91 |
| 5 | 38 | 40 | 1.441 | 1.080 | 5.270 | 6.01 | 4.22 |
| 6 | 10 | 14 | 1.986 | 0.390 | 6.316 | 26.8 | 27.4 |
| 7 | 12 | 16 | 2.065 | 0.320 | 3.995 | 28.4 | 26.3 |
| 8 | 13 | 19 | 1.836 | 0.370 | 3.392 | 21.5 | 23.4 |
| 9 | 46 | 34 | 1.587 | 1.880 | 13.16 | 6.34 | 5.27 |
| 10 | 46 | 32 | 1.659 | 1.710 | 13.09 | 7.61 | 6.88 |
| 11 | 48 | 32 | 1.544 | 1.800 | 14.18 | 6.58 | 5.80 |
| 12 | 44 | 28 | 1.535 | 1.270 | 5.101 | 8.73 | 9.28 |
| 13 | 44 | 28 | 1.578 | 1.880 | 6.560 | 8.50 | 8.63 |
| 14 | 44 | 32 | 1.554 | 1.860 | 4.875 | 6.09 | 6.44 |
| 15 | 38 | 26 | 1.557 | 1.580 | 4.108 | 10.0 | 11.3 |

Table 5: Paired samples t-test analysis on comparing soil infiltration rate determination methods

| Determination methods | Average difference (mm/h) | Standard deviation of difference (mm/h) | p-value | 95% confidence intervals for the difference in means (mm/h) |
|-----------------------|---------------------------|---|---------|---|
| Model vs. field tests | -0.05 | 1.20 | 0.8814 | -0.71, 0.61 |

The Bland-Altman approach [10] was also used to plot the agreement between the soil infiltration rate values measured by field tests with the soil infiltration rate values predicted using the model.

The soil infiltration rate values predicted by model were compared with the soil infiltration rate values measured by field tests and are shown in Table 4. Also, a plot of the soil infiltration rate values determined by model and field tests with the line of equality (1.0: 1.0) is shown in Fig. 1. The mean soil infiltration rate difference between two methods was -0.05 mm/h (95% confidence interval: -0.71 and 0.61 mm/h; $P = 0.8814$). The standard deviation of the soil infiltration rate differences was 1.20 mm/h. The paired samples t-test results showed that the soil infiltration rate values predicted with model were not significantly different than the soil infiltration rate values measured with field tests (Table 5). The soil infiltration rate differences between these two methods were normally distributed and 95% of the soil infiltration rate differences were expected to lie between $\mu - 1.96\sigma$ and $\mu + 1.96\sigma$, known as 95% limits of agreement [10]. The 95% limits of agreement for comparison of soil infiltration rate determined with field tests and model were calculated

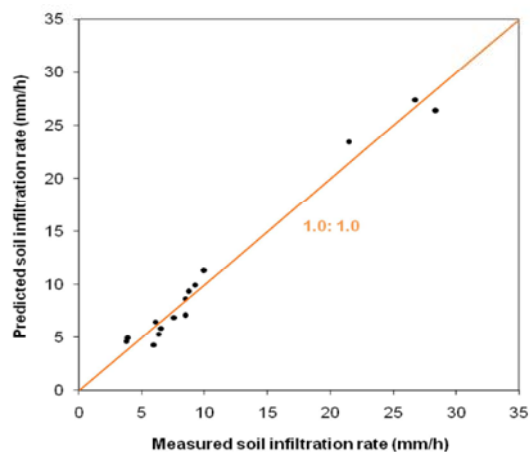


Fig. 1: Soil infiltration rate values measured using field tests (Measured soil infiltration rate) and soil infiltration rate values predicted using the model (Predicted soil infiltration rate) with the line of equality (1.0: 1.0)

at -2.40 and 2.30 mm/h (Fig. 2). Thus, soil infiltration rate predicted by model may be 2.40 mm/h lower or 2.30 mm/h higher than soil infiltration rate measured by field tests.

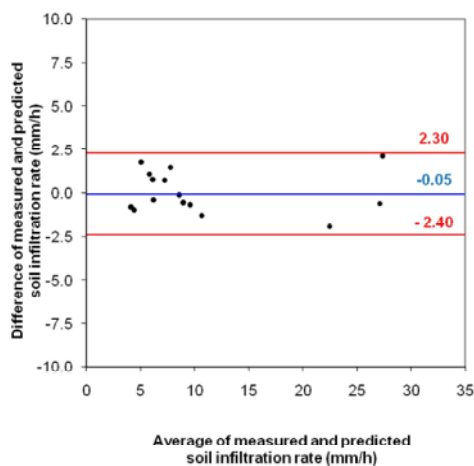


Fig. 2: Bland-Altman plot for the comparison of soil infiltration rate values measured using field tests (Measured soil infiltration rate) and soil infiltration rate values predicted using the model (Predicted soil infiltration rate); the outer lines indicate the 95% limits of agreement (-2.40, 2.30) and the center line shows the average difference (-0.05).

The average percentage difference for soil infiltration rate prediction using model and field tests was 12.2%. These results are in line with those of Smerdon *et al.* [1], Rashidi & Seyfi [2], Mustafa *et al.* [3], Walker *et al.* [4], Walker [5], Holzapfel *et al.* [6] and Walker & Busman [7], who reported that particle size distribution of soil was the most important factor which affected the soil infiltration rate. They also reported that bulk density, organic matter and moisture content of soil had significant effect on the soil infiltration rate.

CONCLUSIONS

A five-variable linear regression model was used to predict soil infiltration rate (IN) based on silt content (SI), clay content (CL), bulk density (BD), organic matter (OM) and moisture content (MC) of soil. The soil infiltration rate values predicted using the model was compared to the soil infiltration rate values measured by field tests. Results of study indicated that the difference between the soil infiltration rate values predicted by model and measured by field tests were not statistically significant. Therefore, the five-variable linear regression model $IN = 28.13 - 0.220 SI - 0.518 CL + 4.592 BD - 1.440 OM + 0.022 MC$ with $R^2 = 0.9092$ provide an easy, economic and brief method to predict soil infiltration rate.

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