

## Modeling of Deflection for Radial-Ply Tire Based on Overall Unloaded Diameter, Inflation Pressure, Vertical Load and Rotational Speed

*Parham Fatehirad, Majid Rashidi and Mohammad Gholami*

Department of Agricultural Machinery,  
Takestan Branch, Islamic Azad University, Takestan, Iran

**Abstract:** This study was conducted to model deflection ( $\delta$ ) of radial-ply tire based on overall unloaded diameter ( $d$ ), inflation pressure ( $P$ ), vertical load ( $W$ ) and rotational speed ( $N$ ). For this purpose, deflection of three radial-ply tires with different overall unloaded diameter were measured at three levels of inflation pressure, four levels of vertical load and six levels of rotational speed. In order to model deflection based on overall unloaded diameter, inflation pressure, vertical load and rotational speed, a four-variable linear regression model was suggested and all the data were subjected to regression analysis. The statistical results of study indicated that the four-variable linear regression model  $\delta = 221.58 - 0.3484 d - 0.3788 P + 0.0651 W - 0.0019 N$  with  $R^2 = 0.8003$  may be suggested to predict deflection of radial-ply tire based on overall unloaded diameter, inflation pressure, vertical load and rotational speed for a limited range of radial-ply tire sizes.

**Key words:** Radial-ply tire • Deflection • Overall unloaded diameter • Inflation pressure • Vertical load • Rotational speed • Modeling

### INTRODUCTION

In the case of tracked vehicles, the contact area between machine and ground surface is relatively constant for varying sinkage in the soil and is calculated as the length of track on hard ground times track width. However, a flexible tire has a smaller contact area on hard surface than it dose on soft ground. A rule of thumb which can be used for estimation of tire contact area is shown by equation 1 [1]:

$$A = bL \quad (1)$$

where:

A = Contact area of tire ( $m^2$ )

b = Section width of tire (m)

L = Contact length of tire (m)

Wong [2] and Bekker [3] gave an approximate method for calculating contact length of tire as given below in equation 2:

$$L = 2(d\delta - \delta^2)^{0.5} \quad (2)$$

where:

$d$  = Overall unloaded diameter of tire (m)

$\delta$  = Deflection of tire (m)

Tire deflection is a key parameter and many equations have been developed based on tire deflection to evaluate the tractive performance of bias-ply and radial-ply tires operating in cohesive-frictional soils. Gross traction, motion resistance, net traction and tractive efficiency are predicted as a function of soil strength, tire load, tire slip, tire size and tire deflection [4].

Fig. 1 shows the tire dimensions ( $b$ ,  $d$  and  $\delta$ ) used. The tire dimensions can be obtained from tire data book or by measuring the tire [4]. The section width ( $b$ ) is the first number in a tire size designation. The overall unloaded diameter ( $d$ ) can be obtained from the tire data handbooks available from off-road tire manufacturers. The tire deflection ( $\delta$ ) on a hard surface is equal to  $d/2$  minus the measured static loaded radius. The static loaded radius for the tire's rated load and inflation pressure is standard tire data from the tire data handbooks. It can also be obtained by measuring the tire [4, 5].

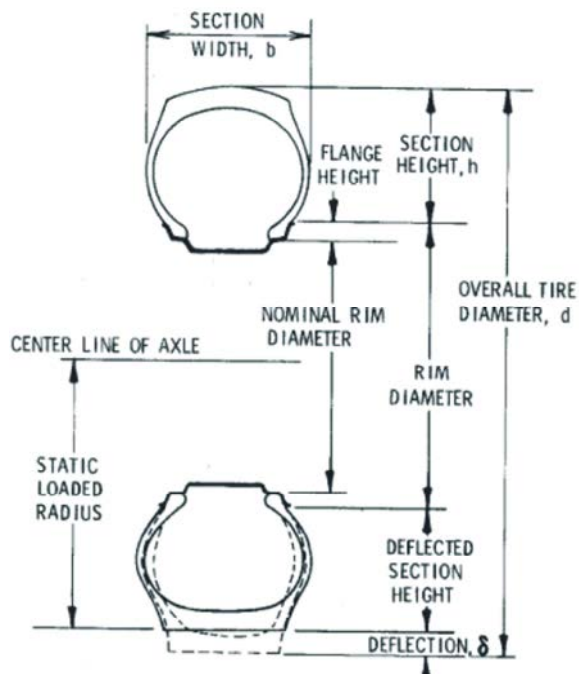


Fig. 1: Tire dimensions, adapted from Brixius [4]



Fig. 2: Tire deflection test apparatus

Table 1: Overall unloaded diameter of three radial-ply tires used in this study

Tire No.	Overall unloaded diameter d (mm)
1	578
2	582
3	605

As deflections for a given tire size, inflation pressure, vertical load and rotational speed may significantly be different between bias-ply and radial-ply tires, this study was conducted to model deflection ( $\delta$ ) of radial-ply tire based on overall unloaded diameter (d), inflation pressure (P), vertical load (W) and rotational speed (N) using a linear regression model.

## MATERIALS AND METHODS

**Tire Deflection Test Apparatus:** A tire deflection test apparatus was designed and constructed to measure deflection of tires with different sizes at diverse levels of inflation pressure, vertical load and rotational speed (Fig. 2).

**Experimental Procedure:** For this purpose, deflection of three radial-ply tires with different overall unloaded diameter were measured at three levels of inflation pressure, four levels of vertical load and six levels of rotational speed. The overall unloaded diameter of three radial-ply tires is given in Table 1. Results of deflection measurement for radial-ply tires No. 1, 2 and 3 are given in Tables 2, 3 and 4, respectively.

**Regression Model:** A typical four-variable linear regression model is shown in equation 3 [6-9]:

$$Y = C_0 + C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4 \quad (3)$$

where:

Y = Dependent variable, for example deflection of radial-ply tire

$X_1, X_2, X_3, X_4$  = Independent variables, for example overall unloaded diameter, inflation pressure, vertical load and rotational speed

$C_0, C_1, C_2, C_3, C_4$  = Regression coefficients

To model deflection based on overall unloaded diameter, inflation pressure, vertical load and rotational speed, a four-variable linear regression model was suggested.

Table 2: Overall unloaded diameter, inflation pressure, vertical load, rotational speed and deflection (three replications) for radial-ply tire No. 1

Overall unloaded diameter d (mm)	Inflation pressure P (psi)	Vertical load W (kg)	Rotational speed N (rev/min)	Deflection $\delta$ (mm)			
				$\delta_1$	$\delta_2$	$\delta_3$	
578	30	100	0	9.500	9.500	9.250	
			600	8.750	8.750	8.500	
			700	8.500	8.500	8.250	
			800	8.250	8.250	8.000	
			900	8.000	8.000	7.500	
		1000	7.500	7.500	7.250		
		150	0	14.50	14.50	14.25	
			600	13.25	13.25	13.00	
			700	13.00	13.00	12.75	
			800	12.75	12.75	12.50	
			900	12.50	12.50	12.25	
		200	1000	12.25	12.25	12.25	
			0	18.50	18.50	18.25	
			600	17.50	17.50	17.25	
			700	17.25	17.25	17.25	
	800		17.00	17.00	16.75		
	250	900	16.75	16.75	16.50		
		1000	16.50	16.50	16.50		
		0	22.00	22.00	21.75		
		600	20.75	20.75	20.50		
		700	20.50	20.50	20.25		
	578	35	100	800	20.25	20.25	20.00
				900	20.00	20.00	19.75
				1000	19.75	19.75	19.75
				0	11.25	11.25	11.00
				600	9.750	9.750	10.00
			700	9.750	9.500	9.750	
			150	800	9.500	9.250	9.250
				900	9.250	9.250	9.250
				1000	8.750	8.750	9.000
0				14.25	14.25	14.50	
600				13.25	13.25	13.50	
200			700	13.00	13.00	13.25	
			800	12.75	12.50	12.75	
			900	12.50	12.25	12.50	
			1000	12.25	12.25	12.25	
		0	18.50	18.75	18.50		
250		600	17.50	17.75	17.50		
		700	17.50	17.50	17.25		
		800	17.25	17.25	17.00		
		900	17.00	17.00	17.00		
		1000	16.25	16.50	16.25		
250		0	21.25	21.00	21.25		
		600	20.25	20.00	20.25		
		700	20.00	20.00	20.00		
		800	19.75	19.75	19.50		
		900	19.50	19.50	19.25		
1000		19.25	19.25	19.25			

Table 2: Continued

Overall unloaded diameter d (mm)	Inflation pressure P (psi)	Vertical load W (kg)	Rotational speed N (rev/min)	Deflection $\delta$ (mm)		
				$\delta_1$	$\delta_2$	$\delta_3$
40	100	0	0	8.250	8.500	8.250
			600	7.250	7.500	7.250
			700	6.750	7.000	6.750
			800	6.750	6.750	6.750
			900	6.500	6.500	6.750
			1000	6.250	6.250	6.000
		150	0	11.75	11.75	12.00
			600	10.75	10.75	11.00
			700	11.00	10.50	10.75
			800	10.75	10.75	10.50
			900	10.25	10.50	10.25
			1000	10.00	10.25	10.00
	200	0	0	15.25	15.00	15.25
			600	14.25	14.00	14.25
			700	14.00	13.75	14.00
			800	13.75	13.25	13.75
			900	13.50	13.25	13.50
			1000	13.25	13.00	13.25
		250	0	19.00	18.75	19.00
			600	18.00	17.75	18.00
			700	17.75	17.50	17.75
			800	17.25	17.00	17.50
			900	17.25	17.00	17.25
			1000	16.75	16.50	16.75

Table 3: Overall unloaded diameter, inflation pressure, vertical load, rotational speed and deflection (three replications) for radial-ply tire No. 2

Overall unloaded diameter d (mm)	Inflation pressure P (psi)	Vertical load W (kg)	Rotational speed N (rev/min)	Deflection $\delta$ (mm)		
				$\delta_1$	$\delta_2$	$\delta_3$
582	30	100	0	17.75	17.75	17.50
			600	17.00	17.00	16.75
			700	16.75	16.75	16.50
			800	16.50	16.50	16.25
			900	16.25	16.25	16.00
			1000	16.00	16.00	16.00
		150	0	21.00	21.00	20.75
			600	20.25	20.25	20.00
			700	20.00	20.00	19.75
			800	19.75	19.75	19.50
			900	19.50	19.50	19.25
			1000	19.25	19.25	19.00
	200	0	0	24.50	24.50	24.00
			600	23.50	23.50	23.25
			700	23.25	23.25	23.00
			800	23.00	23.00	22.75
			900	22.75	22.75	22.50
			1000	22.50	22.50	22.50
		250	0	27.00	27.00	26.75
			600	26.25	26.25	26.00
			700	26.00	26.00	25.75
			800	25.75	25.75	25.50
			900	25.50	25.50	25.25
			1000	25.25	25.25	25.00

Table 3: Continued

Overall unloaded diameter d (mm)	Inflation pressure P (psi)	Vertical load W (kg)	Rotational speed N (rev/min)	Deflection $\delta$ (mm)			
				$\delta_1$	$\delta_2$	$\delta_3$	
35	100	0	0	14.75	14.75	14.50	
			600	14.00	14.00	13.75	
			700	13.75	13.75	13.50	
			800	13.50	13.50	13.25	
			900	13.25	13.25	13.00	
		1000	13.00	13.00	12.75		
		150	0	18.25	18.25	18.50	
			600	17.50	17.50	17.75	
			700	17.25	17.25	17.50	
			800	17.00	17.00	17.25	
	900		16.75	16.75	17.00		
	200	1000	16.50	16.50	16.50		
			0	21.50	21.50	21.25	
			600	20.75	20.75	20.50	
			700	20.50	20.50	20.25	
			800	20.25	20.25	20.00	
		250	900	20.00	20.00	19.75	
			1000	19.75	19.75	19.50	
			0	25.25	25.25	25.00	
			600	24.50	24.50	24.25	
			700	24.25	24.25	24.00	
	40	100	0	0	14.50	14.50	14.25
				600	13.75	13.75	13.50
				700	13.50	13.50	13.25
				800	13.25	13.25	13.00
				900	13.00	13.00	12.75
			1000	12.75	12.75	12.75	
			150	0	17.50	17.50	17.25
				600	16.75	16.75	16.50
				700	16.50	16.50	16.25
800				16.25	16.25	16.00	
900		16.00		16.00	15.75		
200		1000	15.75	15.75	15.50		
			0	19.75	19.75	19.50	
			600	19.00	19.00	18.75	
			700	18.75	18.75	18.50	
			800	18.50	18.50	18.25	
		250	900	18.25	18.25	18.00	
			1000	18.00	18.00	17.75	
			0	22.25	22.25	22.00	
			600	21.50	21.50	21.25	
			700	21.25	21.25	21.00	
40		100	0	800	21.00	21.00	20.75
				900	20.75	20.75	20.50
				1000	20.50	20.50	20.50

Table 4: Overall unloaded diameter, inflation pressure, vertical load, rotational speed and deflection (three replications) for radial-ply tire No. 3

Overall unloaded diameter d (mm)	Inflation pressure P (psi)	Vertical load W (kg)	Rotational speed N (rev/min)	Deflection $\delta$ (mm)			
				$\delta_1$	$\delta_2$	$\delta_3$	
605	30	100	0	5.500	5.500	5.250	
			600	4.750	4.750	4.500	
			700	4.500	4.500	4.250	
			800	4.250	4.250	4.000	
			900	4.000	4.000	3.750	
		1000	3.750	3.750	3.500		
		150	0	9.500	9.500	9.250	
			600	8.250	8.250	8.000	
			700	8.000	8.000	7.750	
			800	7.750	7.750	7.500	
			900	7.500	7.500	7.250	
		200	1000	7.250	7.250	7.000	
			0	12.50	12.50	12.25	
			600	11.25	11.25	11.00	
			700	11.00	11.00	10.75	
	800		10.75	10.75	10.50		
	250	900	10.50	10.50	10.25		
			1000	10.25	10.25	10.50	
			0	17.25	17.25	17.00	
			600	16.00	16.00	15.75	
			700	15.75	15.75	15.50	
		800	15.50	15.50	15.25		
		900	15.25	15.25	15.00		
		1000	15.00	15.00	14.75		
		35	100	0	3.000	3.000	2.750
				600	2.000	2.000	1.750
	700			1.750	1.750	1.500	
	800			1.500	1.500	1.250	
	900			1.250	1.250	1.000	
	1000			1.000	1.000	1.000	
150	0			7.250	7.250	7.000	
	600			6.500	6.500	6.250	
	700			6.250	6.250	6.000	
	800			6.000	6.000	5.750	
	900			5.750	5.750	5.500	
200	1000			5.500	5.500	5.250	
	0			8.750	8.750	8.500	
	600			7.500	7.500	7.250	
	700			7.250	7.250	7.000	
	800		7.000	7.000	6.750		
250	900		6.750	6.750	6.500		
			1000	6.500	6.500	6.250	
			0	13.25	13.25	13.00	
			600	12.25	12.25	12.25	
			700	12.00	12.00	11.75	
	800		11.75	11.75	11.75		
	900		11.75	11.50	11.50		
	1000		11.50	11.25	11.25		

Table 4: Continued

Overall unloaded diameter d (mm)	Inflation pressure P (psi)	Vertical load W (kg)	Rotational speed N (rev/min)	Deflection $\delta$ (mm)				
				$\delta_1$	$\delta_2$	$\delta_3$		
40	100	0	0	2.750	2.750	2.500		
			600	1.750	1.750	1.500		
			700	1.500	1.500	1.250		
			800	1.250	1.250	1.000		
			900	1.000	1.000	0.750		
			1000	0.750	1.000	0.750		
			150	0	0	5.000	5.000	4.750
					600	3.750	3.750	3.500
					700	3.500	3.500	3.250
					800	3.250	3.250	3.000
	900	3.000			3.000	2.750		
	1000	2.750			2.750	2.500		
	200	0			0	8.000	8.000	7.750
					600	7.000	7.000	6.750
					700	6.750	6.750	6.500
					800	6.500	6.500	6.250
			900	6.250	6.250	6.000		
			1000	6.000	6.000	6.750		
			250	0	0	9.250	9.250	9.000
					600	8.000	8.000	7.750
700					7.750	7.750	7.500	
800					7.500	7.500	7.250	
900	7.250	7.250			7.000			
1000	7.000	7.000			6.750			

Table 5: Four-variable linear regression model, p-value of independent variables and coefficient of determination ( $R^2$ )

Model	p-value				$R^2$
	d	P	W	N	
$\delta = 221.58 - 0.3484 d - 0.3788 P + 0.0651 W - 0.0019 N$	5.8E-159	2.53E-37	2.0E-135	5.53E-08	0.8003

### RESULTS AND DISCUSSION

In order to model deflection of radial-ply tire based on overall unloaded diameter, inflation pressure, vertical load and rotational speed, a four-variable linear regression model was suggested and all the data were subjected to regression analysis using the Microsoft Excel 2007. The four-variable linear regression model, p-value of independent variables and coefficient of determination ( $R^2$ ) of the model are shown in Table 5. As it is shown in Table 5, this model has a high  $R^2$  value at 0.8003, indicating good agreement of the experimental data. In addition, the p-value of independent variables (d, P, W and N) is as follows: 5.8E-159, 2.53E-37, 2.0E-135 and 5.53E-08, respectively. Thus, based on the statistical results, this model is initially accepted, which is given by equation 4:

$$\delta = 221.58 - 0.3484 d - 0.3788 P + 0.0651 W - 0.0019 N$$

In this model, deflection of radial-ply tire can be predicted using a four-variable linear regression of overall unloaded diameter, inflation pressure, vertical load and rotational speed.

### CONCLUSION

It can be concluded that the four-variable linear regression model  $\delta = 221.58 - 0.3484 d - 0.3788 P + 0.0651 W - 0.0019 N$  with  $R^2 = 0.8003$  may be suggested to predict deflection of radial-ply tire based on overall unloaded diameter, inflation pressure, vertical load and rotational speed for a limited range of radial-ply tire sizes.

### REFERENCES

1. McKyes, E., 1985. Soil Cutting and Tillage. Elsevier Science Publishing Company Inc., New York, USA.
2. Wong, J.Y., 1978. Theory of Ground Vehicles. John Wiley and Sons, New York, USA.

3. Bekker, M.G., 1985. The effect of tire tread in parametric analyses of tire-soil systems. NRCC Report No. 24146, National Research Council of Canada.
4. Brixius, W.W., 1987. Traction prediction equations for bias ply tires. ASAE Paper No. 871622. St. Joseph, Mich.: ASAE.
5. Goering, C.E., M.L. Stone, D.W. Smith and P.K. Turnquist, 2006. Off-Road Vehicle Engineering Principles. St. Joseph, Mich.: ASABE.
6. Azadeh, S., M. Rashidi and M. Gholami, 2013. Modeling of bias-ply tire deflection based on tire dimensions, tire inflation pressure and vertical load on tire. Middle-East J. Sci. Res., 14(1): 117-121.
7. Mousavi, M., M. Rashidi, I. Ranjbar, M.S. Garmroudi and M. Ghaebi, 2013. Modeling of bias-ply tire contact area based on tire dimensions, tire inflation pressure and vertical load on tire using linear regression models. Am-Euras. J. Agric. & Environ. Sci., 13(5): 627-632.
8. Oroojloo, M., M. Rashidi and M. Gholami, 2013. Modeling of radial-ply tire contact area based on tire dimensions, tire inflation pressure and vertical load on tire. Middle-East J. Sci. Res., 17(7): 949-954.
9. Sheikhi, M.A., M. Rashidi and M. Gholami, 2013. Modeling of radial-ply tire deflection based on tire dimensions, tire inflation pressure and vertical load on tire. Am-Euras. J. Agric. & Environ. Sci., 13(2): 222-226.