

Computation of Makespan Using Genetic Algorithm in a Flowshop

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Abstract: This research paper addresses the scheduling problems with the primary objective of minimizing the makespan in a flow shop with 'N' jobs through 'M' machines. The EPDT (Heuristic approach) and BAT (Meta-Heuristic approach) heuristics are proposed to solve the flow shop scheduling problem in a modern manufacturing environment. These two algorithms are applied along with the Genetic Algorithm (GA) for the further improvement of results in achieving the minimal makespan. The performances of these newer heuristics are evaluated by solving the Taillard benchmark problems in MATLAB environment with various sizes of problems. The proposed GA applied EPDT heuristic and GA applied BAT meta-heuristic for the flow shop problems have been found very effective in solving scheduling problems and finding a better sequence which can reduce the makespan to a great extent. The improvement of EPDT and BAT were obtained by applying the GA yields superior results as well as these results also very close to upper bound than NEH results. The results of the heuristics are tested statistically by ANOVA and it shows that the GA applied heuristics gives a quality solution.

Key words: Genetic algorithm • Mutation • Crossover

INTRODUCTION

A Permutation Flow Shop (PFS) is a shop design of machines arranged in series in which the jobs are processed in a same order without eliminating any machine. Generally, the following assumptions are considered in any flowshop environment,

- Pre-emption is not allowed. Once an operation is started on the machine, it must be completed before another operation can begin on that machine.
- Machines never break down and are available throughout the scheduling period.
- All processing time on the machine are known, deterministic, finite and independent of sequence of the jobs to be processed.
- All the machines are readily available for continuous assignment, without consideration of temporary unavailability such as breakdown or maintenance.
- Each job is processed through each of the 'M' machines once and only once. Also a job does not become available to the next machine until and unless processing of the current machine is completed.

- In-process inventory is allowed. If a next machine in the sequence needed by a job is not available, the job can wait and join the queue of that machine.

Here the scheduling is a vital task which involves organizing, choosing and timing resource used to carry out all the activities necessary to produce the desired output at the desired time, while satisfying a large number of time and relationship constraints among the activities and the resources [1]. This forces researchers to focus their efforts in developing an optimal solution for achieving minimum makespan with newer heuristics.

An algorithm was developed, for flowshop scheduling problems with 'N' jobs through 2 machines [2]. The NP-completeness of the flow shop scheduling problems had been discussed by Quan-Ke Pan and Ling Wang in detail [3]. Palmer [4] was the first to propose a heuristic with a slope index procedure, which was an effective and simple methodology in tracing a better makespan.

A significant work in the development of an effective heuristic was discussed by CDS [5]. Their algorithm consists essentially in splitting the 'M' machine problem into a series of equivalent two-machine flow shop

problems and solving by Johnson's rule. Dannenbring [6] had developed a procedure called 'rapid access', which attempted to combine the advantages of Palmer's slope index and CDS procedures.

Stinson and Simith [7] had proposed a different approach called travelling salesman problem with two steps. The solution was found to be better than Palmer [8] and CDS methods, but with increased computational effort.

Since the problem is NP-hard, the meta-heuristics are required to solve effectively the industry size problems. Thus, the meta-heuristics with search techniques were developed to achieve the near optimal solutions for the PFS problems [9]. For applying a local search technique in a PFS, an initial solution is generated and then it applies a move mechanism to search the neighborhood of the current solution to choose the better one [10]. Schuster and Framinan [11] used the neighborhood search technique which was specially designed for flow shop problems. This technique yields better result compared to others. A step of local search starts with the current feasible solution $x \in X$ to which is applied a function $m \in M(x)$ that transforms x into x' , a new feasible solution ($x' = m(x)$). This transformation is called a move and $\{x': x' = m(x); x, x' \in X; m \in M(x)\}$ is called the neighborhood of x .

These heuristics can be further improved by adding a sub-process called searching technique. There are many searching techniques, some of them are Particle Swarm Optimization [12], two-phase subpopulation genetic algorithm (GA) [13], HAS [14], hybrid genetic algorithm [15].

Among these techniques, the hybrid genetic algorithm performances well [16]. There are various methods to improve the performance of the genetic algorithm. The first possibility is to implement the best configuration of the algorithm itself [17, 18]. Alternatively, we could add in other heuristics as sub-process of the genetic algorithm, called hybrid GA (HGA). The most popular forms of the hybrid GA are to incorporate one or more of hill climbing and/or neighborhood search [19].

This research paper aims to minimize the makespan of a permutation flowshop through the application of hybrid genetic algorithm in a heuristic and meta-heuristic approach.

Methodologies

Method I: EPDT Heuristic: The heuristic distributes a higher class of exponential factor to the processing time of the job based on the machine it passes through.

This helps in developing a mathematical model which is determined from the advancement of a classical algorithm called 'slope index' algorithm.

The exponential value factor added to the job processing time is evaluated through the exponential equation [20], which gives an index value to the job. By sorting the index value of the jobs in descending order, an optimal sequence can be obtained.

Algorithm:

Step 1: Let 'n' number of jobs to be machined through 'm' machines. It is assumed that all jobs are present for processing at time zero. And one job can run on one machine at a time without changing the machine order.

Step 2: The exponential index to be calculated using the exponential equation (1) for 'n' jobs.

$$y_j = \sum_{i=0}^{m-1} (2.61 * m - \exp(i)) * T_{jm-i} \quad (1)$$

where,

Y_j = Exponential index value for j^{th} job,

m = Number of machines

$T_{j(m-i)}$ = Process time of j^{th} job under $(m-i)^{\text{th}}$ machine

Step 3: Sort the exponential index in descending order.

Step 4: Based on the sorted order, the jobs to be sequenced.

Method II: BAT Heuristic: The newly proposed heuristic (BAT heuristic) is to find an optimal makespan using mathematical logics with local search technique [21].

Algorithm

Step 1: Assign the processing time of 'N' jobs in 'M' machines. And frame the PFS problem $N \times M$ matrix.

Step 2: Calculate a_{ij} and b_{ij} values using the equations (2) and (3).

$$a_{ij} = \sum_{i=1}^{k-1} P_{ij} \quad (2)$$

$$b_{ij} = \sum_{i=k+1}^m P_{ij} \quad (3)$$

Step 3: Calculate T_i , A_i and B_i values using the equations (4), (5) and (6).

$$T_i = \sum_{j=1}^n P_{ij} \quad (4)$$

$$A_i = \min(a_{ij}) \quad (5)$$

$$B_i = \min(b_{ij}) \quad (6)$$

Step 4: Calculate the S_i values for 'M' machines using the equation (7).

$$S_i = T_i + A_i + B_i \quad (7)$$

Step 5: Calculate the LB value for the $N \times M$ PFS problem using the equation (8).

$$LB = \max(S_i) \quad (8)$$

Step 6: Identify the Z machine by the below stated condition in equation (9).

$$Z = k; \text{ if } (LB = T_k + A_k + B_k) \quad (9)$$

Step 7: Identify the pivot jobs ZA and ZB is using the condition stated in equation (10) and (11).

$$ZA = j; \text{ if } (A_k = a_{kj}) \quad (10)$$

$$ZB = j; \text{ if } (A_k = b_{kj}) \quad (11)$$

Step 8: Place the ZA and ZB pivoted jobs in the sequence under the condition, if the pivoted job is ZA, ($Z \neq 1$) && ($ZA \neq 1$) then place the ZA at the beginning of the sequence. If the pivoted job is ZB, ($Z \neq M$) && ($ZB \neq N$) then place the ZB at the end of the sequence.

Step 9: After the step 9 is successful, eliminate the ZA and ZB jobs from the $N \times M$ PFS problem.

Step 10: Apply local search technique by repeating the step 3 to step 10.

Step 11: Arrange the jobs in a sequence according to the pivoting conditions.

Genetic Algorithm (GA) for Flow Shop Scheduling:

The genetic algorithm (GA) was proposed by John Holland [22]. However, it has become one of the well-known meta-heuristics after Goldberg [23]. The mechanism of the simple GA is demonstrated in a pseudo

code. Hybrid genetic algorithm (HGA) [24] is a method of searching an optimal solution based on an evolutionary technique which works with a population of solutions. In the proposed GA, a population of solutions was considered and the fitness of each solution was evaluated by using a problem specific objective function after crossover as well as mutation operations. Then the best solution was selected which ensured a better solution. The stages of GA are as follows [25].

Pseudo Code for HGA:

Step 1: Initialize a population from the heuristic proposed sequence.

Step 2: Perform a crossover operation to get offspring based on the probability of crossover.

Step 3: Conduct a mutation based on the probability of mutation.

Step 4: Fitness evaluation for each individual using an objective function of minimum makespan.

Step 5: Randomly select the survived chromosome for the next generation using roulette wheel.

Chromosome representation- A solution to the N-job and M-machine problem was represented as a chromosome. A chromosome consists of 'M' parts; each part corresponding to each machine and consisting of 'n' bits that represent the order of jobs on that machine.

Fitness function- It evaluated the performance measures to be optimized. A fitness value was found for each chromosome or schedule which was the weighted sum of makespan.

Initial population- The initial solution or a population plays a critical role in determining the quality of the final solution. The sequence from the heuristic is taken as initial solution.

Selection- The better chromosome is selected by comparing the parent and daughter chromosomes under each stage or spin.

Crossover- The crossover process was used to breed a pair of children chromosomes from a pair of parent chromosomes. The crossover operator randomly chooses a locus and exchanged the sub-sequences before and after that locus between two chromosomes. Thus two new children chromosomes were developed from two parent chromosomes by crossover.

Mutation- If a random number generated was less than the mutation probability and then mutation would be carried out. Here, the mutation was done by interchanging two bits of a chromosome selected at random.

RESULTS AND DISCUSSION

Statistical Analysis Using Taillard Benchmark Problems:

The benchmark problems proposed by Taillard [26] are tested against the newly proposed EPDT heuristic and BAT heuristic for the various sizes of the problems with 20, 50 & 100 jobs through 5, 10 & 20 machines. The results obtained from the MATLAB environment for the NEH heuristic, EPDT heuristic, GA applied EPDT heuristic [27, 28], BAT heuristic and GA applied BAT heuristic were compared and tabulated in Table 1 to 9. The maximum relative deviation from the upper bound was calculated using the equation (12).

Maximum Relative Deviation (MRD) = (Makespan–upper bound)/makespan*100

(12)

From the Table 1 to 9, it can be seen that the GA based EPDT and BAT heuristics are found improved

when compared with NEH heuristic and the MRD also shows the same. The Table 10 and Figure 1 shows the average results of Table 1-9.

From the Table 10, the average MRD to UB was calculated and it is shown in Table 11 and Figure 2. It is observed that the GA applied BAT heuristic was better compared to others with less computational instances.

Analysis of Variance (ANOVA): The ANOVA is carried out to check the three main hypotheses which are normality, homogeneity of variance and independence of residuals. The residuals resulting from the experimental data were analyzed and all three hypotheses could be accepted [29]. For example, the normality can be checked by the plot of the residuals. Here the One way ANOVA was carried out in MINITAB16 environment, considering the makespan reaching the Upper Bound of the NEH, EPDT, GA applied EPDT, BAT and GA applied BAT heuristics. This analysis has been made to determine the optimal noise level by “smaller as best” concept and the best significant level has been identified for the GA applied BAT heuristic from the Table 12 and has been shown that the p-value is 0.419 which is lesser than f-value of 0.98, at 95% confidence level.

Table 1: 5 machines 20 jobs

Seeds	Upper Bound	Makespan					Maximum Relative deviation from Upper Bound				
		NEH	EPDT	GA EPDT	BAT	GA BAT	NEH	EPDT	GA EPDT	BAT	GA BAT
873654221	1278	1286	1377	1339	1336	1278	0.622	7.190	4.556	4.341	0.000
379008056	1359	1365	1360	1316	1360	1360	0.440	0.074	-3.267	0.074	0.074
1866992158	1081	1159	1236	1176	1185	1081	6.730	12.540	8.078	8.776	0.000
216771124	1293	1325	1564	1356	1338	1299	2.415	17.327	4.646	3.363	0.462
495070989	1236	1305	1342	1291	1273	1235	5.287	7.899	4.260	2.907	-0.081
402959317	1195	1228	1385	1224	1280	1195	2.687	13.718	2.369	6.641	0.000
1369363414	1239	1278	1268	1259	1303	1251	3.052	2.287	1.589	4.912	0.959
2021925980	1206	1223	1504	1237	1313	1206	1.390	19.814	2.506	8.149	0.000
573109518	1230	1291	1434	1372	1239	1230	4.725	14.226	10.350	0.726	0.000
88325120	1108	1151	1298	1203	1170	1108	3.736	14.638	7.897	5.299	0.000

Table 2: 10 machines 20 jobs

Seeds	Upper Bound	Makespan					Maximum Relative deviation from Upper Bound				
		NEH	EPDT	GA EPDT	BAT	GA BAT	NEH	EPDT	GA EPDT	BAT	GA BAT
587595453	1582	1680	1915	1665	1752	1583	5.833	17.389	4.985	9.703	0.063
1401007982	1659	1729	1928	1775	1906	1660	4.049	13.952	6.535	12.959	0.060
873136276	1496	1557	1737	1676	1884	1508	3.918	13.874	10.740	20.594	0.796
268827376	1378	1439	1727	1450	1585	1384	4.239	20.208	4.966	13.060	0.434
1634173168	1419	1502	1713	1485	1597	1430	5.526	17.163	4.444	11.146	0.769
691823909	1397	1453	1618	1488	1518	1414	3.854	13.659	6.116	7.971	1.202
73807235	1484	1562	1870	1515	1628	1484	4.994	20.642	2.046	8.845	0.000
1273398721	1538	1609	1928	1588	1735	1550	4.413	20.228	3.149	11.354	0.774
2065119309	1593	1647	1832	1692	1831	1609	3.279	13.046	5.851	12.998	0.994
1672900551	1591	1653	2035	1661	1855	1614	3.751	21.818	4.214	14.232	1.425

Table 3: 20 machines 20 jobs

Seeds	Upper Bound	Makespan					Maximum Relative deviation from Upper Bound				
		NEH	EPDT	GA EPDT	BAT	GA BAT	NEH	EPDT	GA EPDT	BAT	GA BAT
479340445	2297	2410	2606	2409	2571	2305	4.689	11.857	4.649	10.657	0.347
268827376	2100	2150	2516	2287	2236	2105	2.326	16.534	8.177	6.082	0.238
1958948863	2326	2411	2575	2546	2510	2342	3.526	9.670	8.641	7.331	0.683
918272953	2223	2262	2561	2329	2438	2233	1.724	13.198	4.551	8.819	0.448
555010963	2291	2397	2513	2444	2452	2307	4.422	8.834	6.260	6.566	0.694
2010851491	2226	2349	2697	2398	2370	2235	5.236	17.464	7.173	6.076	0.403
1519833303	2273	2362	2687	2396	2398	2273	3.768	15.408	5.134	5.213	0.000
1748670931	2200	2249	2676	2387	2383	2212	2.179	17.788	7.834	7.679	0.542
1923497586	2237	2320	2553	2412	2392	2255	3.578	12.378	7.255	6.480	0.798
1829909967	2178	2277	2372	2339	2372	2186	4.348	8.179	6.883	8.179	0.366

Table 4: 5 machines 50 jobs

Seeds	Upper Bound	Makespan					Maximum Relative deviation from Upper Bound				
		NEH	EPDT	GA EPDT	BAT	GA BAT	NEH	EPDT	GA EPDT	BAT	GA BAT
1328042058	2724	2733	2906	2735	2735	2724	0.329	6.263	0.402	0.402	0.000
200382020	2836	2843	3055	2987	2987	2838	0.246	7.169	5.055	5.055	0.070
496319842	2621	2640	2902	2789	2789	2621	0.720	9.683	6.024	6.024	0.000
1203030903	2751	2782	3052	2898	2898	2751	1.114	9.862	5.072	5.072	0.000
1730708564	2863	2868	3125	3013	3013	2864	0.174	8.384	4.978	4.978	0.035
450926852	2829	2850	3067	2852	2852	2829	0.737	7.760	0.806	0.806	0.000
1303135678	2725	2758	2858	2878	2878	2725	1.197	4.654	5.316	5.316	0.000
1273398721	2683	2721	2984	2745	2745	2683	1.397	10.087	2.259	2.259	0.000
587288402	2554	2576	2830	2800	2634	2554	0.854	9.753	8.786	3.037	0.000
248421594	2782	2790	2970	2906	2820	2782	0.287	6.330	4.267	1.348	0.000

Table 5: 10 machines 50 jobs

Seeds	Upper Bound	Makespan					Maximum Relative deviation from Upper Bound				
		NEH	EPDT	GA EPDT	BAT	GA BAT	NEH	EPDT	GA EPDT	BAT	GA BAT
1958948863	3037	3135	3717	3422	3122	3045	3.126	18.294	11.251	2.723	0.263
575633267	2911	3032	3429	3256	3256	2927	3.991	15.106	10.596	10.596	0.547
655816003	2873	2986	3402	3251	3251	2871	3.784	15.550	11.627	11.627	-0.070
1977864101	3067	3198	3325	3220	3220	3078	4.096	7.759	4.752	4.752	0.357
93805469	3025	3160	3726	3197	3118	3031	4.272	18.814	5.380	2.983	0.198
1803345551	3021	3178	3846	3356	3356	3020	4.940	21.451	9.982	9.982	-0.033
49612559	3124	3277	3624	3244	3222	3148	4.669	13.797	3.699	3.042	0.762
1899802599	3048	3123	3640	3213	3102	3063	2.402	16.264	5.135	1.741	0.490
2013025619	2913	3002	3662	3101	3101	2936	2.965	20.453	6.063	6.063	0.783
578962478	3114	3257	3655	3465	3440	3131	4.391	14.802	10.130	9.477	0.543

Table 6: 20 machines 50 jobs

Seeds	Upper Bound	Makespan					Maximum Relative deviation from Upper Bound				
		NEH	EPDT	GA EPDT	BAT	GA BAT	NEH	EPDT	GA EPDT	BAT	GA BAT
1539989115	3886	4082	4610	4268	4268	3936	4.802	15.705	8.950	8.950	1.270
691823909	3733	3921	4338	4087	4087	3813	4.795	13.947	8.662	8.662	2.098
655816003	3689	3927	4513	4160	4160	3733	6.061	18.258	11.322	11.322	1.179
1315102446	3755	3969	4557	4062	4062	3832	5.392	17.599	7.558	7.558	2.009
1949668355	3655	3835	4603	4095	4095	3701	4.694	20.595	10.745	10.745	1.243
1923497586	3719	3914	4478	4020	4013	3787	4.982	16.950	7.488	7.326	1.796
1805594913	3730	3952	4642	4134	4134	3843	5.617	19.647	9.773	9.773	2.940
1861070898	3744	3938	4534	4033	4033	3778	4.926	17.424	7.166	7.166	0.900
715643788	3790	3952	4417	4157	4157	3845	4.099	14.195	8.828	8.828	1.430
464843328	3791	4079	4646	4115	4115	3857	7.061	18.403	7.874	7.874	1.711

Table 7: 5 machines 100 jobs

Seeds	Upper Bound	Makespan					Maximum Relative deviation from Upper Bound				
		NEH	EPDT	GA EPDT	BAT	GA BAT	NEH	EPDT	GA EPDT	BAT	GA BAT
896678084	5493	5519	5838	5828	5495	5493	0.471	5.910	5.748	0.036	0.000
1179439976	5274	5348	5536	5442	5389	5268	1.384	4.733	3.087	2.134	-0.114
1122278347	5175	5219	5674	5414	5340	5175	0.843	8.795	4.414	3.090	0.000
416756875	5018	5023	5425	5271	5225	5023	0.100	7.502	4.800	3.962	0.100
267829958	5250	5266	6165	5311	5311	5255	0.304	14.842	1.149	1.149	0.095
1835213917	5135	5139	5520	5233	5233	5135	0.078	6.975	1.873	1.873	0.000
1328833962	5247	5259	5497	5361	5342	5246	0.228	4.548	2.126	1.778	-0.019
1418570761	5106	5120	5754	5528	5303	5094	0.273	11.262	7.634	3.715	-0.236
161033112	5454	5489	5738	5686	5686	5448	0.638	4.949	4.080	4.080	-0.110
304212574	5328	5341	5587	5342	5342	5325	0.243	4.636	0.262	0.262	-0.056

Table 8: 10 machines 100 jobs

Seeds	Upper Bound	Makespan					Maximum Relative deviation from Upper Bound				
		NEH	EPDT	GA EPDT	BAT	GA BAT	NEH	EPDT	GA EPDT	BAT	GA BAT
1539989115	5776	5846	6339	5937	5937	5800	1.197	8.882	2.712	2.712	0.414
655816003	5362	5453	6298	5523	5523	5362	1.669	14.862	2.915	2.915	0.000
960914243	5679	5824	6497	6134	6134	5681	2.490	12.590	7.418	7.418	0.035
1915696806	5820	5929	6742	6089	6089	5841	1.838	13.675	4.418	4.418	0.360
2013025619	5491	5679	6617	6019	6019	5503	3.310	17.017	8.772	8.772	0.218
1168140026	5308	5375	6279	5633	5633	5328	1.247	15.464	5.770	5.770	0.375
1923497586	5602	5704	6476	5738	5738	5627	1.788	13.496	2.370	2.370	0.444
167698528	5640	5760	6279	6541	6279	5646	2.083	10.177	13.775	10.177	0.106
1528387973	5891	6032	6524	6420	6420	5925	2.338	9.703	8.240	8.240	0.574
993794175	5860	5918	6468	6338	6338	5903	0.980	9.400	7.542	7.542	0.728

Table 9: 20 machines 100 jobs

Seeds	Upper Bound	Makespan					Maximum Relative deviation from Upper Bound				
		NEH	EPDT	GA EPDT	BAT	GA BAT	NEH	EPDT	GA EPDT	BAT	GA BAT
450926852	6345	6541	7240	6769	6769	6420	2.996	12.362	6.264	6.264	1.168
1462772409	6323	6523	7584	6922	6922	6386	3.066	16.627	8.654	8.654	0.987
1021685265	6385	6639	7668	7030	7030	6445	3.826	16.732	9.175	9.175	0.931
83696007	6331	6557	7616	6907	6907	6410	3.447	16.872	8.339	8.339	1.232
508154254	6405	6695	7590	6730	6730	6465	4.332	15.613	4.829	4.829	0.928
1861070898	6487	6664	7430	7159	7159	6548	2.656	12.692	9.387	9.387	0.932
26482542	6393	6632	7730	7075	7075	6405	3.604	17.296	9.640	9.640	0.187
444956424	6514	6739	7589	7225	7225	6605	3.339	14.165	9.841	9.841	1.378
2115448041	6386	6677	7433	7095	7095	6439	4.358	14.086	9.993	9.993	0.823
118254244	6544	6677	7769	6893	6893	6602	1.992	15.768	5.063	5.063	0.879

Table 10: Comparison of heuristics based on MRD to UB

	NEH	EPDT	GA EPDT	BAT	GA BAT
20 Jobs, 5 M/C	3.10839	10.97127	4.298336	4.518815	0.141368
20 Jobs, 10 M/C	4.385454	17.19799	5.304582	12.28634	0.651782
20 Jobs, 20 M/C	3.579464	13.13088	6.65574	7.308178	0.451852
50 Jobs, 5 M/C	0.705455	7.994422	4.296603	3.429801	0.010539
50 Jobs, 10 M/C	3.863544	16.229	7.861442	6.298369	0.384036
50 Jobs, 20 M/C	5.242789	17.27227	8.836489	8.820352	1.657697
100 Jobs, 5 M/C	0.456174	7.41504	3.517344	2.207878	-0.03403
100 Jobs, 10 M/C	1.894042	12.52658	6.393066	6.033279	0.32548
100 Jobs, 20 M/C	3.361546	15.22128	8.118412	8.118412	0.944455

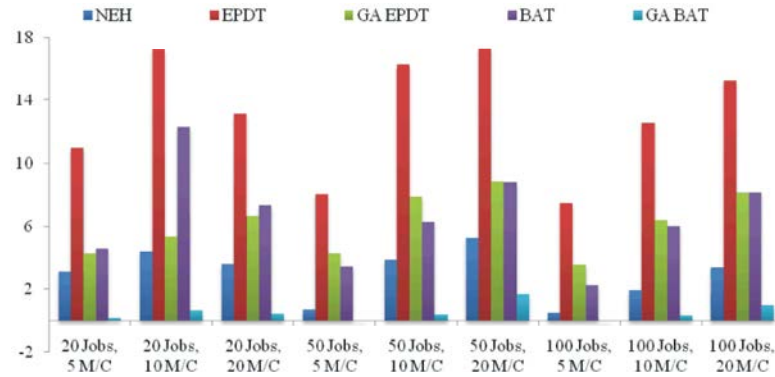


Fig. 1: Comparison of heuristics based on MRD to UB

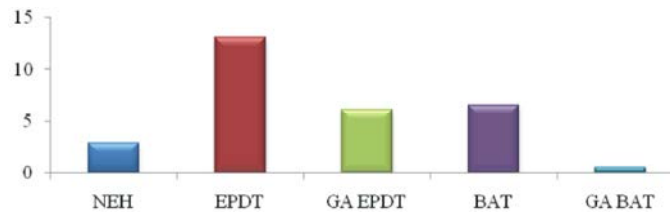


Fig. 2: Comparison of heuristics based on the overall MRD

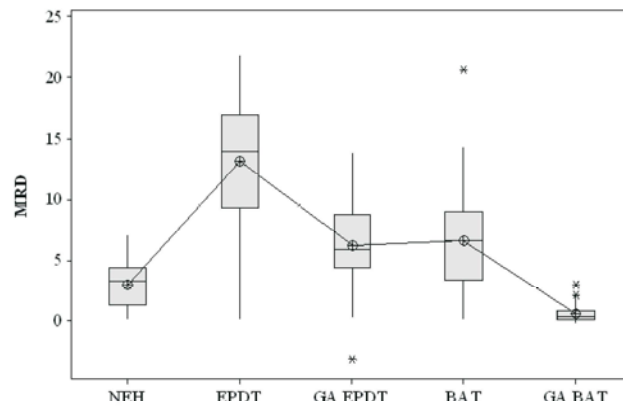


Fig. 3: Boxplot of NEH heuristic, EPDT heuristic, GA applied EPDT, BAT heuristic and GA applied BAT heuristic

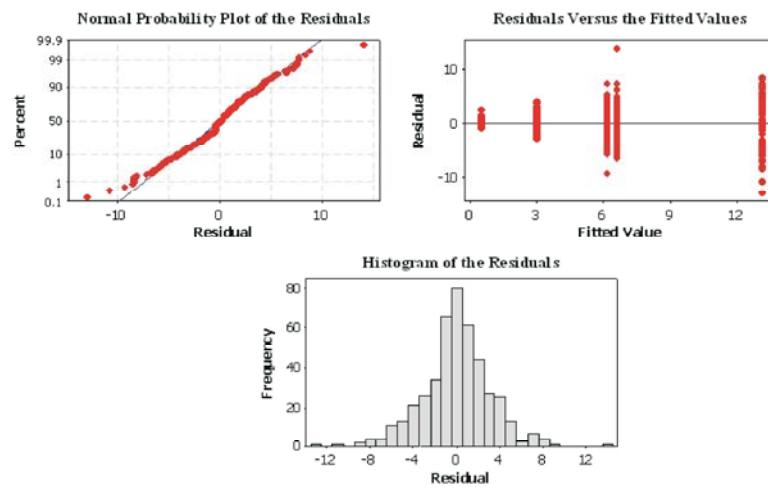


Fig. 4: Residual plots of CDS, NEH, BAT and GA applied BAT heuristics

Table 11: Comparison of heuristics based on the overall MRD

NEH	EPDT	GA EPDT	BAT	GA BAT
2.955207	13.10653	6.142446	6.557937	0.503686

Table 12: ANOVA analyze

Source	DF	SS	MS	F	P
Factor	4	13898732	3474683	0.98	0.419
Error	445	1579489708	3549415		
Total	449	1593388440			
S = 3.200		R-Sq = 64.04%		R-Sq(adj) = 63.72%	

Table 13: CIs for mean based on pooled standard deviation

Level	N	Mean	St. Dev.
NEH	90	2.955	1.811
EPDT	90	13.107	4.883
GA EPDT	90	6.142	3.086
BAT	90	6.558	3.77
GA BAT	90	0.504	0.597

Table 14: Hsu's MCB

Level	Lower	Center	Upper
NEH	0	2.452	3.482
EPDT	0	12.603	13.634
GA EPDT	0	5.639	6.669
BAT	0	6.054	7.085
GA BAT	-3.482	-2.452	0

The results of heuristics by benchmark problem are evaluated based on mean and Standard Deviation of 90 values with a constraint of "smaller, the best" and it is shown in Table 13. Even the GA applied BAT was better compared to others, the mean and Standard Deviation of other heuristics are also closer to the best results so the BAT and GA applied EPDT also good in the level of optimal makespan compared to NEH. And once from this Table 13, it has been proved that the GA applied BAT heuristic is better in finding minimum makespan compared to others.

The Hsu's MCB (Multiple Comparisons with the Best) based on "smaller the best" is shown in Table 14. From the Table 14, the proposed GA applied BAT is minimum at all levels compared to others and it is represented graphically in Fig. 3.

The residual plots of NEH, EPDT, GA applied EPDT, BAT and GA applied BAT heuristics was shown in Fig. 4. From the Figure 4, the GA applied BAT performs well in all three hypotheses that are (i) the range of makespan is normally distributed, (ii) the result are unique and well fitted to the upper bound and (iii) the residuals are independent. Since all three hypotheses are achievable, the GA applied BAT is concluded to be acceptable.

CONCLUSION

The newly proposed heuristics performed well in achieving the primary objective of minimizing the makespan. With the application of GA the EPDT and BAT heuristics are reduces the makespan compared to EPDT and BAT heuristics. This work was evaluated through a set of benchmark problems in MATLAB environment and compared with results of NEH. The maximum relative deviation (MRD) from the upper bound of the heuristics was examined. A statistical analysis tool called ANOVA (one way stacked) was used to evaluate the heuristics in MINITAB platform. By this analysis, it is noticed that the BAT, GA applied EPDT and GA applied BAT are lies equally in residual plot which are closer and better compared to NEH. Among these approaches, the GA applied BAT gained a p-value of 0.419 which is lesser than f-value and it satisfy all three hypotheses; so it is considered to be acceptable. The GA applied BAT yields about 0.5 MRD from the upper bound so it is superior in finding the minimal makespan than others heuristics.

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