American-Eurasian Journal of Scientific Research 6 (1): 01-05, 2011
ISSN 1818-6785

Graph Colouring, an Approach to Nurses Scheduling, Case Study: Ejura District Hospital, Ashanti Region, Ghana

S.K. Amponsah, E. Agyeman and K.G. Okrah

Department of Mathematics, KNUST, Kumasi, Ghana

Abstract: The ever-increasing rate of infant and maternal mortality in Ghana has made the optimal assignment of nurses to shifts, a problem of primary importance. In this paper we shall investigate the various soft and hard constraints of nurses at the maternity ward of Ejura District Hospital of Ashanti Region of Ghana. The investigation will lead to the creation of conflict graphs for the nurses. We shall then apply the hard constraints to schedule the nurses using Graph Colouring.

Key words: Graph Colouring · Conflict Graphs · Nurse Scheduling · Greedy Algorithm

INTRODUCTION

Graph colouring originally rose from the well-known four-colour theorem, which was posed as a conjecture in the 1850s [1, 2]. The four-colour theorem asks the question; whether the regions of any map could be coloured with four colours so that suburbs with a common border have different colours. Many incorrect proofs of the four-colour theorem were published often with hard-to-find errors. However, it was finally proved by the Appel and Haken [3].

Graph colouring has been studied extensively for the past decades and there are several interesting practical and feasible problems that can be modeled using graph colouring. The surge in recent times has resulted in countless real-life problem applications, which include; time tabling scheduling problems, frequency assignment, register allocation, printed circuit board testing I and II (colours and clique), analysis of biological and archaeological data and pattern matching etc.

Related Works: The nurses' rostering problem has been studied by many researchers [4-6]. With all these, graph colouring was not used as a tool. Likewise graph colouring and its applications have gone under a series of investigations [7-11]. This paper blends graph colouring and nurses' scheduling problem.

Statement of the Problem: In this paper, we shall consider the scheduling problem for the maternity ward at Ejura District Hospital of the Ashanti Region of Ghana. The staff of the ward are stratified based on their qualifications, experience and job descriptions, which includes principal midwifery officers (PMO), senior midwifery officers (SMO), midwifery officers (MO), senior staff midwives (SSM), staff midwives (SM), part-time midwives (PTM), student midwives (SM) and midwifery aids (MA).

There are three basic shifts in a day, the Morning (M), the Afternoon (A) and the Night shifts (N). Nurses are also entitled to different types of off-duties, namely day-off (D/O), Night-off (N/O) and public holiday-off (H/O). Nurses are assigned to duties periodically taking into account a number of constraints and requirements.

We shall consider hard constraints, which are the constraints that must be satisfied in order to have a feasible schedule due to physical resource restrictions and legislation. When requirements are desirable but not obligatory they are referred to as soft constraints. Soft constraints are often used to evaluate the quality of feasible schedules. In generating the roster, we ensure that every planning decision made is coherent with the following hard rules.

- Each nurse is required to work a shift per day.
- Each nurse gets at least one D/O shift per week.
- Every nurse is entitled to three (3) days off after a night shift.
- Every night shift is taken continuously for four consecutive days.
- The minimum number of nurses for morning shift should not be less than three.

Corresponding Author: S.K. Amponsah, Department of Mathematics, KNUST, Kumasi, Ghana

- The number of nurses for both afternoon and night shifts should be at least two.
- Only the Principal Midwifery Officer (PMO) is entitled to holiday off duty.
- Principal midwifery officer is scheduled for only morning shifts and has day off duties on both Saturdays and Sundays.

The Soft Rules May Be Violated:

- Nurses should take turn to be assigned on N shift. To be exact, if there are k nurses of a particular rank to be scheduled for N shift, each nurse in that group should have N shift every k days.
- Immediately after an N shift, a nurse prefers to take on D/O shift. If it is not possible, a nurse prefers to take an A shift. If it is possible, a nurse accepts a M shift.
- Nurses want to have consecutive holidays.
- Nurses should have equal opportunities of having D/O shifts.

The Scheduling Model: The model involves creating a conflict graph from the assembled data from the maternity ward of Ejura District Hospital. We colour the conflict graph and transform it into a conflict free table of nurses. From these conflicts free nurses table, we then assign the nurses to different shifts based on the hard constraints of the ward.

We assume in our scheduling model that a set of nurses that are available to be scheduled are to be assigned appropriately shift slots, subject to a number of essential and preferential scheduling constraints.

Each nurse to be scheduled constitutes a data entry, containing the required or optional information. Such information is expected to be supplied by either the nurse, the matron of the ward or the hospital administrator.

The nurses table contains data of nurses available at the ward to be scheduled and their which serves as a primary key for the nurse table, year-experience field, contains data of the number of years a nurse has served in the ward, nurse-name field has the names of the nurses, nurse-rank field has the rank of each nurse to be scheduled and no-of-nurses for the total number of nurses available in the ward to be scheduled.

The shift table contains data on different shifts that are available and has fields, morning shift, which contains nurses scheduled for morning shift, afternoon-shift for the afternoon and night-shift field for nurses for the night shift.

The duty-off field include day-off, which contains nurses on a day off duty, holiday-off has nurses who do not go to work on public holidays and night-off has nurses who received a number of rest days after a four continuous night shifts.

Figure 1 depicts the diagrammatic representation of all the tables.

Nurses Conflicts Graph: In a conflict graph, the vertices represent the items of interest and in our case the vertices represent nurses and an edge connecting each pair representing conflicting nurses based on their soft conditions.

Suppose a set of n nurses $\{n_1,n_2,...,n_m$ are to be scheduled. Each nurse n_j will be represented by exactly one vertex v_j in G and the set of vertices

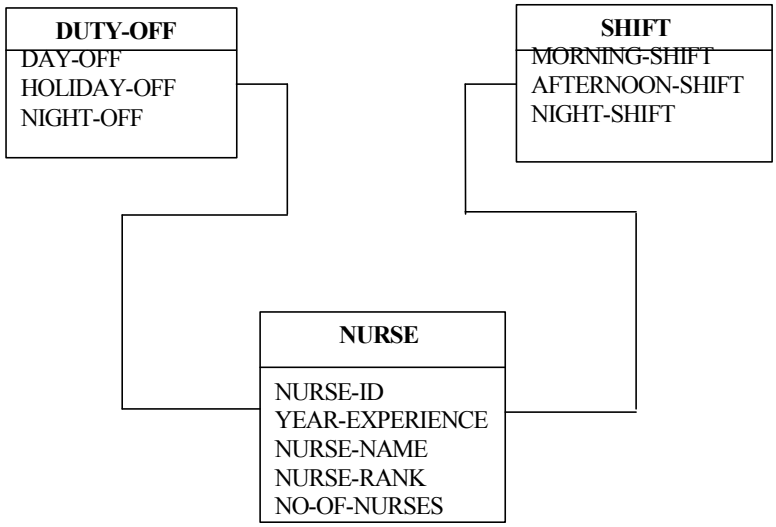

Fig. 1: Representation of all the tables

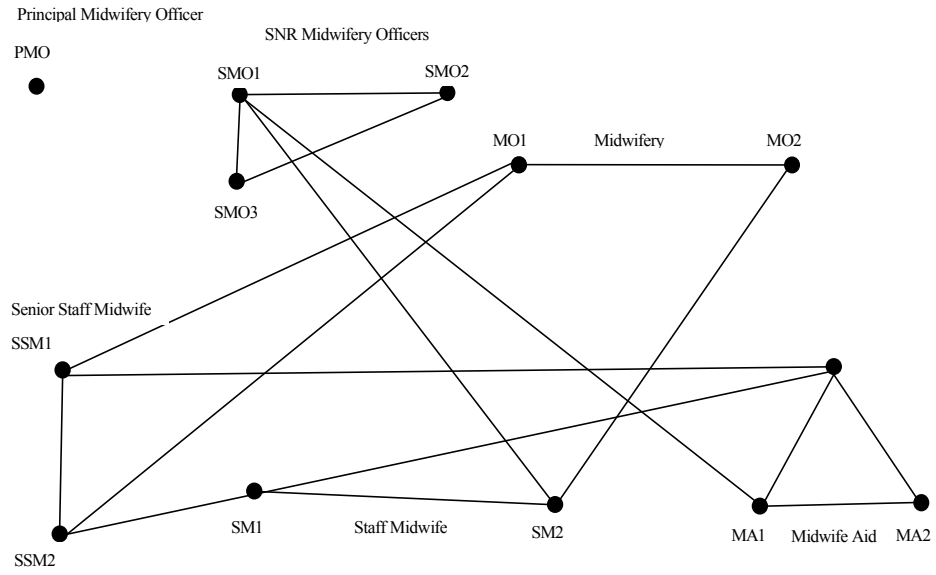

Fig. 2: Conflict graph of nurses in the maternity ward of Ejura District Hospital

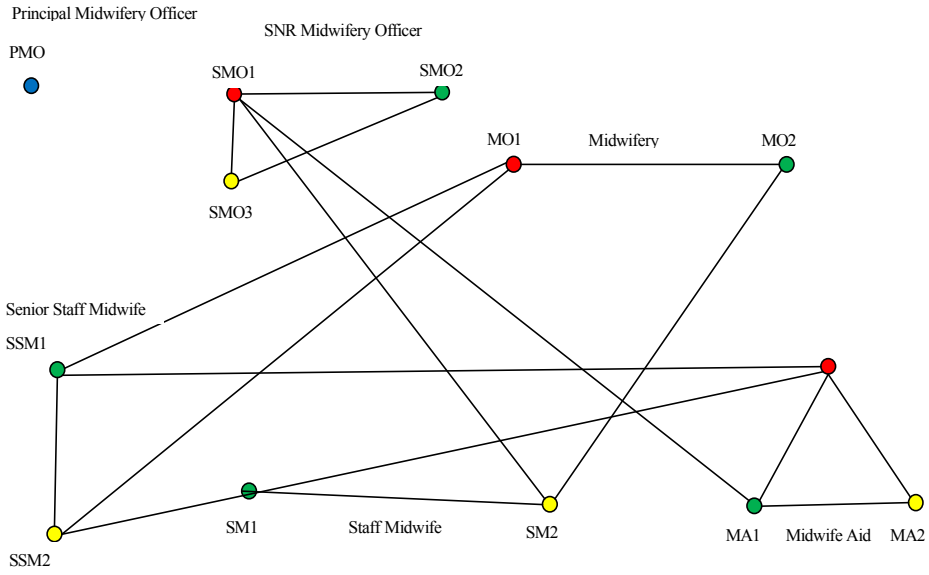

Fig. 3: Conflict graph

$V(G) = \{v_1, v_2, ..., v_n\}$. Each of the n vertices belongs to a particular group. We begin by grouping the gathered data by the ranks of the nurses. Edges can be added to the conflict graph G in the following manner:

If the shift or days off of a nurse n_j and the shift or days off of another nurse n_j are the same, then we must add an edge between vertex v_i and vertex v_j since nurse n_i and n_j cannot be scheduled for the same shift slot.

Figure 2 depicts the conflict graph of nurses in the said maternity ward of Ejura District Hospital.

Colouring the Conflict Graph: We proceed to colour the vertices of the graph once the conflict graph has been carefully and properly constructed to satisfy the soft constraints. In a proper vertex colouring of a graph G, a pair of vertices v_i and v_j are coloured with different colours if they are adjacent to each other. Vertices that do not share an edge may be coloured with different colours, or they may be coloured the same colour. Figure 3 depicts the diagrammatic representation of the coloured conflict graph for nurses at the maternity ward under consideration.

Table 1: Shift Table

	Colours			
Nurses	Red	Green	Yellow	Blue
SMO	SMO1	SMO2	SMO3	-
MO	MO1	MO2	-	-
SSM	-	SSM1	SSM2	-
SM	-	SM1	SM2	-
MA	MA1	MA3	MA2	-
PMO	-	-	-	PMO

Table 2: Schedule Table

		Week Days						
Colours	Nurses	Mo	Tu	We	Th	Fr	Sa	Su
Blue	PMO	H/O	M	M	M	M	D/O	D/O
Red	SMO1	A	A	A	A	A	A	A
	MO1	A	A	A	A	A	A	A
	MA1	A	A	A	A	A	A	A
Green	SMO2	M	M	M	M	M	M	M
	MO2	M	M	M	M	M	M	M
	SSM1	M	M	M	M	M	M	M
	SM1	M	M	M	M	M	M	M
	MA2	M	M	M	M	M	M	M
Yellow	SMO3	N	N	N	N	N/O	N/O	N/O
	SSM2	N	N	N	N	N/O	N/O	N/O
	SM2	N	N	N	N	N/O	N/O	N/O
	MA3	N	N	N	N	N/O	N/O	N/O

Transforming the Colouring to Shift Table: A technique has been devised to transform the coloured conflict graph into a near-perfect shift table. Table 1 depicts nurses under the same colours. Nurses that fall under the same colour are scheduled for the same shift slot.

For example, with the red colour, nurses SMO1, MO1 and MA1 can be scheduled for the same shift slot without conflict.

It can be seen that the principal midwifery officer does not conflict with any other nurse, simply because she is the matron of the ward and is entitled to only morning shifts.

Our allocation follows a format, which is going to be adhered to throughout.

We shall denote the days of the week i.e. Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday by Mo, Tu, We, Th, Fr, Sa, Su respectively.

The shift table can be constructed by representing the columns with days of the week and rows with the nurses. To start with, we assign all nurses under the Red colour on Afternoon shift, Green colour on Morning shift and Yellow colour on Night shift.

The Proposed Shift Table: The first and second weeks shift table for the maternity ward under consideration is constructed after applying all the hard constraints. Table 1.2 is the shift table in terms of the ranks of the nurses.

CONCLUSION

The aim of this research paper is to satisfy the various applications of graph colouring and to come out with a model in solving the schedule problem for nurses using graph colouring techniques. The schedule officer may very well wish to use our proposed model multiple times, incorporating different sets of preferential conditions each time, until finally arriving at a shift table that is ultimately most suitable.

Table 2: The Proposed Shift Table

	Week Days													
Nurses	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su
PMO	H/O	M	M	M	M	D/O	D/O	M	M	M	M	M	D/O	D/O
SMO1	A	A	D/O	D/O	N	N	N	N	N/O	N/O	N/O	M	M	M
SMO2	D/O	M	M	A	A	M	M	D/O	N	N	N	N	N/O	N/O
SMO3	N	N	N	N	N/O	N/O	N/O	D/O	M	M	M	D/O	A	A
MO1	D/O	A	A	D/O	N	N	N	N	N/O	N/O	N/O	M	M	M
MO2	M	D/O	D/O	M	M	M	M	A	A	A	A	D/O	N	N
SSM1	D/O	M	D/O	M	M	M	M	A	A	A	A	D/O	N	N
SSM2	N	N	N	N	N/O	N/O	N/O	M	D/O	M	M	A	D/O	A
SM1	M	D/O	M	A	A	A	A	D/O	N	N	N	N	N/O	N/O
SM2	N	N	N	N	N/O	N/O	N/O	M	M	D/O	M	A	A	D/O
MA1	A	D/O	A	D/O	N	N	N	N	N/O	N/O	N/O	M	M	M
MA2	M	M	D/O	A	A	A	A	D/O	N	N	N	N	N/O	N/O
MA3	N	N	N	N	N/O	N/O	N/O	M	M	M	D/O	A	A	D/O

We can finally conclude that the scheduling-by-graph-colouring model incorporating such specific graph colouring methods and our proposed technique will ultimately produce more satisfactory nurse's shifts table for the wards any hospital.

ACKNOWLEDGEMENT

The authors are grateful to Prof I.A Adetunde, the Dean of Faculty of Engineering, University of Mines and Technology, Tarkwa, Ghana for his valuable advice, suggestion and making this paper publishable.

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